Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.4-Improper/50-1.2.4.2-d-x-^m-a-x^q+b-x^n+c-x^-2-n-q-^p

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September 28, 2022 Compiled on September 28, 2022 at 2:15am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [140]. This is test number [50].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (140)	0.00 (0)
Mathematica	99.29 (139)	0.71 (1)
Maple	97.14 (136)	2.86 (4)
Fricas	92.86 (130)	7.14 (10)
Giac	77.86 (109)	22.14 (31)
Mupad	51.43 (72)	48.57 (68)
Sympy	33.57 (47)	66.43 (93)
Maxima	17.14 (24)	82.86 (116)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

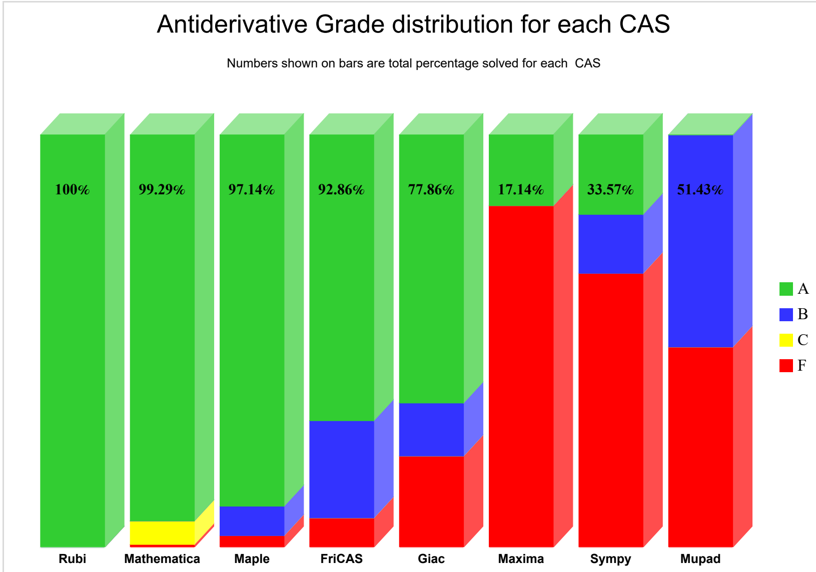
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

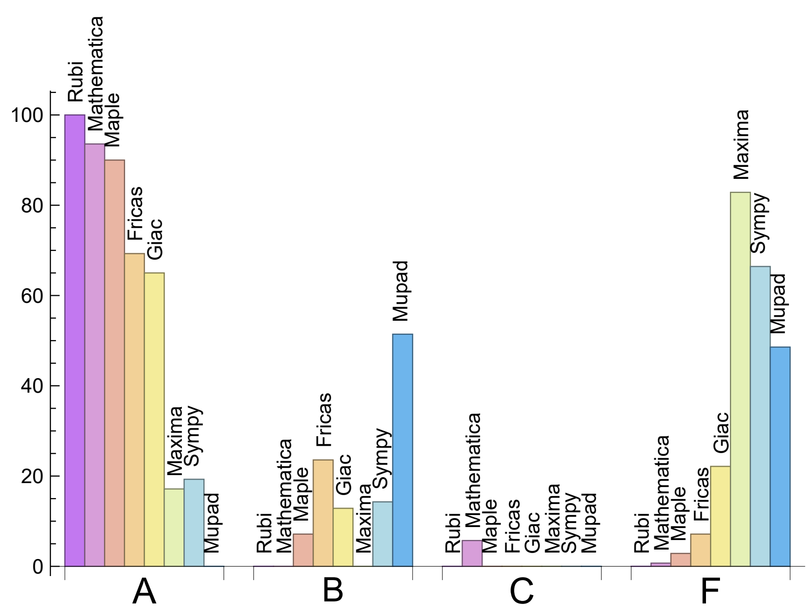
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.57	0.00	5.71	0.71
Maple	90.00	7.14	0.00	2.86
Fricas	69.29	23.57	0.00	7.14
Giac	65.00	12.86	0.00	22.14
Sympy	19.29	14.29	0.00	66.43
Maxima	17.14	0.00	0.00	82.86
Mupad	N/A	51.43	0.00	48.57

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	100.00 %	0.00 %	0.00 %
Maple	4	100.00 %	0.00 %	0.00 %
Fricas	10	40.00 %	0.00 %	60.00 %
Giac	31	38.71 %	48.39 %	12.90 %
Maxima	116	82.76 %	0.00 %	17.24 %
Sympy	93	68.82 %	29.03 %	2.15 %
Mupad	68	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

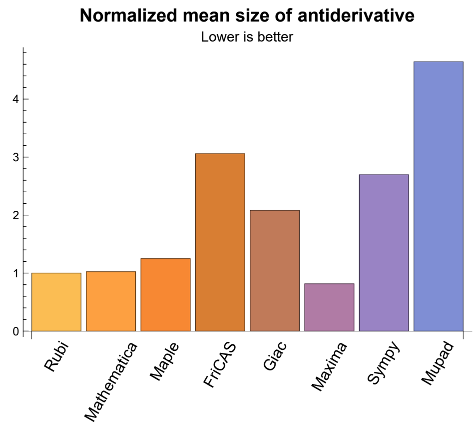
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

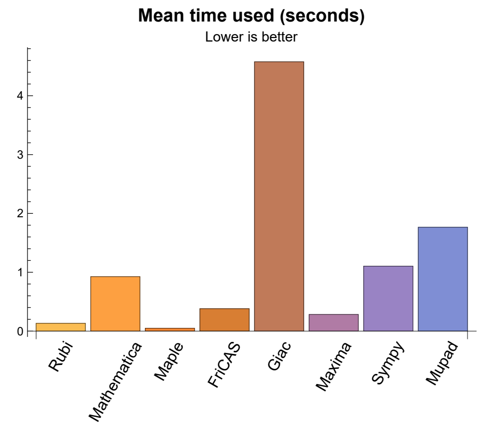
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	139.59	1.00	103.50	1.00
Mathematica	0.93	136.45	1.02	108.00	0.98
Maple	0.05	199.37	1.25	132.00	1.15
Maxima	0.28	32.71	0.81	28.50	0.81
Fricas	0.38	468.12	3.06	270.50	2.72
Sympy	1.10	199.02	2.69	124.00	0.94
Giac	4.58	360.27	2.08	79.00	1.10
Mupad	1.77	716.32	4.64	172.00	2.35

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {104}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

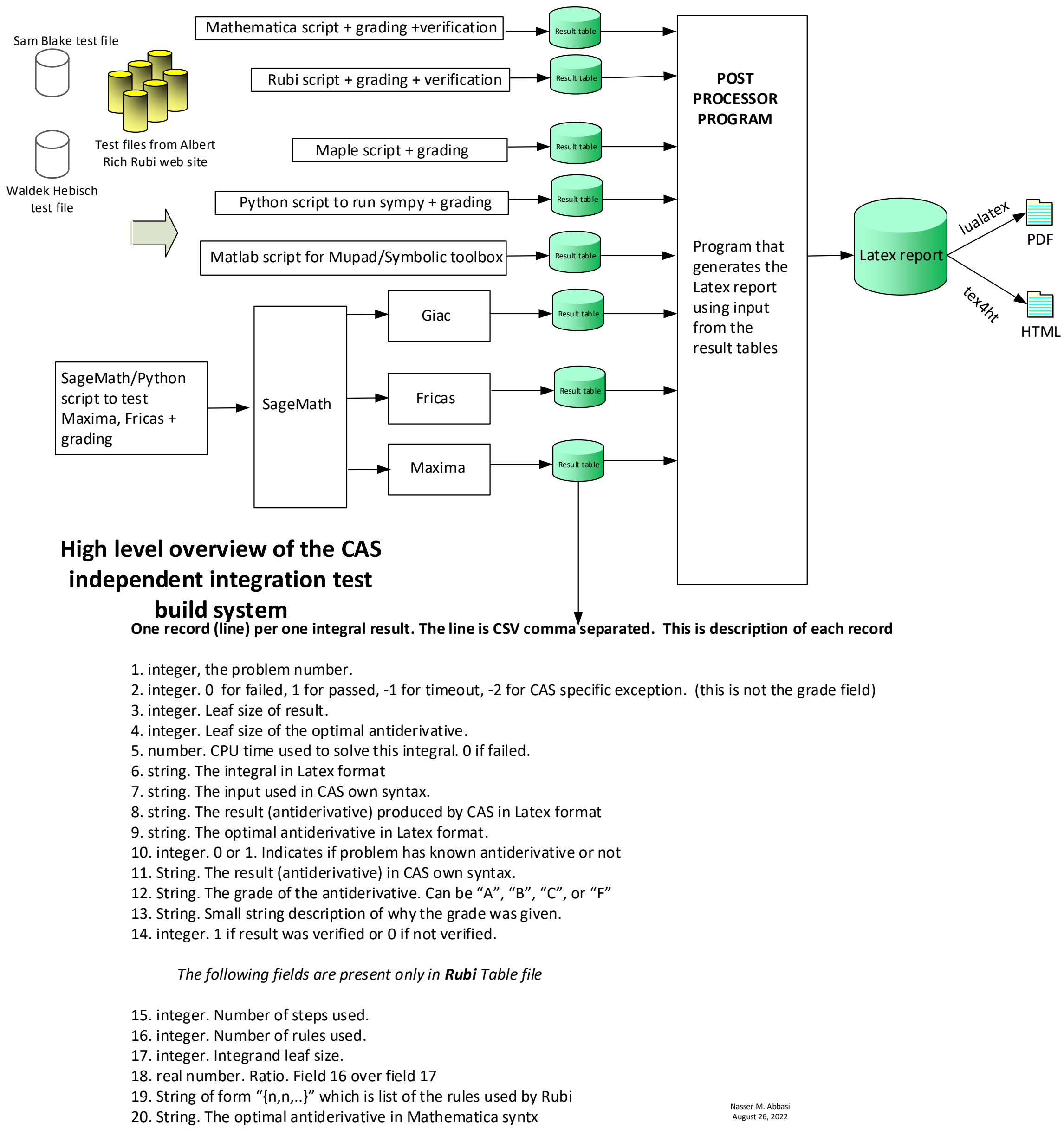
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 108, 109, 111, 113, 115, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { }

C grade: { 105, 107, 110, 112, 114, 116, 117, 119 }

F grade: { 140 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 35, 37, 47, 48, 72, 105, 110, 112, 118, 119 }

C grade: { }

F grade: { 104, 121, 122, 140 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 137 }

B grade: { }

C grade: { }

F grade: { 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 106, 108, 109, 111, 113, 114, 115, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 60, 72, 79, 81, 83, 85, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 118 }

C grade: { }

F grade: { 104, 105, 107, 110, 112, 116, 117, 119, 122, 140 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 79, 81, 83, 85, 87, 94 }

B grade: { 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 65, 72, 78, 80, 82, 84, 86, 93, 95, 97 }

C grade: { }

F grade: { 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 88, 89, 90, 91, 92, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 49, 50, 51, 52, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 80, 82, 84, 86, 88, 89, 91, 93, 95, 97, 99, 101, 103, 106, 113, 115, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade: { 60, 65, 72, 79, 81, 83, 85, 87, 90, 92, 94, 96, 98, 100, 102, 109, 111, 118 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 43, 44, 45, 46, 47, 48, 53, 54, 61, 62, 63, 64, 104, 105, 107, 108, 110, 112, 114, 116, 117, 119, 120, 121, 122, 140 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 58, 59, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 129, 133, 137 }

C grade: { }

F grade: { 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 139, 140 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.030	0.286	0.319	0.013	3.584	0.032

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.001	0.027	0.280	0.338	0.017	2.938	0.032

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.002	0.000	0.025	0.290	0.316	0.017	3.557	0.032

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.004	0.001	0.037	0.291	0.319	0.012	3.210	0.031

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.004	0.001	0.009	0.283	0.316	0.006	3.313	0.025

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.040	0.006	0.049	0.282	0.328	0.011	2.837	0.034

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.020	0.005	0.046	0.272	0.322	0.011	2.986	0.024

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.89	0.85	0.83
time (sec)	N/A	0.017	0.006	0.041	0.276	0.317	0.010	3.511	0.022

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	49	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.91	0.85	0.83
time (sec)	N/A	0.021	0.006	0.055	0.277	0.338	0.011	3.935	0.025

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	48	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.89	0.85	0.83
time (sec)	N/A	0.021	0.006	0.049	0.281	0.343	0.011	4.010	0.024

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	84	98	0	297	381	86	112
N.S.	1	1.00	0.94	1.10	0.00	3.34	4.28	0.97	1.26
time (sec)	N/A	0.063	0.078	0.032	0.000	0.338	0.456	4.120	0.141

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	73	75	0	235	306	67	172
N.S.	1	1.00	1.04	1.07	0.00	3.36	4.37	0.96	2.46
time (sec)	N/A	0.042	0.042	0.033	0.000	0.355	0.351	4.954	2.033

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	56	0	185	216	55	112
N.S.	1	1.00	1.02	1.00	0.00	3.30	3.86	0.98	2.00
time (sec)	N/A	0.028	0.021	0.020	0.000	0.327	0.167	4.366	0.133

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	120	124	34	46
N.S.	1	1.00	1.12	1.03	0.00	3.53	3.65	1.00	1.35
time (sec)	N/A	0.019	0.005	0.015	0.000	0.352	0.097	4.116	0.035

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	61	61	0	211	564	62	213
N.S.	1	1.00	0.98	0.98	0.00	3.40	9.10	1.00	3.44
time (sec)	N/A	0.034	0.044	0.023	0.000	0.344	4.580	4.404	2.298

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	81	0	269	0	79	339
N.S.	1	1.00	0.95	1.00	0.00	3.32	0.00	0.98	4.19
time (sec)	N/A	0.071	0.055	0.039	0.000	0.354	0.000	3.940	2.504

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	102	128	0	358	0	105	447
N.S.	1	1.00	0.98	1.23	0.00	3.44	0.00	1.01	4.30
time (sec)	N/A	0.101	0.099	0.040	0.000	0.376	0.000	2.865	0.587

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	157	0	445	0	136	524
N.S.	1	1.00	0.96	1.15	0.00	3.25	0.00	0.99	3.82
time (sec)	N/A	0.138	0.075	0.043	0.000	0.411	0.000	3.349	2.596

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	132	198	0	837	842	161	261
N.S.	1	1.00	0.88	1.32	0.00	5.58	5.61	1.07	1.74
time (sec)	N/A	0.114	0.130	0.064	0.000	0.363	1.063	3.231	2.456

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	109	169	0	635	729	125	279
N.S.	1	1.00	0.96	1.48	0.00	5.57	6.39	1.10	2.45
time (sec)	N/A	0.070	0.100	0.047	0.000	0.349	0.765	3.574	2.491

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	97	0	387	280	88	135
N.S.	1	1.00	1.21	1.45	0.00	5.78	4.18	1.31	2.01
time (sec)	N/A	0.032	0.062	0.029	0.000	0.347	0.321	3.689	2.129

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	69	70	0	338	253	76	110
N.S.	1	1.00	1.05	1.06	0.00	5.12	3.83	1.15	1.67
time (sec)	N/A	0.026	0.043	0.030	0.000	0.362	0.336	4.780	2.183

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	68	0	341	265	76	119
N.S.	1	1.00	1.06	1.03	0.00	5.17	4.02	1.15	1.80
time (sec)	N/A	0.023	0.050	0.028	0.000	0.356	0.324	3.213	0.084

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	177	0	781	0	126	620
N.S.	1	1.00	0.99	1.64	0.00	7.23	0.00	1.17	5.74
time (sec)	N/A	0.100	0.127	0.053	0.000	0.398	0.000	3.703	2.871

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	131	205	0	975	0	171	775
N.S.	1	1.00	0.89	1.39	0.00	6.59	0.00	1.16	5.24
time (sec)	N/A	0.131	0.180	0.056	0.000	0.455	0.000	3.592	2.833

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	175	255	0	1226	0	229	914
N.S.	1	1.00	0.87	1.26	0.00	6.07	0.00	1.13	4.52
time (sec)	N/A	0.176	0.235	0.058	0.000	0.520	0.000	4.062	2.956

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	218	295	0	1407	0	282	1120
N.S.	1	1.00	0.87	1.17	0.00	5.58	0.00	1.12	4.44
time (sec)	N/A	0.220	0.210	0.064	0.000	0.620	0.000	5.164	3.064

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	272	359	0	1640	0	347	1260
N.S.	1	1.00	0.86	1.13	0.00	5.16	0.00	1.09	3.96
time (sec)	N/A	0.268	0.248	0.065	0.000	0.791	0.000	3.455	3.143

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	184	310	0	390	0	283	-1
N.S.	1	1.00	0.72	1.21	0.00	1.52	0.00	1.10	-0.00
time (sec)	N/A	0.386	0.409	0.044	0.000	0.361	0.000	3.741	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	150	265	0	326	0	230	-1
N.S.	1	1.00	0.73	1.29	0.00	1.59	0.00	1.12	-0.00
time (sec)	N/A	0.236	0.295	0.035	0.000	0.347	0.000	4.995	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	121	167	0	260	0	166	-1
N.S.	1	1.00	0.74	1.02	0.00	1.60	0.00	1.02	-0.01
time (sec)	N/A	0.039	0.256	0.032	0.000	0.333	0.000	5.164	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	101	146	0	220	0	125	-1
N.S.	1	1.00	0.85	1.23	0.00	1.85	0.00	1.05	-0.01
time (sec)	N/A	0.050	0.191	0.029	0.000	0.357	0.000	6.290	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	133	126	0	638	0	0	-1
N.S.	1	1.00	0.77	0.73	0.00	3.69	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.187	0.027	0.000	0.363	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	125	174	0	653	0	0	-1
N.S.	1	1.00	0.72	1.01	0.00	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.197	0.034	0.000	0.363	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	111	207	0	226	0	0	-1
N.S.	1	1.00	0.97	1.82	0.00	1.98	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.259	0.054	0.000	0.361	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	130	234	0	272	0	0	-1
N.S.	1	1.00	0.84	1.51	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.449	0.045	0.000	0.350	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	159	387	0	336	0	0	-1
N.S.	1	1.00	0.78	1.89	0.00	1.64	0.00	0.00	-0.00
time (sec)	N/A	0.249	0.580	0.044	0.000	0.367	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	304	649	0	664	0	521	-1
N.S.	1	1.00	0.72	1.54	0.00	1.57	0.00	1.23	-0.00
time (sec)	N/A	0.750	0.904	0.045	0.000	0.374	0.000	5.399	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	239	479	0	558	0	429	-1
N.S.	1	1.00	0.66	1.32	0.00	1.53	0.00	1.18	-0.00
time (sec)	N/A	0.665	0.698	0.039	0.000	0.375	0.000	4.666	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	211	431	0	474	0	365	-1
N.S.	1	1.00	0.73	1.50	0.00	1.65	0.00	1.27	-0.00
time (sec)	N/A	0.344	0.564	0.037	0.000	0.374	0.000	4.967	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	161	289	0	384	0	284	-1
N.S.	1	1.00	0.81	1.46	0.00	1.94	0.00	1.43	-0.01
time (sec)	N/A	0.118	0.406	0.033	0.000	0.359	0.000	3.053	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	130	265	0	320	0	232	-1
N.S.	1	1.00	0.79	1.61	0.00	1.94	0.00	1.41	-0.01
time (sec)	N/A	0.080	0.340	0.033	0.000	0.377	0.000	3.126	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	166	222	0	791	0	0	-1
N.S.	1	1.00	0.73	0.98	0.00	3.48	0.00	0.00	-0.00
time (sec)	N/A	0.160	0.532	0.028	0.000	0.416	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	156	254	0	757	0	0	-1
N.S.	1	1.00	0.71	1.16	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.162	0.527	0.049	0.000	0.431	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	160	338	0	757	0	0	-1
N.S.	1	1.00	0.73	1.54	0.00	3.46	0.00	0.00	-0.00
time (sec)	N/A	0.159	0.554	0.042	0.000	0.395	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	172	435	0	815	0	0	-1
N.S.	1	1.00	0.67	1.69	0.00	3.17	0.00	0.00	-0.00
time (sec)	N/A	0.231	0.689	0.045	0.000	0.418	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	141	501	0	332	0	0	-1
N.S.	1	1.00	0.72	2.54	0.00	1.69	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.749	0.047	0.000	0.391	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	176	534	0	394	0	0	-1
N.S.	1	1.00	0.71	2.14	0.00	1.58	0.00	0.00	-0.00
time (sec)	N/A	0.339	1.027	0.063	0.000	0.431	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	107	144	0	226	0	140	-1
N.S.	1	1.00	0.75	1.01	0.00	1.58	0.00	0.98	-0.01
time (sec)	N/A	0.115	0.234	0.034	0.000	0.356	0.000	6.042	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	88	88	0	188	0	94	-1
N.S.	1	1.00	0.85	0.85	0.00	1.83	0.00	0.91	-0.01
time (sec)	N/A	0.052	0.030	0.028	0.000	0.382	0.000	3.651	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	67	65	0	129	0	61	-1
N.S.	1	1.00	0.94	0.92	0.00	1.82	0.00	0.86	-0.01
time (sec)	N/A	0.024	0.078	0.021	0.000	0.357	0.000	4.663	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	68	66	0	130	0	59	-1
N.S.	1	1.00	1.51	1.47	0.00	2.89	0.00	1.31	-0.02
time (sec)	N/A	0.011	0.074	0.023	0.000	0.374	0.000	4.756	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	87	88	0	194	0	0	-1
N.S.	1	1.00	1.13	1.14	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.146	0.031	0.000	0.345	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	152	0	232	0	0	-1
N.S.	1	1.00	0.93	1.28	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.247	0.036	0.000	0.386	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	185	283	0	616	0	326	-1
N.S.	1	1.00	0.71	1.08	0.00	2.35	0.00	1.24	-0.00
time (sec)	N/A	0.333	0.654	0.059	0.000	0.427	0.000	3.752	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	143	199	0	486	0	237	-1
N.S.	1	1.00	0.71	0.99	0.00	2.42	0.00	1.18	-0.00
time (sec)	N/A	0.202	0.463	0.061	0.000	0.415	0.000	5.204	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	109	166	0	414	0	170	-1
N.S.	1	1.00	0.71	1.08	0.00	2.71	0.00	1.11	-0.01
time (sec)	N/A	0.111	0.396	0.031	0.000	0.384	0.000	4.420	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	53	0	73	0	69	75
N.S.	1	1.00	0.92	1.32	0.00	1.82	0.00	1.72	1.88
time (sec)	N/A	0.025	0.244	0.030	0.000	0.350	0.000	4.243	2.122

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	52	0	72	0	74	75
N.S.	1	1.00	0.92	1.33	0.00	1.85	0.00	1.90	1.92
time (sec)	N/A	0.024	0.245	0.029	0.000	0.364	0.000	3.798	2.034

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	164	0	411	0	199	-1
N.S.	1	1.00	1.15	1.74	0.00	4.37	0.00	2.12	-0.01
time (sec)	N/A	0.045	0.428	0.031	0.000	0.403	0.000	5.283	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	134	201	0	496	0	0	-1
N.S.	1	1.00	0.93	1.40	0.00	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.515	0.052	0.000	0.410	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	180	292	0	630	0	0	-1
N.S.	1	1.00	0.86	1.40	0.00	3.01	0.00	0.00	-0.00
time (sec)	N/A	0.194	0.722	0.059	0.000	0.450	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	224	340	0	716	0	0	-1
N.S.	1	1.00	0.83	1.25	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	0.302	0.975	0.064	0.000	0.538	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	271	446	0	866	0	0	-1
N.S.	1	1.00	0.79	1.30	0.00	2.52	0.00	0.00	-0.00
time (sec)	N/A	0.404	1.417	0.070	0.000	0.603	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	34	47	37	71	280	107	89
N.S.	1	1.00	0.92	1.27	1.00	1.92	7.57	2.89	2.41
time (sec)	N/A	0.010	0.039	0.014	0.265	0.344	0.291	5.286	2.084

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.044	0.274	0.342	0.009	3.281	0.029

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.005	0.002	0.010	0.267	0.346	0.008	3.060	0.031

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	19	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.76	0.76	0.76
time (sec)	N/A	0.003	0.000	0.025	0.265	0.332	0.007	3.661	0.028

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	15	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.004	0.001	0.010	0.269	0.338	0.006	3.391	0.027

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	17	20	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	0.81	0.95	0.81
time (sec)	N/A	0.006	0.001	0.026	0.261	0.321	0.023	4.191	0.025

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	20	12	16	16
N.S.	1	1.00	1.00	0.94	0.89	1.11	0.67	0.89	0.89
time (sec)	N/A	0.005	0.001	0.012	0.268	0.351	0.023	3.568	0.030

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	300	85	241	1377	399	271
N.S.	1	1.00	0.92	3.95	1.12	3.17	18.12	5.25	3.57
time (sec)	N/A	0.037	0.118	0.014	0.264	0.349	0.702	3.853	2.195

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	44	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.81	0.81	0.94	0.85	0.83
time (sec)	N/A	0.024	0.005	0.069	0.273	0.323	0.011	3.115	0.034

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.034	0.006	0.052	0.261	0.352	0.015	3.541	0.024

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	48	44	51	46	45
N.S.	1	1.00	1.00	0.83	0.89	0.81	0.94	0.85	0.83
time (sec)	N/A	0.018	0.005	0.031	0.272	0.326	0.011	3.442	0.024

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	44	44	46	46	45
N.S.	1	1.00	0.89	0.83	0.81	0.81	0.85	0.85	0.83
time (sec)	N/A	0.030	0.006	0.046	0.277	0.348	0.011	3.558	0.024

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	41	41	48	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.84	0.98	0.88	0.86
time (sec)	N/A	0.019	0.004	0.011	0.283	0.323	0.015	3.957	0.023

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	105	0	313	391	92	842
N.S.	1	1.00	0.93	1.05	0.00	3.13	3.91	0.92	8.42
time (sec)	N/A	0.085	0.059	0.046	0.000	0.351	1.977	3.693	2.203

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	217	0	1564	194	2457	2500
N.S.	1	1.00	1.23	1.07	0.00	7.70	0.96	12.10	12.32
time (sec)	N/A	0.385	0.099	0.043	0.000	0.407	12.597	5.127	2.718

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	83	0	254	316	75	655
N.S.	1	1.00	0.96	1.02	0.00	3.14	3.90	0.93	8.09
time (sec)	N/A	0.062	0.031	0.033	0.000	0.356	1.307	4.295	2.438

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	169	0	1059	129	2109	3026
N.S.	1	1.00	1.13	0.94	0.00	5.92	0.72	11.78	16.91
time (sec)	N/A	0.161	0.073	0.036	0.000	0.366	1.446	5.105	2.584

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	197	223	59	118
N.S.	1	1.00	0.98	0.95	0.00	3.13	3.54	0.94	1.87
time (sec)	N/A	0.046	0.017	0.020	0.000	0.350	0.711	4.495	0.166

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	149	0	559	75	503	416
N.S.	1	1.00	1.10	0.99	0.00	3.73	0.50	3.35	2.77
time (sec)	N/A	0.070	0.058	0.031	0.000	0.344	0.769	4.187	2.209

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	129	131	35	41
N.S.	1	1.00	1.08	1.00	0.00	3.58	3.64	0.97	1.14
time (sec)	N/A	0.029	0.006	0.016	0.000	0.357	0.275	3.865	2.040

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	117	0	613	87	1026	763
N.S.	1	1.00	0.86	0.78	0.00	4.09	0.58	6.84	5.09
time (sec)	N/A	0.061	0.051	0.027	0.000	0.350	0.677	3.763	2.479

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	65	0	223	253	68	1014
N.S.	1	1.00	1.64	0.94	0.00	3.23	3.67	0.99	14.70
time (sec)	N/A	0.049	0.044	0.028	0.000	0.370	9.075	2.969	2.701

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	159	0	1116	148	1839	2997
N.S.	1	1.00	1.10	0.91	0.00	6.41	0.85	10.57	17.22
time (sec)	N/A	0.151	0.263	0.040	0.000	0.362	1.988	4.904	2.859

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	85	0	293	0	94	2033
N.S.	1	1.00	1.52	0.96	0.00	3.29	0.00	1.06	22.84
time (sec)	N/A	0.092	0.083	0.033	0.000	0.361	0.000	3.520	3.910

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	151	209	0	868	0	161	1473
N.S.	1	1.00	0.91	1.26	0.00	5.23	0.00	0.97	8.87
time (sec)	N/A	0.159	0.130	0.062	0.000	0.357	0.000	5.883	0.528

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	319	0	2856	0	3339	2500
N.S.	1	1.00	0.99	0.96	0.00	8.63	0.00	10.09	7.55
time (sec)	N/A	0.496	0.432	0.068	0.000	0.562	0.000	4.613	3.756

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	179	0	663	0	152	1336
N.S.	1	1.00	0.92	1.36	0.00	5.02	0.00	1.15	10.12
time (sec)	N/A	0.105	0.118	0.068	0.000	0.365	0.000	4.449	2.943

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	279	0	2257	0	2736	2500
N.S.	1	1.00	1.04	1.03	0.00	8.33	0.00	10.10	9.23
time (sec)	N/A	0.350	0.334	0.048	0.000	0.434	0.000	6.750	3.858

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	407	282	96	187
N.S.	1	1.00	1.19	1.33	0.00	5.22	3.62	1.23	2.40
time (sec)	N/A	0.047	0.061	0.034	0.000	0.341	0.866	7.800	2.197

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	230	0	1668	296	2132	2500
N.S.	1	1.00	0.99	0.97	0.00	7.04	1.25	9.00	10.55
time (sec)	N/A	0.253	0.270	0.044	0.000	0.377	8.882	9.377	3.635

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	360	269	82	178
N.S.	1	1.00	1.05	1.03	0.00	4.80	3.59	1.09	2.37
time (sec)	N/A	0.048	0.046	0.033	0.000	0.350	0.829	6.136	0.144

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	271	0	1680	0	1970	2500
N.S.	1	1.00	1.00	1.23	0.00	7.60	0.00	8.91	11.31
time (sec)	N/A	0.168	0.285	0.077	0.000	0.386	0.000	7.959	3.363

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	361	267	82	172
N.S.	1	1.00	1.07	1.01	0.00	4.88	3.61	1.11	2.32
time (sec)	N/A	0.047	0.055	0.036	0.000	0.368	0.792	7.906	2.160

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	320	0	2309	0	2682	2500
N.S.	1	1.00	0.96	1.27	0.00	9.16	0.00	10.64	9.92
time (sec)	N/A	0.321	0.281	0.072	0.000	0.432	0.000	13.392	3.847

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	185	0	813	0	166	2500
N.S.	1	1.00	1.70	1.52	0.00	6.66	0.00	1.36	20.49
time (sec)	N/A	0.128	0.222	0.050	0.000	0.437	0.000	8.448	6.312

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	294	0	2912	0	3087	2500
N.S.	1	1.00	0.98	0.95	0.00	9.45	0.00	10.02	8.12
time (sec)	N/A	0.930	0.405	0.064	0.000	0.561	0.000	7.109	2.639

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	213	0	1007	0	182	2500
N.S.	1	1.00	1.53	1.31	0.00	6.22	0.00	1.12	15.43
time (sec)	N/A	0.168	0.179	0.058	0.000	0.516	0.000	8.714	6.766

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	344	334	0	3435	0	3651	2500
N.S.	1	1.00	0.95	0.93	0.00	9.52	0.00	10.11	6.93
time (sec)	N/A	2.078	0.470	0.082	0.000	0.796	0.000	7.371	4.907

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	328	263	0	1242	0	274	2500
N.S.	1	1.00	1.50	1.20	0.00	5.67	0.00	1.25	11.42
time (sec)	N/A	0.225	0.243	0.068	0.000	0.525	0.000	7.067	7.473

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	170	0	0	0	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	10.068	0.008	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	486	1042	0	0	0	0	-1
N.S.	1	1.00	1.28	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	7.317	0.068	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	123	157	0	232	0	127	-1
N.S.	1	1.00	0.95	1.22	0.00	1.80	0.00	0.98	-0.01
time (sec)	N/A	0.060	0.180	0.036	0.000	0.348	0.000	5.272	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	452	508	0	0	0	0	-1
N.S.	1	1.00	1.30	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.137	6.604	0.036	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	154	136	0	666	0	0	-1
N.S.	1	1.00	0.79	0.70	0.00	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.196	0.027	0.000	0.393	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	181	369	0	396	0	662	-1
N.S.	1	1.00	0.74	1.51	0.00	1.62	0.00	2.71	-0.00
time (sec)	N/A	0.230	0.433	0.044	0.000	0.383	0.000	4.059	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	609	1878	0	0	0	0	-1
N.S.	1	1.00	1.25	3.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.297	11.450	0.053	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	150	295	0	332	0	518	-1
N.S.	1	1.00	0.85	1.67	0.00	1.88	0.00	2.93	-0.01
time (sec)	N/A	0.087	0.352	0.040	0.000	0.356	0.000	4.084	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	540	1394	0	0	0	0	-1
N.S.	1	1.00	1.27	3.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.272	10.420	0.038	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	72	0	135	0	60	-1
N.S.	1	1.00	0.98	0.88	0.00	1.65	0.00	0.73	-0.01
time (sec)	N/A	0.038	0.085	0.020	0.000	0.357	0.000	3.375	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	193	177	0	122	0	0	-1
N.S.	1	1.00	1.60	1.46	0.00	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.030	10.089	0.023	0.000	0.085	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	-1
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	-0.02
time (sec)	N/A	0.020	0.081	0.039	0.000	0.356	0.000	4.331	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	303	508	0	0	0	0	-1
N.S.	1	1.00	0.92	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.121	10.329	0.038	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	463	533	0	0	0	0	-1
N.S.	1	1.00	1.18	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	10.697	0.026	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	123	179	0	424	0	193	-1
N.S.	1	1.00	1.19	1.74	0.00	4.12	0.00	1.87	-0.01
time (sec)	N/A	0.049	0.432	0.038	0.000	0.414	0.000	3.700	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	468	519	1136	0	0	0	0	-1
N.S.	1	1.00	1.11	2.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	10.891	0.048	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	159	220	0	508	0	0	-1
N.S.	1	1.00	1.03	1.43	0.00	3.30	0.00	0.00	-0.01
time (sec)	N/A	0.113	0.524	0.077	0.000	0.393	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	0	0	83	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	1.63	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.049	0.016	0.000	0.412	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	239	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	15.117	0.013	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	66	58	0	55	0	60	-1
N.S.	1	1.00	1.47	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.006	0.081	0.217	0.000	0.357	0.000	5.938	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	66	58	0	55	0	60	-1
N.S.	1	1.00	1.47	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.009	0.002	0.197	0.000	0.335	0.000	6.378	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	66	58	0	55	0	60	-1
N.S.	1	1.00	1.47	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.009	0.002	0.184	0.000	0.405	0.000	5.517	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	81	0	70	0	69	-1
N.S.	1	1.00	0.93	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.028	0.071	0.126	0.000	0.348	0.000	3.723	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	81	0	70	0	69	-1
N.S.	1	1.00	0.93	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.030	0.002	0.107	0.000	0.338	0.000	3.192	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	80	81	0	70	0	69	-1
N.S.	1	1.00	0.93	0.94	0.00	0.81	0.00	0.80	-0.01
time (sec)	N/A	0.028	0.002	0.112	0.000	0.331	0.000	3.281	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	35	0	111	0	35	34
N.S.	1	1.00	0.97	0.92	0.00	2.92	0.00	0.92	0.89
time (sec)	N/A	0.011	0.012	0.240	0.000	0.368	0.000	3.710	0.080

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	68	64	0	130	0	59	-1
N.S.	1	1.00	1.51	1.42	0.00	2.89	0.00	1.31	-0.02
time (sec)	N/A	0.017	0.016	0.027	0.000	0.369	0.000	4.252	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	70	64	0	131	0	53	-1
N.S.	1	1.00	1.49	1.36	0.00	2.79	0.00	1.13	-0.02
time (sec)	N/A	0.049	0.022	0.019	0.000	0.408	0.000	4.341	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	72	66	0	139	0	58	-1
N.S.	1	1.00	1.47	1.35	0.00	2.84	0.00	1.18	-0.02
time (sec)	N/A	0.062	0.019	0.018	0.000	0.404	0.000	4.095	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	0	124	0	38	44
N.S.	1	1.00	0.93	0.89	0.00	2.82	0.00	0.86	1.00
time (sec)	N/A	0.026	0.003	0.038	0.000	0.355	0.000	4.595	2.226

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	78	72	0	135	0	62	-1
N.S.	1	1.00	1.59	1.47	0.00	2.76	0.00	1.27	-0.02
time (sec)	N/A	0.011	0.012	0.055	0.000	0.372	0.000	5.015	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	80	72	0	137	0	56	-1
N.S.	1	1.00	1.57	1.41	0.00	2.69	0.00	1.10	-0.02
time (sec)	N/A	0.044	0.017	0.023	0.000	0.335	0.000	4.032	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	82	74	0	145	0	61	-1
N.S.	1	1.00	1.55	1.40	0.00	2.74	0.00	1.15	-0.02
time (sec)	N/A	0.052	0.014	0.023	0.000	0.346	0.000	4.413	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	33	31	20	47	0	55	33
N.S.	1	1.00	0.82	0.78	0.50	1.18	0.00	1.38	0.82
time (sec)	N/A	0.018	0.003	0.194	0.460	0.345	0.000	4.896	0.427

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	66	58	0	55	0	60	-1
N.S.	1	1.00	1.47	1.29	0.00	1.22	0.00	1.33	-0.02
time (sec)	N/A	0.009	0.004	0.179	0.000	0.358	0.000	4.122	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	61	50	0	49	0	47	-1
N.S.	1	1.00	1.42	1.16	0.00	1.14	0.00	1.09	-0.02
time (sec)	N/A	0.032	0.075	0.022	0.000	0.377	0.000	4.155	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.442	0.027	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [140] had the largest ratio of [36]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	20	0.050
2	A	2	1	1.00	18	0.056
3	A	1	0	1.00	16	0.000
4	A	2	1	1.00	20	0.050
5	A	2	1	1.00	20	0.050
6	A	3	2	1.00	22	0.091
7	A	3	2	1.00	20	0.100
8	A	3	2	1.00	18	0.111
9	A	3	2	1.00	22	0.091
10	A	3	2	1.00	22	0.091
11	A	7	6	1.00	22	0.273
12	A	6	6	1.00	22	0.273
13	A	5	5	1.00	22	0.227
14	A	3	3	1.00	22	0.136
15	A	7	7	1.00	20	0.350
16	A	8	7	1.00	18	0.389
17	A	8	7	1.00	22	0.318
18	A	8	7	1.00	22	0.318
19	A	8	7	1.00	22	0.318
20	A	7	7	1.00	22	0.318
21	A	4	4	1.00	22	0.182
22	A	4	4	1.00	22	0.182
23	A	4	4	1.00	22	0.182
24	A	8	7	1.00	22	0.318
25	A	8	7	1.00	22	0.318

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	7	1.00	20	0.350
27	A	8	7	1.00	18	0.389
28	A	8	7	1.00	22	0.318
29	A	8	6	1.00	24	0.250
30	A	7	6	1.00	22	0.273
31	A	5	5	1.00	20	0.250
32	A	4	4	1.00	24	0.167
33	A	7	6	1.00	24	0.250
34	A	7	6	1.00	24	0.250
35	A	5	5	1.00	24	0.208
36	A	6	5	1.00	24	0.208
37	A	7	5	1.00	24	0.208
38	A	10	7	1.00	22	0.318
39	A	10	7	1.00	20	0.350
40	A	8	7	1.00	24	0.292
41	A	6	5	1.00	24	0.208
42	A	5	4	1.00	24	0.167
43	A	8	7	1.00	24	0.292
44	A	8	7	1.00	24	0.292
45	A	8	7	1.00	24	0.292
46	A	9	8	1.00	24	0.333
47	A	7	6	1.00	24	0.250
48	A	8	6	1.00	24	0.250
49	A	6	6	1.00	24	0.250
50	A	4	4	1.00	24	0.167
51	A	3	3	1.00	22	0.136
52	A	2	2	1.00	20	0.100
53	A	3	3	1.00	24	0.125
54	A	5	5	1.00	24	0.208
55	A	8	6	1.00	24	0.250
56	A	7	6	1.00	24	0.250
57	A	6	6	1.00	24	0.250
58	A	1	1	1.00	24	0.042
59	A	1	1	1.00	24	0.042
60	A	3	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	5	1.00	22	0.227
62	A	6	5	1.00	20	0.250
63	A	7	5	1.00	24	0.208
64	A	8	5	1.00	24	0.208
65	A	2	1	1.00	18	0.056
66	A	2	1	1.00	18	0.056
67	A	2	1	1.00	16	0.062
68	A	1	0	1.00	14	0.000
69	A	2	1	1.00	18	0.056
70	A	2	1	1.00	18	0.056
71	A	2	1	1.00	18	0.056
72	A	3	2	1.00	20	0.100
73	A	3	2	1.00	20	0.100
74	A	4	3	1.00	18	0.167
75	A	3	2	1.00	16	0.125
76	A	4	3	1.00	20	0.150
77	A	3	2	1.00	20	0.100
78	A	8	7	1.00	20	0.350
79	A	6	5	1.00	20	0.250
80	A	7	7	1.00	20	0.350
81	A	5	4	1.00	20	0.200
82	A	6	6	1.00	20	0.300
83	A	4	3	1.00	20	0.150
84	A	4	4	1.00	20	0.200
85	A	4	3	1.00	18	0.167
86	A	8	8	1.00	16	0.500
87	A	5	4	1.00	20	0.200
88	A	9	8	1.00	20	0.400
89	A	9	8	1.00	20	0.400
90	A	7	5	1.00	20	0.250
91	A	8	8	1.00	20	0.400
92	A	6	5	1.00	20	0.250
93	A	5	5	1.00	20	0.250
94	A	5	4	1.00	20	0.200
95	A	5	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	20	0.200
97	A	5	5	1.00	20	0.250
98	A	5	4	1.00	20	0.200
99	A	9	8	1.00	18	0.444
100	A	6	5	1.00	16	0.312
101	A	9	8	1.00	20	0.400
102	A	7	5	1.00	20	0.250
103	A	9	8	1.00	20	0.400
104	A	3	3	1.00	20	0.150
105	A	5	5	1.00	24	0.208
106	A	5	5	1.00	24	0.208
107	A	5	5	1.00	24	0.208
108	A	8	7	1.00	24	0.292
109	A	8	8	1.00	24	0.333
110	A	6	6	1.00	24	0.250
111	A	6	5	1.00	24	0.208
112	A	6	6	1.00	24	0.250
113	A	4	4	1.00	24	0.167
114	A	2	2	1.00	24	0.083
115	A	2	2	1.00	24	0.083
116	A	6	6	1.00	24	0.250
117	A	5	5	1.00	24	0.208
118	A	3	3	1.00	24	0.125
119	A	6	6	1.00	24	0.250
120	A	5	5	1.00	24	0.208
121	A	1	1	1.00	34	0.029
122	A	7	4	1.00	27	0.148
123	A	2	2	1.00	18	0.111
124	A	3	3	1.00	18	0.167
125	A	3	3	1.00	17	0.176
126	A	5	5	1.00	18	0.278
127	A	6	6	1.00	18	0.333
128	A	6	6	1.00	17	0.353
129	A	2	2	1.00	18	0.111
130	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	22	0.136
132	A	3	3	1.00	24	0.125
133	A	3	3	1.00	20	0.150
134	A	3	3	1.00	20	0.150
135	A	3	3	1.00	24	0.125
136	A	3	3	1.00	26	0.115
137	A	3	3	1.00	18	0.167
138	A	3	3	1.00	18	0.167
139	A	3	3	1.00	20	0.150
140	A	2	2	1.00	36	0.056

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^2(ax^2+bx^3+cx^4)\,dx$

Optimal. Leaf size=25

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.050$, Rules used = $\{14\}$

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3+cx^4\right)\,dx&=\int \left(ax^4+bx^5+cx^6\right)\,dx\\&=\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6 + (c*x^7)/7
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{x^5(30c\,x^2+35bx+42a)}{210}$	20
default	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
norman	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20
risch	$\frac{1}{5}a\,x^5+\frac{1}{6}b\,x^6+\frac{1}{7}c\,x^7$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6+1/7*c*x^7
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Fricas [A]
time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{6}\,bx^6+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^5}{5}+\frac{bx^6}{6}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6 + c*x**7/7
```

Giac [A]
time = 3.58, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7 + \frac{1}{6}\,bx^6 + \frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/6*b*x^6 + 1/5*a*x^5
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^5\,\left(30\,c\,x^2+35\,b\,x+42\,a\right)}{210}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (x^5*(42*a + 35*b*x + 30*c*x^2))/210
```

3.2 $\int x(ax^2+bx^3+cx^4)\,dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{14\}$

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)\,dx &= \int \left(ax^3+bx^4+cx^5\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (a*x^4)/4 + (b*x^5)/5 + (c*x^6)/6
```

Maple [A]
time = 0.03, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{x^4(10c\,x^2+12bx+15a)}{60}$	20
default	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
norman	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20
risch	$\frac{1}{4}a\,x^4+\frac{1}{5}b\,x^5+\frac{1}{6}c\,x^6$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5+1/6*c*x^6
```

Maxima [A]
time = 0.28, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Sympy [A]
time = 0.02, size = 19, normalized size = 0.76

$$\frac{ax^4}{4}+\frac{bx^5}{5}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5 + c*x**6/6
```

Giac [A]
time = 2.94, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6 + \frac{1}{5}\,bx^5 + \frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/5*b*x^5 + 1/4*a*x^4
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^4\,\left(10\,c\,x^2+12\,b\,x+15\,a\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (x^4*(15*a + 12*b*x + 10*c*x^2))/60
```

3.3 $\int (ax^2 + bx^3 + cx^4) \, dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Rubi steps

$$\int (ax^2 + bx^3 + cx^4) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3} + \frac{bx^4}{4} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3 + c*x^4,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4 + (c*x^5)/5
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{x^3 (12c x^2+15bx+20a)}{60}$	20
default	$\frac{1}{3}a x^3 + \frac{1}{4}b x^4 + \frac{1}{5}c x^5$	20

norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4+\frac{1}{5}c\,x^5$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^4+b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4+1/5*c*x^5
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.76

$$\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Fricas [A]
time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A]
time = 0.02, size = 19, normalized size = 0.76

$$\frac{ax^3}{3}+\frac{bx^4}{4}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**4+b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4 + c*x**5/5
```

Giac [A]
time = 3.56, size = 19, normalized size = 0.76

$$\frac{1}{5}\,cx^5+\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^4+b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^3\,\left(12\,c\,x^2+15\,b\,x+20\,a\right)}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3 + c*x^4,x)
```

```
[Out] (x^3*(20*a + 15*b*x + 12*c*x^2))/60
```

3.4 $\int \frac{ax^2+bx^3+cx^4}{x} dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{14\}$

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2+bx^3+cx^4}{x}\,dx &= \int \left(ax+bx^2+cx^3\right)\,dx\\ &= \frac{ax^2}{2}+\frac{bx^3}{3}+\frac{cx^4}{4}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^3}{3} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3 + (c*x^4)/4
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.80

method	result	size
gosper	$\frac{x^{2}\left(3 c \,x^{2}+4 b x +6 a \right)}{12}$	20
default	$\frac{1}{2} a \,x^{2}+\frac{1}{3} b \,x^{3}+\frac{1}{4} c \,x^{4}$	20
norman	$\frac{1}{2} a \,x^{2}+\frac{1}{3} b \,x^{3}+\frac{1}{4} c \,x^{4}$	20
risch	$\frac{1}{2} a \,x^{2}+\frac{1}{3} b \,x^{3}+\frac{1}{4} c \,x^{4}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3+1/4*c*x^4
```

Maxima [A]
time = 0.29, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^{4} + \frac{1}{3}\,bx^{3} + \frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Fricas [A]
time = 0.32, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^{4} + \frac{1}{3}\,bx^{3} + \frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^{2}}{2} + \frac{bx^{3}}{3} + \frac{cx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x,x)
```

```
[Out] a*x**2/2 + b*x**3/3 + c*x**4/4
```

Giac [A]
time = 3.21, size = 19, normalized size = 0.76

$$\frac{1}{4}\,cx^4+\frac{1}{3}\,bx^3+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x,x, algorithm="giac")
[Out] 1/4*c*x^4 + 1/3*b*x^3 + 1/2*a*x^2
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{x^2\,\left(3\,c\,x^2+4\,b\,x+6\,a\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x,x)
[Out] (x^2*(6*a + 4*b*x + 3*c*x^2))/12
```

3.5 $\int \frac{ax^2+bx^3+cx^4}{x^2} \, dx$

Optimal. Leaf size=20

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {14}

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3+cx^4}{x^2} \, dx &= \int \left(a+bx+cx^2\right) \, dx \\ &= ax+\frac{bx^2}{2}+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
default	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
risch	$ax+\frac{1}{2}b\,x^{2}+\frac{1}{3}c\,x^{3}$	17
gosper	$\frac{x\left(2c\,x^{2}+3bx+6a\right)}{6}$	18
norman	$\frac{a\,x^{2}+\frac{1}{2}b\,x^{3}+\frac{1}{3}c\,x^{4}}{x}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)/x^2,x,method=_RETURNVERBOSE)
[Out] a*x+1/2*b*x^2+1/3*c*x^3
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.80

$$\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="maxima")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Fricas [A]
time = 0.32, size = 16, normalized size = 0.80

$$\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="fricas")
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.75

$$ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3
```

Giac [A]
time = 3.31, size = 16, normalized size = 0.80

$$\frac{1}{3}\,cx^3 + \frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.80

$$\frac{c\,x^3}{3} + \frac{b\,x^2}{2} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3
```

3.6 $\int x^2(ax^2+bx^3+cx^4)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.091$, Rules used = {1599, 712}

$$\frac{a^2x^7}{7}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{1}{4}abx^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3+cx^4\right)^2\,dx&=\int x^6\left(a+bx+cx^2\right)^2\,dx\\&=\int\left(a^2x^6+2abx^7+\left(b^2+2ac\right)x^8+2bcx^9+c^2x^{10}\right)\,dx\\&=\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{1}{5}bcx^{10}+\frac{c^2x^{11}}{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^7}{7} + \frac{1}{4}abx^8 + \frac{1}{9}\left(b^2 + 2ac\right)x^9 + \frac{1}{5}bcx^{10} + \frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + ((b^2 + 2*a*c)*x^9)/9 + (b*c*x^10)/5 + (c^2*x^1
1)/11
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^7}{7}+\frac{ab\,x^8}{4}+\frac{(2ac+b^2)x^9}{9}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{bc\,x^{10}}{5}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{ab\,x^8}{4}+\frac{a^2x^7}{7}$	46
gosper	$\frac{x^7\left(1260c^2x^4+2772bc\,x^3+3080ac\,x^2+1540b^2x^2+3465abx+1980a^2\right)}{13860}$	47
risch	$\frac{1}{7}a^2x^7+\frac{1}{4}ab\,x^8+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{1}{5}bc\,x^{10}+\frac{1}{11}c^2x^{11}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*(2*a*c+b^2)*x^9+1/5*b*c*x^10+1/11*c^2*x^11
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{1}{11}\,c^2x^{11} + \frac{1}{5}\,bcx^{10} + \frac{1}{4}\,abx^8 + \frac{1}{9}\,\left(b^2 + 2\,ac\right)x^9 + \frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

Fricas [A]
time = 0.33, size = 44, normalized size = 0.81

$$\frac{1}{11}\,c^2x^{11} + \frac{1}{5}\,bcx^{10} + \frac{1}{4}\,abx^8 + \frac{1}{9}\,\left(b^2 + 2\,ac\right)x^9 + \frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/4*a*b*x^8 + 1/9*(b^2 + 2*a*c)*x^9 + 1/7*a^
2*x^7
```

Sympy [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{a^2x^7}{7}+\frac{abx^8}{4}+\frac{bcx^{10}}{5}+\frac{c^2x^{11}}{11}+x^9\cdot\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b*c*x**10/5 + c**2*x**11/11 + x**9*(2*a*c/9 + b*
*2/9)
```

Giac [A]
time = 2.84, size = 46, normalized size = 0.85

$$\frac{1}{11}\,c^2x^{11}+\frac{1}{5}\,bcx^{10}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{1}{4}\,abx^8+\frac{1}{7}\,a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 1/5*b*c*x^10 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 1/4*a*b*x^8 + 1/
7*a^2*x^7
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.83

$$x^9\,\left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^7}{7}+\frac{c^2\,x^{11}}{11}+\frac{a\,b\,x^8}{4}+\frac{b\,c\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^7)/7 + (c^2*x^11)/11 + (a*b*x^8)/4 + (b*c*
x^10)/5
```

3.7 $\int x(ax^2+bx^3+cx^4)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {1599, 712}

$$\frac{a^2x^6}{6}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x\left(ax^2+bx^3+cx^4\right)^2\,dx &= \int x^5\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^5+2abx^6+\left(b^2+2ac\right)x^7+2bcx^8+c^2x^9\right)dx\\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{2}{9}bcx^9+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^8)/8 + (2*b*c*x^9)/9 + (c^2*
x^10)/10
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^6}{6}+\frac{2ab\,x^7}{7}+\frac{\left(2ac+b^2\right)x^8}{8}+\frac{2bc\,x^9}{9}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{2bc\,x^9}{9}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{2ab\,x^7}{7}+\frac{a^2x^6}{6}$	46
gosper	$\frac{x^6\left(252c^2x^4+560bc\,x^3+630ac\,x^2+315b^2x^2+720abx+420a^2\right)}{2520}$	47
risch	$\frac{1}{6}a^2x^6+\frac{2}{7}ab\,x^7+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{2}{9}bc\,x^9+\frac{1}{10}c^2x^{10}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*(2*a*c+b^2)*x^8+2/9*b*c*x^9+1/10*c^2*x^10
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

Fricas [A]
time = 0.32, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{2}{7}\,abx^7+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 2/7*a*b*x^7 + 1/8*(b^2 + 2*a*c)*x^8 + 1/6*a^2
*x^6
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.91

$$\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{10}}{10}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + 2*b*c*x**9/9 + c**2*x**10/10 + x**8*(a*c/4 + b
**2/8)
```

Giac [A]
time = 2.99, size = 46, normalized size = 0.85

$$\frac{1}{10}\,c^2x^{10}+\frac{2}{9}\,bcx^9+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 2/9*b*c*x^9 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 2/7*a*b*x^7 + 1/6
*a^2*x^6
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^6}{6}+\frac{c^2\,x^{10}}{10}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^6)/6 + (c^2*x^10)/10 + (2*a*b*x^7)/7 + (2*b*
c*x^9)/9
```

3.8 $\int \left(ax^2+bx^3+cx^4\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1608, 712}

$$\frac{a^2x^5}{5}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3+cx^4\right)^2\,dx&=\int x^4\left(a+bx+cx^2\right)^2\,dx\\&=\int \left(a^2x^4+2abx^5+\left(b^2+2ac\right)x^6+2bcx^7+c^2x^8\right)\,dx\\&=\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{1}{4}bcx^8+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^7)/7 + (b*c*x^8)/4 + (c^2*x^9)
/9
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{ab\,x^6}{3}+\frac{(2ac+b^2)x^7}{7}+\frac{bc\,x^8}{4}+\frac{c^2x^9}{9}$	45
norman	$\frac{c^2x^9}{9}+\frac{bc\,x^8}{4}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{ab\,x^6}{3}+\frac{a^2x^5}{5}$	46
gosper	$\frac{x^5(140c^2x^4+315bc\,x^3+360ac\,x^2+180b^2x^2+420abx+252a^2)}{1260}$	47
risch	$\frac{1}{5}a^2x^5+\frac{1}{3}ab\,x^6+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{1}{4}bc\,x^8+\frac{1}{9}c^2x^9$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*(2*a*c+b^2)*x^7+1/4*b*c*x^8+1/9*c^2*x^9
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.89

$$\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{7}\,b^2x^7+\frac{1}{5}\,a^2x^5+\frac{1}{21}\,\left(6\,cx^7+7\,bx^6\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 1/5*a^2*x^5 + 1/21*(6*c*x^7 + 7*b
*x^6)*a
```

Fricas [A]
time = 0.32, size = 44, normalized size = 0.81

$$\frac{1}{9}\,c^2x^9+\frac{1}{4}\,bcx^8+\frac{1}{3}\,abx^6+\frac{1}{7}\,\left(b^2+2\,ac\right)x^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/3*a*b*x^6 + 1/7*(b^2 + 2*a*c)*x^7 + 1/5*a^2*x
^5
```

Sympy [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{bcx^8}{4} + \frac{c^2x^9}{9} + x^7 \cdot \left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**5/5 + a*b*x**6/3 + b*c*x**8/4 + c**2*x**9/9 + x**7*(2*a*c/7 + b**2/
7)
```

Giac [A]
time = 3.51, size = 46, normalized size = 0.85

$$\frac{1}{9}\,c^2x^9 + \frac{1}{4}\,bcx^8 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 1/4*b*c*x^8 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 1/3*a*b*x^6 + 1/5*a
^2*x^5
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^7\left(\frac{b^2}{7} + \frac{2\,a\,c}{7}\right) + \frac{a^2\,x^5}{5} + \frac{c^2\,x^9}{9} + \frac{a\,b\,x^6}{3} + \frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^5)/5 + (c^2*x^9)/9 + (a*b*x^6)/3 + (b*c*x^
8)/4
```

3.9 $\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1599, 712}

$$\frac{a^2x^4}{4}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3+cx^4\right)^2}{x}\,dx &= \int x^3\left(a+bx+cx^2\right)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+\left(b^2+2ac\right)x^5+2bcx^6+c^2x^7\right)dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{2}{7}bcx^7+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^6)/6 + (2*b*c*x^7)/7 + (c^2*
x^8)/8
```

Maple [A]
time = 0.06, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{2ab\,x^5}{5}+\frac{(2ac+b^2)x^6}{6}+\frac{2bc\,x^7}{7}+\frac{c^2x^8}{8}$	45
norman	$\frac{c^2x^8}{8}+\frac{2bc\,x^7}{7}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{2ab\,x^5}{5}+\frac{a^2x^4}{4}$	46
gosper	$\frac{x^4\left(105c^2x^4+240bc\,x^3+280ac\,x^2+140b^2x^2+336abx+210a^2\right)}{840}$	47
risch	$\frac{1}{4}a^2x^4+\frac{2}{5}ab\,x^5+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{2}{7}bc\,x^7+\frac{1}{8}c^2x^8$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*(2*a*c+b^2)*x^6+2/7*b*c*x^7+1/8*c^2*x^8
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.81

$$\frac{1}{8}\,c^2x^8+\frac{2}{7}\,bcx^7+\frac{2}{5}\,abx^5+\frac{1}{6}\,\left(b^2+2\,ac\right)x^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 2/5*a*b*x^5 + 1/6*(b^2 + 2*a*c)*x^6 + 1/4*a^2*x
^4
```

Sympy [A]
time = 0.01, size = 49, normalized size = 0.91

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{2bcx^7}{7} + \frac{c^2x^8}{8} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + 2*b*c*x**7/7 + c**2*x**8/8 + x**6*(a*c/3 + b**
2/6)
```

Giac [A]
time = 3.94, size = 46, normalized size = 0.85

$$\frac{1}{8}\,c^2x^8 + \frac{2}{7}\,bcx^7 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 2/7*b*c*x^7 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 2/5*a*b*x^5 + 1/4*a
^2*x^4
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6} + \frac{a\,c}{3}\right) + \frac{a^2\,x^4}{4} + \frac{c^2\,x^8}{8} + \frac{2\,a\,b\,x^5}{5} + \frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^4)/4 + (c^2*x^8)/8 + (2*a*b*x^5)/5 + (2*b*c*
x^7)/7
```

3.10 $\int \frac{(ax^2+bx^3+cx^4)^2}{x^2} \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Rubi [A]

time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1599, 712}

$$\frac{a^2x^3}{3}+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{(ax^2+bx^3+cx^4)^2}{x^2} \, dx &= \int x^2\left(a+bx+cx^2\right)^2 \, dx\\
&= \int \left(a^2x^2+2abx^3+\left(b^2+2ac\right)x^4+2bcx^5+c^2x^6\right) \, dx\\
&= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{1}{3}bcx^6+\frac{c^2x^7}{7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{1}{3}bcx^6 + \frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^5)/5 + (b*c*x^6)/3 + (c^2*x^7)
/7
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^3}{3} + \frac{ab\,x^4}{2} + \frac{(2ac+b^2)x^5}{5} + \frac{bc\,x^6}{3} + \frac{c^2x^7}{7}$	45
gosper	$\frac{x^3\left(30c^2x^4+70bc\,x^3+84ac\,x^2+42b^2x^2+105abx+70a^2\right)}{210}$	47
risch	$\frac{1}{3}a^2x^3 + \frac{1}{2}ab\,x^4 + \frac{2}{5}ac\,x^5 + \frac{1}{5}b^2x^5 + \frac{1}{3}bc\,x^6 + \frac{1}{7}c^2x^7$	47
norman	$\frac{\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{a^2x^4}{3}+\frac{c^2x^8}{7}+\frac{ab\,x^5}{2}+\frac{bc\,x^7}{3}}{x}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*(2*a*c+b^2)*x^5+1/3*b*c*x^6+1/7*c^2*x^7
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,\left(b^2 + 2\,ac\right)x^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Fricas [A]
time = 0.34, size = 44, normalized size = 0.81

$$\frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{2}\,abx^4 + \frac{1}{5}\,\left(b^2 + 2\,ac\right)x^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/2*a*b*x^4 + 1/5*(b^2 + 2*a*c)*x^5 + 1/3*a^2*x
^3
```

Sympy [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{bcx^6}{3} + \frac{c^2x^7}{7} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**2/x**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b*c*x**6/3 + c**2*x**7/7 + x**5*(2*a*c/5 + b**2/
5)
```

Giac [A]
time = 4.01, size = 46, normalized size = 0.85

$$\frac{1}{7}\,c^2x^7 + \frac{1}{3}\,bcx^6 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 1/3*b*c*x^6 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 1/2*a*b*x^4 + 1/3*a
^2*x^3
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^5\,\left(\frac{b^2}{5}+\frac{2\,a\,c}{5}\right)+\frac{a^2\,x^3}{3}+\frac{c^2\,x^7}{7}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^6}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^2/x^2,x)
```

```
[Out] x^5*((2*a*c)/5 + b^2/5) + (a^2*x^3)/3 + (c^2*x^7)/7 + (a*b*x^4)/2 + (b*c*x^
6)/3
```

3.11 $\int \frac{x^5}{ax^2+bx^3+cx^4} dx$

Optimal. Leaf size=89

$$-\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}$$

```
[Out] -b*x/c^2+1/2*x^2/c+1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/c^3+b*(-3*a*c+b^2)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1599, 715, 648, 632, 212, 642}

$$\frac{(b^2-ac)\log(a+bx+cx^2)}{2c^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}-\frac{bx}{c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^2/(2*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 -
4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x + c*x^2])/(2*c^
3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^3}{a+bx+cx^2}\,dx\\
&= \int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+\left(b^2-ac\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\int \frac{ab+\left(b^2-ac\right)x}{a+bx+cx^2}\,dx}{c^2}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}-\frac{\left(b\left(b^2-3ac\right)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2c^3}+\frac{\left(b^2-ac\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^3}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}+\frac{\left(b\left(b^2-3ac\right)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,\right.}{c^3}\\
&= -\frac{bx}{c^2}+\frac{x^2}{2c}+\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx+cx^2\right)}{2c^3}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 84, normalized size = 0.94

$$\frac{cx(-2b+cx)-\frac{2b\left(b^2-3ac\right)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\left(b^2-ac\right)\log(a+x(b+cx))}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (c*x*(-2*b + c*x) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + x*(b + c*x)])/(2*c^3)
```

Maple [A]
time = 0.03, size = 98, normalized size = 1.10

method	result
default	$-\frac{-\frac{1}{2}c\,x^2+bx}{c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c^2}$
risch	$\frac{x^2}{2c}-\frac{bx}{c^2}-\frac{2\ln\left(12a^2b\,c^2-7a\,b^3c+b^5-2\sqrt{-b^2\left(4ac-b^2\right)\left(3ac-b^2\right)^2}\,cx-\sqrt{-b^2\left(4ac-b^2\right)\left(3ac-b^2\right)^2}\,b\right)a^2}{c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/2*c*x^2+b*x)+1/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^2+b*x+a)+2*(a*b-1/2*
(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.34, size = 297, normalized size = 3.34

$$\left[\frac{(b^2c^2-4\,ac^3)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\,(b^3c-4\,abc^2)x+(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^2+bx+a\right)}{2\,(b^2c^3-4\,ac^4)},\ \frac{(b^2c^2-4\,ac^3)x^2+2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-2\,(b^3c-4\,abc^2)x+(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^2+bx+a\right)}{2\,(b^2c^3-4\,ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2 - 4*a*c^3)*x^2 - (b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2
+ b*x + a))/(b^2*c^3 - 4*a*c^4), 1/2*((b^2*c^2 - 4*a*c^3)*x^2 + 2*(b^3 - 3*
a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - 2*(b^3*c - 4*a*b*c^2)*x + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*log(c*x^2 +
b*x + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(83) = 166$.
time = 0.46, size = 381, normalized size = 4.28

$$-\frac{bx}{c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)+\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)\log\left(x+\frac{2a^2c-ab^2+4ac^3\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)-b^2c^2\left(\frac{b\sqrt{-4ac+b^2}\cdot(3ac-b^2)}{2c^3\cdot(4ac-b^2)}-\frac{ac-b^2}{2c^3}\right)}{3abc-b^3}\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -b*x/c**2 + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
- (a*c - b**2)/(2*c**3))*log(x + (2*a**2*c - a*b**2 + 4*a*c**3*(-b*sqrt(-4*
a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))
 - b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c - b**2))
 - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*
c - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3))*log(x + (2*a**2*
c - a*b**2 + 4*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(2*c**3*(4*a*c
- b**2)) - (a*c - b**2)/(2*c**3)) - b**2*c**2*(b*sqrt(-4*a*c + b**2)*(3*a*c
 - b**2)/(2*c**3*(4*a*c - b**2)) - (a*c - b**2)/(2*c**3)))/(3*a*b*c - b**3)
) + x**2/(2*c)
```

Giac [A]
time = 4.12, size = 86, normalized size = 0.97

$$\frac{cx^2-2\,bx}{2\,c^2}+\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,c^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(c*x^2 - 2*b*x)/c^2 + 1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/c^3 - (b^3 -
 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [B]
time = 0.14, size = 112, normalized size = 1.26

$$\frac{x^2}{2\,c}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(4\,a^2\,c^2-5\,a\,b^2\,c+b^4\right)}{2\,\left(4\,a\,c^4-b^2\,c^3\right)}-\frac{b\,x}{c^2}+\frac{b\,\mathrm{atan}\left(\frac{b+2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)\,\left(3\,a\,c-b^2\right)}{c^3\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x^2/(2*c) - (log(a + b*x + c*x^2)*(b^4 + 4*a^2*c^2 - 5*a*b^2*c))/(2*(4*a*c^
4 - b^2*c^3)) - (b*x)/c^2 + (b*atan((b + 2*c*x)/(4*a*c - b^2)^(1/2))*(3*a*c
 - b^2))/(c^3*(4*a*c - b^2)^(1/2))
```

3.12 $\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=70

$$\frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

```
[Out] x/c-1/2*b*ln(c*x^2+b*x+a)/c^2-(-2*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1599, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2
- 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x^2}{a+bx+cx^2}\,dx\\
&= \frac{x}{c}+\frac{\int \frac{-a-bx}{a+bx+cx^2}\,dx}{c}\\
&= \frac{x}{c}-\frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2}+\frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2c^2}\\
&= \frac{x}{c}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}-\frac{(b^2-2ac)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{c^2}\\
&= \frac{x}{c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 73, normalized size = 1.04

$$\frac{x}{c}+\frac{(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}}-\frac{b\log\left(a+bx+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2
 + 4*a*c]) - (b*Log[a + b*x + c*x^2])/(2*c^2)
```

Maple [A]
time = 0.03, size = 75, normalized size = 1.07

method	result
default	$\frac{x}{c}+\frac{-\frac{b\ln\left(c\,x^{2}+bx+a\right)}{2c}+\frac{2\left(-a+\frac{b^{2}}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{c}$
risch	$\frac{x}{c}-\frac{2\ln\left(-8a^{2}c^{2}+6a\,b^{2}c-b^{4}-2\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,cx-\sqrt{-\left(4ac-b^{2}\right)\left(2ac-b^{2}\right)^{2}}\,b\right)ab}{c(4ac-b^{2})}+\frac{\ln\left(-8a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] x/c+1/c*(-1/2*b/c*ln(c*x^2+b*x+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 235, normalized size = 3.36

$$\left[-\frac{(b^{2}-2\,ac)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{2}+2\,bcx+b^{2}-2\,ac+\sqrt{b^{2}-4\,ac}\,(2\,cx+b)}{cx^{2}+bx+a}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})},-\frac{2\,(b^{2}-2\,ac)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^{2}+4\,ac}\,(2\,cx+b)}{b^{2}-4\,ac}\right)-2\,(b^{2}c-4\,ac^{2})x+(b^{3}-4\,abc)\log\left(cx^{2}+bx+a\right)}{2\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 2*(b^2*c - 4*a*c^2
```

```
)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(b^2*c^2 - 4*a*c^3), -1/2*(2*(b
^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2
- 4*a*c)) - 2*(b^2*c - 4*a*c^2)*x + (b^3 - 4*a*b*c)*log(c*x^2 + b*x + a))/(
b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(65) = 130$.
time = 0.35, size = 306, normalized size = 4.37

$$\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)\log\left(x+\frac{-ab-4ac^2\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)+b^2c\left(-\frac{b}{2c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{2c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*
log(x + (-a*b - 4*a*c**2*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/
(2*c**2*(4*a*c - b**2))) + b**2*c*(-b/(2*c**2) - sqrt(-4*a*c + b**2)*(2*a*c
 - b**2)/(2*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(2*c**2) + sqrt(-4
*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)))*log(x + (-a*b - 4*a*c*
*2*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*c - b**2)
)) + b**2*c*(-b/(2*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(2*c**2*(4*a*
c - b**2))))/(2*a*c - b**2)) + x/c
```

Giac [A]
time = 4.95, size = 67, normalized size = 0.96

$$\frac{x}{c}-\frac{b\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/c - 1/2*b*log(c*x^2 + b*x + a)/c^2 + (b^2 - 2*a*c)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 2.03, size = 172, normalized size = 2.46

$$\frac{x}{c}+\frac{b^3\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^3-b^2\,c^2\right)}-\frac{2\,a\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}+\frac{b^2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c^2\,\sqrt{4\,a\,c-b^2}}-\frac{2\,a\,b\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^3-b^2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] x/c + (b^3*log(a + b*x + c*x^2))/(2*(4*a*c^3 - b^2*c^2)) - (2*a*atan(b/(4*a
*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) + (
b^2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c^2*(4*a*c
- b^2)^(1/2)) - (2*a*b*c*log(a + b*x + c*x^2))/(4*a*c^3 - b^2*c^2)
```

3.13 $\int \frac{x^3}{ax^2+bx^3+cx^4} \, dx$

Optimal. Leaf size=56

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

```
[Out] 1/2*ln(c*x^2+b*x+a)/c+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1599, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 - 4*a*c]) + Log[a +
b*x + c*x^2]/(2*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax^2+bx^3+cx^4}\,dx &= \int \frac{x}{a+bx+cx^2}\,dx\\ &= \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c} - \frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2c}\\ &= \frac{\log\left(a+bx+cx^2\right)}{2c} + \frac{b\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{c}\\ &= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}} + \frac{\log\left(a+bx+cx^2\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 57, normalized size = 1.02

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + \log(a+x(b+cx))}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a +
 x*(b + c*x)])/(2*c)
```

Maple [A]
time = 0.02, size = 56, normalized size = 1.00

method	result

default	$\frac{\ln(c\,x^2+bx+a)}{2c}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(-2\sqrt{-b^2\,(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-b^2\,(4ac-b^2)}\,b\right)a}{4ac-b^2}-\frac{\ln\left(-2\sqrt{-b^2\,(4ac-b^2)}\,cx-4abc+b^3-\sqrt{-}\right.}{2c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*x^2+b*x+a)/c-b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.33, size = 185, normalized size = 3.30

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)},\frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+(b^2-4\,ac)\log\left(cx^2+bx+a\right)}{2\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a))/(b^2*c - 4*a*c^2), 1/2*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-sqrt(-b^2 + 4*a
*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^2 + b*x + a))/(b^2*c
 - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(49) = 98$.
time = 0.17, size = 216, normalized size = 3.86

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(-\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2c\,(4ac-b^2)}+\frac{1}{2c}\right)\log\left(x+\frac{-4ac\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)+2a+b^2\left(\frac{b\sqrt{-4ac+b^2}}{2c(4ac-b^2)}+\frac{1}{2c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(-b
*sqrt(-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(-b*sqrt(
-4*a*c + b**2)/(2*c*(4*a*c - b**2)) + 1/(2*c)))/b) + (b*sqrt(-4*a*c + b**2)
/(2*c*(4*a*c - b**2)) + 1/(2*c))*log(x + (-4*a*c*(b*sqrt(-4*a*c + b**2)/(2*
c*(4*a*c - b**2)) + 1/(2*c)) + 2*a + b**2*(b*sqrt(-4*a*c + b**2)/(2*c*(4*a*
c - b**2)) + 1/(2*c)))/b)
```

Giac [**A**]
time = 4.37, size = 55, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^2+bx+a\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/2*log(
c*x^2 + b*x + a)/c
```

Mupad [**B**]
time = 0.13, size = 112, normalized size = 2.00

$$\frac{2\,a\,c\,\ln\left(c\,x^2+b\,x+a\right)}{4\,a\,c^2-b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{c\,\sqrt{4\,a\,c-b^2}}-\frac{b^2\,\ln\left(c\,x^2+b\,x+a\right)}{2\,\left(4\,a\,c^2-b^2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*a*c*log(a + b*x + c*x^2))/(4*a*c^2 - b^2*c) - (b*atan(b/(4*a*c - b^2)^(1
/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(c*(4*a*c - b^2)^(1/2)) - (b^2*log(a +
b*x + c*x^2))/(2*(4*a*c^2 - b^2*c))
```

3.14 $\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -2*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1599, 632, 212}

$$-\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (-2*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^2}{ax^2+bx^3+cx^4}\,dx = \int \frac{1}{a+bx+cx^2}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4),x]
[Out] (2*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.02, size = 35, normalized size = 1.03

method	result	size
default	$\frac{2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	35
risch	$-\frac{\ln\left(b+2cx+\sqrt{-4ac+b^2}\right)}{\sqrt{-4ac+b^2}}+\frac{\ln\left(-b-2cx+\sqrt{-4ac+b^2}\right)}{\sqrt{-4ac+b^2}}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 120, normalized size = 3.53

$$\left[\frac{\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c
*x^2 + b*x + a))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(34) = 68$.
time = 0.10, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*c)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))
```

Giac [A]
time = 4.12, size = 34, normalized size = 1.00

$$\frac{2\,\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 2*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 0.03, size = 46, normalized size = 1.35

$$\frac{2\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (2*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x)/(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)
^(1/2)
```

3.15 $\int \frac{x}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=62

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(c*x^2+b*x+a)/a+b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 719, 29, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqrt[b^2 - 4*a*c]) + Log[x]/a
 - Log[a + b*x + c*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{ax^2+bx^3+cx^4}\,dx &= \int \frac{1}{x\left(a+bx+cx^2\right)}\,dx \\
&= \frac{\int \frac{1}{x}\,dx}{a}+\frac{\int \frac{-b-cx}{a+bx+cx^2}\,dx}{a} \\
&= \frac{\log(x)}{a}-\frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a}-\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{2a} \\
&= \frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{a} \\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx+cx^2\right)}{2a}
\end{aligned}
$$

Mathematica [A]

time = 0.04, size = 61, normalized size = 0.98

$$-\frac{\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2\log(x)+\log(a+x(b+cx))}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4),x]
```

```
[Out] -1/2*((2*b*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 2*L
og[x] + Log[a + x*(b + c*x)])/a
```

Maple [A]
time = 0.02, size = 61, normalized size = 0.98

method	result	siz
default	$\frac{-\frac{\ln\left(c\,x^2+bx+a\right)}{2}-\frac{b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a}+\frac{\ln(x)}{a}$	61
risch	$\frac{\ln(x)}{a}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2c-a\,b^2\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(6ac-2b^2\right)\_R+3c\right)x-ab\_R+b\right)\right)$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(-1/2*ln(c*x^2+b*x+a)-b/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^
(1/2)))+ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.34, size = 211, normalized size = 3.40

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(x\right)}{2\,(ab^2-4\,a^2c)},\ \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-(b^2-4\,ac)\log\left(cx^2+bx+a\right)+2\,(b^2-4\,ac)\log\left(x\right)}{2\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^2 - 4*a*c)*log(c*x^2 + b*x +
 a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/2*(2*sqrt(-b^2 + 4*a*c)*
b*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^2 - 4*a*c)*log
(c*x^2 + b*x + a) + 2*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(54) = 108$.
time = 4.58, size = 564, normalized size = 9.10

$$\left(-\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2+3a^2b^2c\left(-\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)-12a^2c^2+11ab^2c-2b^4}{9abc^2-2b^3c}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{2a\left(4ac-b^2\right)}-\frac{1}{2a}\right)\log\left(x+\frac{24a^4c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-14a^3b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2-12a^3c^2\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)+2a^2b^4\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)^2+3a^2b^2c\left(\frac{b\sqrt{-4ac+b^2}}{2a(4ac-b^2)}-\frac{1}{2a}\right)-12a^2c^2+11ab^2c-2b^4}{9abc^2-2b^3c}\right)+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**
2*c*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c*
*2*(-b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(-
b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c - b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a
*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b**3*c)) + (b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a))*log(x + (24*a**4*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 14*a**3*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*
(4*a*c - b**2)) - 1/(2*a))**2 - 12*a**3*c**2*(b*sqrt(-4*a*c + b**2)/(2*a*(4
*a*c - b**2)) - 1/(2*a)) + 2*a**2*b**4*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
 b**2)) - 1/(2*a))**2 + 3*a**2*b**2*c*(b*sqrt(-4*a*c + b**2)/(2*a*(4*a*c -
b**2)) - 1/(2*a)) - 12*a**2*c**2 + 11*a*b**2*c - 2*b**4)/(9*a*b*c**2 - 2*b*
*3*c)) + log(x)/a
```

Giac [A]
time = 4.40, size = 62, normalized size = 1.00

$$-\frac{b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\;a}-\frac{\log\left(cx^2+bx+a\right)}{2\,a}+\frac{\log\left(|x|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/2*log(
c*x^2 + b*x + a)/a + log(abs(x))/a
```

Mupad [B]
time = 2.30, size = 213, normalized size = 3.44

$$\frac{\ln(x)}{a}-\ln\left(b\,c-\left(x\,\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)+3\,c^2\,x\right)\,\left(\frac{1}{2\,a}-\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-\ln\left(\left(x\,\left(6\,a\,c^2-2\,b^2\,c\right)-a\,b\,c\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)-b\,c-3\,c^2\,x\right)\,\left(\frac{1}{2\,a}+\frac{b\,\sqrt{b^2-4\,a\,c}}{2\,\left(a\,b^2-4\,a^2\,c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] log(x)/a - log(b*c - (x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a) - (b*(b^2 - 4
*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) + 3*c^2*x)*(1/(2*a) - (b*(b^2 - 4*a*c)^
(1/2))/(2*(a*b^2 - 4*a^2*c))) - log((x*(6*a*c^2 - 2*b^2*c) - a*b*c)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c))) - b*c - 3*c^2*x)*(1/(2*a
) + (b*(b^2 - 4*a*c)^(1/2))/(2*(a*b^2 - 4*a^2*c)))
```

3.16 $\int \frac{1}{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{1}{ax}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] -1/a/x-b*ln(x)/a^2+1/2*b*ln(c*x^2+b*x+a)/a^2-(-2*a*c+b^2)*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1608, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx+cx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
```

```
[Out] -(1/(a*x)) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sq
rt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x + c*x^2])/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{ax^2+bx^3+cx^4}\,dx = \int \frac{1}{x^2(a+bx+cx^2)}\,dx$$
$$= -\frac{1}{ax} + \frac{\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx}{a}$$
$$= -\frac{1}{ax} + \frac{\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx}{a}$$
$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx}{a^2}$$
$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} + \frac{(b^2-2ac)\int \frac{1}{a+bx+cx^2}\,dx}{2a^2}$$
$$= -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2} - \frac{(b^2-2ac)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{a^2}$$
$$= -\frac{1}{ax} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx+cx^2)}{2a^2}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.95

$$\frac{-\frac{2a}{x} + \frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 2b\log(x) + b\log(a+x(b+cx))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-1),x]
[Out] ((-2*a)/x + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-
b^2 + 4*a*c] - 2*b*Log[x] + b*Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A]
time = 0.04, size = 81, normalized size = 1.00

method	result
default	$\frac{\frac{b\ln(c\,x^2+bx+a)}{2} + \frac{2\left(-ac+\frac{b^2}{2}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^2} - \frac{1}{ax} - \frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{ax}+\frac{2\ln\left(\left(\left(-4a^3c^3+18a^2b^2c^2-12a\,b^4c+2b^6+4\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,abc-2\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)}\right.\right.\right.}{a(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*b*ln(c*x^2+b*x+a)+2*(-a*c+1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x+b)/(4*a*c-b^2)^(1/2)))-1/a/x-b*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.35, size = 269, normalized size = 3.32

$$\left[-\frac{\left(b^2-2\,ac\right)\sqrt{b^2-4\,ac}\,x\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)+2\left(b^3-4\,abc\right)x\log\left(x\right)}{2\left(a^2b^2-4\,a^3c\right)x},-\frac{2\left(b^2-2\,ac\right)\sqrt{-b^2+4\,ac}\,x\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,ab^2-8\,a^2c-\left(b^3-4\,abc\right)x\log\left(cx^2+bx+a\right)+2\left(b^3-4\,abc\right)x\log\left(x\right)}{2\left(a^2b^2-4\,a^3c\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2
*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*a*b^2 - 8*a^2*
c - (b^3 - 4*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((
a^2*b^2 - 4*a^3*c)*x), -1/2*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x*arctan(-s
qrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a*b^2 - 8*a^2*c - (b^3 - 4
*a*b*c)*x*log(c*x^2 + b*x + a) + 2*(b^3 - 4*a*b*c)*x*log(x))/((a^2*b^2 - 4*
a^3*c)*x)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.94, size = 79, normalized size = 0.98

$$\frac{b\log\left(cx^2+bx+a\right)}{2\,a^2}-\frac{b\log\left(|x|\right)}{a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b*log(c*x^2 + b*x + a)/a^2 - b*log(abs(x))/a^2 + (b^2 - 2*a*c)*arctan((
2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) - 1/(a*x)
```

Mupad [B]
time = 2.50, size = 339, normalized size = 4.19

$$\frac{\ln\left(2\,a\,b^3+2\,b^4\,x-2\,a\,b^2\,\sqrt{b^2-4\,a\,c}+a^2\,c\,\sqrt{b^2-4\,a\,c}-2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x+4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(a\left(2\,b\,c-c\,\sqrt{b^2-4\,a\,c}\right)-\frac{b^3}{2}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{1}{a\,x}-\frac{\ln\left(2\,a\,b^3+2\,b^4\,x+2\,a\,b^2\,\sqrt{b^2-4\,a\,c}-a^2\,c\,\sqrt{b^2-4\,a\,c}+2\,b^3\,x\,\sqrt{b^2-4\,a\,c}+2\,a^2\,c^2\,x-7\,a^2\,b\,c-8\,a\,b^2\,c\,x-4\,a\,b\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2}-a\left(2\,b\,c+c\,\sqrt{b^2-4\,a\,c}\right)+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2}\right)}{4\,a^3\,c-a^2\,b^2}-\frac{b\,\ln\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4),x)
```

```
[Out] (log(2*a*b^3 + 2*b^4*x - 2*a*b^2*(b^2 - 4*a*c)^(1/2) + a^2*c*(b^2 - 4*a*c)^
(1/2) - 2*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x
 + 4*a*b*c*x*(b^2 - 4*a*c)^(1/2))*(a*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) - b^3/
2 + (b^2*(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - 1/(a*x) - (log(2*a*
b^3 + 2*b^4*x + 2*a*b^2*(b^2 - 4*a*c)^(1/2) - a^2*c*(b^2 - 4*a*c)^(1/2) + 2
*b^3*x*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*x - 7*a^2*b*c - 8*a*b^2*c*x - 4*a*b*
c*x*(b^2 - 4*a*c)^(1/2))*(b^3/2 - a*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + (b^2*
(b^2 - 4*a*c)^(1/2))/2))/(4*a^3*c - a^2*b^2) - (b*log(x))/a^2
```

3.17 $\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx$

Optimal. Leaf size=104

$$-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log(x)}{a^3}-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}$$

```
[Out] -1/2/a/x^2+b/a^2/x+(-a*c+b^2)*ln(x)/a^3-1/2*(-a*c+b^2)*ln(c*x^2+b*x+a)/a^3+
b*(-3*a*c+b^2)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-ac)\log\left(a+bx+cx^2\right)}{2a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2
- 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[x])/a^3 - ((b^2 - a*c
)*Log[a + b*x + c*x^2])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x(ax^2+bx^3+cx^4)}\,dx = \int \frac{1}{x^3(a+bx+cx^2)}\,dx$$
$$= -\frac{1}{2ax^2} + \frac{\int \frac{-b-cx}{x^2(a+bx+cx^2)}\,dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{\int \left(-\frac{b}{ax^2} + \frac{b^2-ac}{a^2x} + \frac{-b(b^2-2ac)-c(b^2-ac)x}{a^2(a+bx+cx^2)}\right)dx}{a}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} + \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x}{a+bx+cx^2}\,dx}{a^3}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b(b^2-3ac))\int \frac{1}{a+bx+cx^2}\,dx}{2a^3} - \frac{(b^2-ac)\int}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-ac)\log(a+bx+cx^2)}{2a^3} + \frac{(b(b^2-3ac)}{}$$
$$= -\frac{1}{2ax^2} + \frac{b}{a^2x} + \frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}} + \frac{(b^2-ac)\log(x)}{a^3} - \frac{(b^2-}{}$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 0.98

$$\frac{-\frac{a^2}{x^2} + \frac{2ab}{x} - \frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} + 2(b^2-ac)\log(x) + (-b^2+ac)\log(a+x(b+cx))}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] (-(a^2/x^2) + (2*a*b)/x - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + 2*(b^2 - a*c)*Log[x] + (-b^2 + a*c)*Log[a +
x*(b + c*x)])/(2*a^3)
```

Maple [A]
time = 0.04, size = 128, normalized size = 1.23

method	result
default	$\frac{\frac{(c^2a-b^2c)\ln(c\,x^2+bx+a)}{2c} + \frac{2\left(2abc-b^3-\frac{(c^2a-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^3} - \frac{1}{2a\,x^2} + \frac{(-ac+b^2)\ln(x)}{a^3} + \frac{b}{a^2x}$

risch	$\frac{\frac{bx}{a^2}-\frac{1}{2a}}{x^2}-\frac{\ln(x)c}{a^2}+\frac{\ln(x)b^2}{a^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((4c\,a^4-b^2a^3)\_Z^2+(-4a^2c^2+5a\,b^2c-b^4)\_Z+c^3\right)}\_R\ln\left(\left((6a^5c-2a^4b^2\right)\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/2*(a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(2*a*b*c-b^3-1/2*(a*c^2-b^2*c)
*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2+(-a*
c+b^2)*ln(x)/a^3+b/a^2/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 358, normalized size = 3.44

$$\left[-\frac{(b^3-3\,abc)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+a^2b^2-4\,a^3c+(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(cx^2+bx+a\right)-2\,(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(x\right)-2\,(ab^3-4\,a^2bc)x}{2\,(a^3b^2-4\,a^4c)x^2},\ \frac{2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)-a^2b^2+4\,a^3c-(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(cx^2+bx+a\right)+2\,(b^4-5\,ab^2c+4\,a^2c^2)x^2\log\left(x\right)+2\,(ab^3-4\,a^2bc)x}{2\,(a^3b^2-4\,a^4c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^3 - 3*a*b*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^2 + 2*b*c*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + a^2*b^2 - 4*
a^3*c + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) - 2*(b^4 - 5
*a*b^2*c + 4*a^2*c^2)*x^2*log(x) - 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a
^4*c)*x^2), 1/2*(2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-sqrt(-b^2
 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - a^2*b^2 + 4*a^3*c - (b^4 - 5*a*b^2*c
 + 4*a^2*c^2)*x^2*log(c*x^2 + b*x + a) + 2*(b^4 - 5*a*b^2*c + 4*a^2*c^2)*x^
2*log(x) + 2*(a*b^3 - 4*a^2*b*c)*x)/((a^3*b^2 - 4*a^4*c)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.86, size = 105, normalized size = 1.01

$$-\frac{(b^2-ac)\log\left(cx^2+bx+a\right)}{2\,a^3}+\frac{(b^2-ac)\log\left(|x|\right)}{a^3}-\frac{(b^3-3\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -1/2*(b^2 - a*c)*log(c*x^2 + b*x + a)/a^3 + (b^2 - a*c)*log(abs(x))/a^3 - (
b^3 - 3*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a
^3) + 1/2*(2*a*b*x - a^2)/(a^3*x^2)
```

Mupad [B]
time = 0.59, size = 447, normalized size = 4.30

$$\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2+2\,a\,b^3\,\sqrt{b^2-4\,a\,c}+2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x-3\,a^2\,b\,c\,\sqrt{b^2-4\,a\,c}+9\,a^2\,b\,c^2\,x+3\,a^2\,c^2\,x\,\sqrt{b^2-4\,a\,c}-6\,a\,b^2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^4}{2}-a\left(\frac{5\,b^2\,c}{2}+\frac{3\,b\,c\,\sqrt{b^2-4\,a\,c}}{2}\right)+\frac{b^3\,\sqrt{b^2-4\,a\,c}}{2}+2\,a^2\,c^2\right)}{4\,a^4\,c-a^3\,b^2}-\frac{\ln\left(2\,a\,b^4+2\,b^5\,x+6\,a^3\,c^2-2\,a\,b^3\,\sqrt{b^2-4\,a\,c}-2\,b^4\,x\,\sqrt{b^2-4\,a\,c}-9\,a^2\,b^2\,c-10\,a\,b^3\,c\,x+3\,a^2\,b\,c\,\sqrt{b^2-4\,a\,c}+9\,a^2\,b\,c^2\,x-3\,a^2\,c^2\,x\,\sqrt{b^2-4\,a\,c}+6\,a\,b^2\,c\,x\,\sqrt{b^2-4\,a\,c}\right)\left(a\left(\frac{5\,b^2\,c}{2}-\frac{3\,b\,c\,\sqrt{b^2-4\,a\,c}}{2}\right)-\frac{b^4}{2}+\frac{b^3\,\sqrt{b^2-4\,a\,c}}{2}-2\,a^2\,c^2\right)}{4\,a^4\,c-a^3\,b^2}-\frac{\frac{1}{2\,a}-\frac{b\,x}{a^2}}{x^2}-\frac{\ln\left(x\right)\,\left(a\,c-b^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] (log(2*a*b^4 + 2*b^5*x + 6*a^3*c^2 + 2*a*b^3*(b^2 - 4*a*c)^(1/2) + 2*b^4*x*
(b^2 - 4*a*c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x - 3*a^2*b*c*(b^2 - 4*a*c)^
(1/2) + 9*a^2*b*c^2*x + 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c*x*(b^2
- 4*a*c)^(1/2))*(b^4/2 - a*((5*b^2*c)/2 + (3*b*c*(b^2 - 4*a*c)^(1/2))/2) +
(b^3*(b^2 - 4*a*c)^(1/2))/2 + 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (log(2*a*b^
4 + 2*b^5*x + 6*a^3*c^2 - 2*a*b^3*(b^2 - 4*a*c)^(1/2) - 2*b^4*x*(b^2 - 4*a*
c)^(1/2) - 9*a^2*b^2*c - 10*a*b^3*c*x + 3*a^2*b*c*(b^2 - 4*a*c)^(1/2) + 9*a
^2*b*c^2*x - 3*a^2*c^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c*x*(b^2 - 4*a*c)^(1
/2))*(a*((5*b^2*c)/2 - (3*b*c*(b^2 - 4*a*c)^(1/2))/2) - b^4/2 + (b^3*(b^2 -
 4*a*c)^(1/2))/2 - 2*a^2*c^2))/(4*a^4*c - a^3*b^2) - (1/(2*a) - (b*x)/a^2)/
x^2 - (log(x)*(a*c - b^2))/a^3
```

3.18 $\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx$

Optimal. Leaf size=137

$$-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2-ac}{a^3x}-\frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{b(b^2-2ac)\log(x)}{a^4}+\frac{b(b^2-2ac)\log(\text{...}}{2a^4}$$

[Out] -1/3/a/x^3+1/2*b/a^2/x^2+(a*c-b^2)/a^3/x-b*(-2*a*c+b^2)*ln(x)/a^4+1/2*b*(-2*a*c+b^2)*ln(c*x^2+b*x+a)/a^4-(2*a^2*c^2-4*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(1/2)

Rubi [A]
time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 723, 814, 648, 632, 212, 642}

$$\frac{b(b^2-2ac)\log(a+bx+cx^2)}{2a^4}-\frac{b\log(x)(b^2-2ac)}{a^4}-\frac{b^2-ac}{a^3x}+\frac{b}{2a^2x^2}-\frac{(2a^2c^2-4ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]

[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - (b^2 - a*c)/(a^3*x) - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^4*Sqrt[b^2 - 4*a*c]) - (b*(b^2 - 2*a*c)*Log[x])/a^4 + (b*(b^2 - 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x^2(ax^2+bx^3+cx^4)}\,dx = \int \frac{1}{x^4(a+bx+cx^2)}\,dx$$
$$= -\frac{1}{3ax^3} + \frac{\int \frac{-b-cx}{x^3(a+bx+cx^2)}\,dx}{a}$$
$$= -\frac{1}{3ax^3} + \frac{\int \left(-\frac{b}{ax^3} + \frac{b^2-ac}{a^2x^2} + \frac{-b^3+2abc}{a^3x} + \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a^3(a+bx+cx^2)}\right)dx}{a}$$
$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{\int \frac{b^4-3ab^2c+a^2c^2+bc(b^2-2ac)x}{a+bx+cx^2}\,dx}{a^4}$$
$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{(b(b^2-2ac))\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^4}$$
$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{b(b^2-2ac)\log(x)}{a^4} + \frac{b(b^2-2ac)\log(a+bx+cx}{2a^4}$$
$$= -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2-ac}{a^3x} - \frac{(b^4-4ab^2c+2a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}} - \frac{b(}{}$$

Mathematica [A]
time = 0.08, size = 131, normalized size = 0.96

$$\frac{-\frac{2a^3}{x^3} + \frac{3a^2b}{x^2} + \frac{6a(-b^2+ac)}{x} + \frac{6(b^4-4ab^2c+2a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - 6(b^3-2abc)\log(x) + 3(b^3-2abc)\log(a+x(b+cx))}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x]
```

```
[Out] ((-2*a^3)/x^3 + (3*a^2*b)/x^2 + (6*a*(-b^2 + a*c))/x + (6*(b^4 - 4*a*b^2*c
+ 2*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - 6
*(b^3 - 2*a*b*c)*Log[x] + 3*(b^3 - 2*a*b*c)*Log[a + x*(b + c*x)])/(6*a^4)
```

Maple [A]
time = 0.04, size = 157, normalized size = 1.15

method	result
default	$\frac{\frac{(-2abc^2+b^3c)\ln(cx^2+bx+a)}{2c} + \frac{2\left(a^2c^2-3ab^2c+b^4-\frac{(-2abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a^4} - \frac{1}{3ax^3} - \frac{-ac+b^2}{a^3x} + \frac{b(2ac-b^2)\ln}{a^4}$

risch	$\frac{\frac{(ac-b^2)x^2}{a^3}+\frac{bx}{2a^2}-\frac{1}{3a}}{x^3}+\frac{2b\ln(x)c}{a^3}-\frac{b^3\ln(x)}{a^4}+\left(\sum_{\_R=\mathrm{RootOf}\left((4a^5c-a^4b^2)\_Z^2+(8a^2b\,c^2-6a\,b^3c+b^5)\_Z+c^4\right)}\_R\ln\left(((6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/2*(-2*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(a^2*c^2-3*a*b^2*c+b^4-1/
2*(-2*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1
/2)))-1/3/a/x^3-(-a*c+b^2)/a^3/x+b*(2*a*c-b^2)/a^4*ln(x)+1/2*b/a^2/x^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.41, size = 445, normalized size = 3.25

$$\left[\frac{3\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{b^2-4\,ac}\,x^3\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)-2\,a^3b^2+8\,a^4c+3\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(cx^2+bx+a\right)-6\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(x\right)-6\,(ab^4-5\,a^2b^2c+4\,a^3c^2)x^2+3\,(a^2b^3-4\,a^3bc)x}{6\,(a^4b^2-4\,a^5c)x^3},-\frac{6\,(b^4-4\,ab^2c+2\,a^2c^2)\sqrt{-b^2+4\,ac}\,x^3\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,a^3b^2-8\,a^4c-3\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(cx^2+bx+a\right)+6\,(b^5-6\,ab^3c+8\,a^2bc^2)x^3\log\left(x\right)+6\,(ab^4-5\,a^2b^2c+4\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{6\,(a^4b^2-4\,a^5c)x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^2
+ 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a))
 - 2*a^3*b^2 + 8*a^4*c + 3*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 +
b*x + a) - 6*(b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(x) - 6*(a*b^4 - 5*a^2*
b^2*c + 4*a^3*c^2)*x^2 + 3*(a^2*b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^
3), -1/6*(6*(b^4 - 4*a*b^2*c + 2*a^2*c^2)*sqrt(-b^2 + 4*a*c)*x^3*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*a^3*b^2 - 8*a^4*c - 3*(b^5
- 6*a*b^3*c + 8*a^2*b*c^2)*x^3*log(c*x^2 + b*x + a) + 6*(b^5 - 6*a*b^3*c +
8*a^2*b*c^2)*x^3*log(x) + 6*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*x^2 - 3*(a^2*
b^3 - 4*a^3*b*c)*x)/((a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.35, size = 136, normalized size = 0.99

$$\frac{(b^3 - 2\,abc)\log\left(cx^2 + bx + a\right)}{2\,a^4} - \frac{(b^3 - 2\,abc)\log\left(|x|\right)}{a^4} + \frac{(b^4 - 4\,ab^2c + 2\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}\,a^4} + \frac{3\,a^2bx - 2\,a^3 - 6\,(ab^2 - a^2c)x^2}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 2*a*b*c)*log(c*x^2 + b*x + a)/a^4 - (b^3 - 2*a*b*c)*log(abs(x))/
a^4 + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/
(sqrt(-b^2 + 4*a*c)*a^4) + 1/6*(3*a^2*b*x - 2*a^3 - 6*(a*b^2 - a^2*c)*x^2)/
(a^4*x^3)
```

Mupad [B]
time = 2.60, size = 524, normalized size = 3.82

$$\ln\left(2\,a\,b^4\,\sqrt{b^2-4\,a\,c}-2\,b^6\,x-2\,a\,b^5+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}+11\,a^2\,b^3\,c-13\,a^3\,b\,c^2+2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}-17\,a^2\,b^2\,c^2\,x+12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2\,a^4}-\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}+\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\ln\left(2\,a\,b^5+2\,b^6\,x+2\,a\,b^4\,\sqrt{b^2-4\,a\,c}+2\,b^5\,x\,\sqrt{b^2-4\,a\,c}-11\,a^2\,b^3\,c+13\,a^3\,b\,c^2-2\,a^3\,c^3\,x+a^3\,c^2\,\sqrt{b^2-4\,a\,c}+17\,a^2\,b^2\,c^2\,x-12\,a\,b^4\,c\,x-5\,a^2\,b^2\,c\,\sqrt{b^2-4\,a\,c}-8\,a\,b^3\,c\,x\,\sqrt{b^2-4\,a\,c}+7\,a^2\,b\,c^2\,x\,\sqrt{b^2-4\,a\,c}\right)\left(\frac{b^3}{2\,a^4}+\frac{b^2\,\sqrt{b^2-4\,a\,c}}{2\,a^4}-\frac{b\,c}{a^3}-\frac{a^2\,c^2\,\sqrt{b^2-4\,a\,c}}{4\,a^5\,c-a^4\,b^2}\right)+\frac{\frac{x^2\,(a\,c-b^2)}{a^3}-\frac{1}{3\,a}+\frac{b\,x}{2\,a^2}}{x^3}+\frac{b\,\ln\left(x\right)\,\left(2\,a\,c-b^2\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)),x)
```

```
[Out] log(2*a*b^4*(b^2 - 4*a*c)^(1/2) - 2*b^6*x - 2*a*b^5 + 2*b^5*x*(b^2 - 4*a*c)
^(1/2) + 11*a^2*b^3*c - 13*a^3*b*c^2 + 2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^
(1/2) - 17*a^2*b^2*c^2*x + 12*a*b^4*c*x - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) -
 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/
(2*a^4) - (b^2*(b^2 - 4*a*c)^(1/2))/(2*a^4) - (b*c)/a^3 + (a^2*c^2*(b^2 - 4
*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + log(2*a*b^5 + 2*b^6*x + 2*a*b^4*(b^2 -
4*a*c)^(1/2) + 2*b^5*x*(b^2 - 4*a*c)^(1/2) - 11*a^2*b^3*c + 13*a^3*b*c^2 -
2*a^3*c^3*x + a^3*c^2*(b^2 - 4*a*c)^(1/2) + 17*a^2*b^2*c^2*x - 12*a*b^4*c*x
 - 5*a^2*b^2*c*(b^2 - 4*a*c)^(1/2) - 8*a*b^3*c*x*(b^2 - 4*a*c)^(1/2) + 7*a^
2*b*c^2*x*(b^2 - 4*a*c)^(1/2))*(b^3/(2*a^4) + (b^2*(b^2 - 4*a*c)^(1/2))/(2*
a^4) - (b*c)/a^3 - (a^2*c^2*(b^2 - 4*a*c)^(1/2))/(4*a^5*c - a^4*b^2)) + ((x
^2*(a*c - b^2))/a^3 - 1/(3*a) + (b*x)/(2*a^2))/x^3 + (b*log(x)*(2*a*c - b^2
))/a^4
```

3.19 $\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{2(b^2-3ac)\,x}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}-\frac{b\log}{}$$

```
[Out] 2*(-3*a*c+b^2)*x/c^2/(-4*a*c+b^2)-b*x^2/c/(-4*a*c+b^2)+x^3*(b*x+2*a)/(-4*a*
c+b^2)/(c*x^2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+
b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-b*ln(c*x^2+b*x+a)/c^3
```

Rubi [A]

time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 752, 814, 648, 632, 212, 642}

$$-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}+\frac{2x(b^2-3ac)}{c^2\,(b^2-4ac)}-\frac{bx^2}{c\,(b^2-4ac)}+\frac{x^3(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{b\log\left(a+bx+cx^2\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*(b^2 - 3*a*c)*x)/(c^2*(b^2 - 4*a*c)) - (b*x^2)/(c*(b^2 - 4*a*c)) + (x^3*
(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^
2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(3/2)) -
(b*Log[a + b*x + c*x^2])/c^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^8}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{x^4}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$
$$= \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \left(-\frac{2(b^2-3ac)}{c^2}+\frac{2bx}{c}+\frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right)dx}{-b^2+4ac}$$
$$= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2\int \frac{a(b^2-3ac)+b(b^2-4a}{a+bx+cx^2}}{c^2(b^2-4ac)}$$
$$= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{c^3} + \frac{(}{}$$
$$= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{b\log(a+bx+cx^2)}{c^3}$$
$$= \frac{2(b^2-3ac)\,x}{c^2(b^2-4ac)} - \frac{bx^2}{c(b^2-4ac)} + \frac{x^3(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2}{c^3(b}$$

Mathematica [A]
time = 0.13, size = 132, normalized size = 0.88

$$\frac{cx+\frac{-b^4x-ab^2(b-4cx)+a^2c(3b-2cx)}{(b^2-4ac)(a+x(b+cx))}-\frac{2(b^4-6ab^2c+6a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}-b\log(a+x(b+cx))}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (c*x + (-(b^4*x) - a*b^2*(b - 4*c*x) + a^2*c*(3*b - 2*c*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sq
rt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + x*(b + c*x)])/c^3
```

Maple [A]
time = 0.06, size = 198, normalized size = 1.32

method	result
default	$\frac{x}{c^2}-\frac{\frac{-\frac{(2a^2c^2-4ab^2c+b^4)x}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{cx^2+bx+a}+\frac{\frac{(4abc-b^3)\ln(cx^2+bx+a)}{c}+\frac{4\left(3a^2c-ab^2-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{c^2}$

risch	$\frac{x}{c^2}+\frac{\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{c\left(4ac-b^2\right)}-\frac{ba\left(3ac-b^2\right)}{c\left(4ac-b^2\right)}}{c^2\left(c\,x^2+bx+a\right)}-\frac{16\ln\left(-24a^3c^3+30a^2b^2c^2-10a\,b^4c+b^6-2\sqrt{-\left(4ac-b^2\right)\left(6a^2c^2-6a\,b^2c+\right.}\right.}{c\left(4ac-b^2\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x+b*a/c*(3*a*c-b^2)/
(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*ln(c*x^2+b*x+
a)+2*(3*a^2*c-a*b^2-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(146) = 292$.
time = 0.36, size = 837, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^3 - (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2
*b*c^2)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + (b^6 - 9*a*b^4*c + 26*a^2*b^
2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x)*log(c*x
^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x), -(a*b^5
 - 7*a^2*b^3*c + 12*a^3*b*c^2 - (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^3 -
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c
```

```
^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^2 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2
)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c
)) + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x + (a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2 + (b^6 - 8*a*b
^4*c + 16*a^2*b^2*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^2 + (b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(141) = 282$.
time = 1.06, size = 842, normalized size = 5.61

$$\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}-\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)\log\left(x+\frac{-10a^2bc-16a^2c^4\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)+2ab^3+8ab^2c^3\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)-b^4c^2\left(-\frac{b}{c^3}+\frac{\sqrt{-\left(4ac-b^2\right)^3}\cdot\left(6a^2c^2-6ab^2c+b^4\right)}{c^3\cdot\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}\right)}{12a^2c^2-12ab^2c+2b^4}\right)+\frac{-3a^2bc+ab^3+x\left(2a^2c^2-4ab^2c+b^4\right)}{4a^2c^4-ab^2c^3+x^2\cdot\left(4ac^5-b^2c^4\right)+x\left(4abc^4-b^3c^3\right)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**
2*b*c - 16*a**2*c**4*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a
*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**
6))) + 2*a*b**3 + 8*a*b**2*c**3*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2
*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6))) - b**4*c**2*(-b/c**3 - sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**
2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*
c - b**6))))/(12*a**2*c**2 - 12*a*b**2*c + 2*b**4)) + (-b/c**3 + sqrt(-(4*a
*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3*(64*a**3*c**3 - 48*a
**2*b**2*c**2 + 12*a*b**4*c - b**6)))*log(x + (-10*a**2*b*c - 16*a**2*c**4*
(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4)/(c**3
*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) + 2*a*b**3 + 8*a
*b**2*c**3*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c +
b**4)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))) - b**
4*c**2*(-b/c**3 + sqrt(-(4*a*c - b**2)**3)*(6*a**2*c**2 - 6*a*b**2*c + b**4
)/(c**3*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6))))/(12*a**2
*c**2 - 12*a*b**2*c + 2*b**4)) + (-3*a**2*b*c + a*b**3 + x*(2*a**2*c**2 - 4
*a*b**2*c + b**4))/(4*a**2*c**4 - a*b**2*c**3 + x**2*(4*a*c**5 - b**2*c**4)
 + x*(4*a*b*c**4 - b**3*c**3)) + x/c**2
```

Giac [A]
time = 3.23, size = 161, normalized size = 1.07

$$\frac{2\,(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3-4\,ac^4)\sqrt{-b^2+4\,ac}}+\frac{x}{c^2}-\frac{b\log\left(cx^2+bx+a\right)}{c^3}-\frac{\frac{(b^4-4\,ab^2c+2\,a^2c^2)x}{c}+\frac{ab^3-3\,a^2bc}{c}}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")

[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + x/c^2 - b*log(c*x^2 + b*x + a)/c^3 - ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*x/c + (a*b^3 - 3*a^2*b*c)/c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)

Mupad [B]
time = 2.46, size = 261, normalized size = 1.74

$$\frac{x}{c^2}+\frac{\frac{a\,\left(b^3-3\,a\,b\,c\right)}{c\,\left(4\,a\,c-b^2\right)}+\frac{x\,\left(2\,a^2\,c^2-4\,a\,b^2\,c+b^4\right)}{c\,\left(4\,a\,c-b^2\right)}}{c^3\,x^2+b\,c^2\,x+a\,c^2}+\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-128\,a^3\,b\,c^3+96\,a^2\,b^3\,c^2-24\,a\,b^5\,c+2\,b^7\right)}{2\,\left(64\,a^3\,c^6-48\,a^2\,b^2\,c^5+12\,a\,b^4\,c^4-b^6\,c^3\right)}-\frac{2\,\mathrm{atan}\left(\frac{2\,c\,x}{\sqrt{4\,a\,c-b^2}}-\frac{b^3\,c^2-4\,a\,b\,c^3}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(6\,a^2\,c^2-6\,a\,b^2\,c+b^4\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] x/c^2 + ((a*(b^3 - 3*a*b*c))/(c*(4*a*c - b^2)) + (x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/(c*(4*a*c - b^2)))/(a*c^2 + c^3*x^2 + b*c^2*x) + (log(a + b*x + c*x^2)*(2*b^7 - 128*a^3*b*c^3 + 96*a^2*b^3*c^2 - 24*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (2*atan((2*c*x)/(4*a*c - b^2)^(1/2) - (b^3*c^2 - 4*a*b*c^3)/(c^2*(4*a*c - b^2)^(3/2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^2)^(3/2))

3.20 $\int \frac{x^7}{(ax^2+bx^3+cx^4)^2} dx$

Optimal. Leaf size=114

$$-\frac{bx}{c(b^2-4ac)}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx+cx^2)}{2c^2}$$

```
[Out] -b*x/c/(-4*a*c+b^2)+x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)
*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/2*ln(c*x^2+
b*x+a)/c^2
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 752, 787, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)}-\frac{bx}{c(b^2-4ac)}+\frac{\log(a+bx+cx^2)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b*x)/(c*(b^2 - 4*a*c))) + (x^2*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*
x^2)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(c^2*(b^2
- 4*a*c)^(3/2)) + Log[a + b*x + c*x^2]/(2*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^7}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{x^3}{(a+bx+cx^2)^2}\,dx$$
$$= \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx}{-b^2+4ac}$$
$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx}{c(b^2-4ac)}$$
$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2c^2} - \frac{(b(b^2-6ac))\int \frac{}{a\text{-}}}{2c^2(b^2-4}$$
$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{\log(a+bx+cx^2)}{2c^2} + \frac{(b(b^2-6ac)}{}$$
$$= -\frac{bx}{c(b^2-4ac)} + \frac{x^2(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c^2(b^2-4ac)^{3/2}}$$

Mathematica [A]
time = 0.10, size = 109, normalized size = 0.96

$$\frac{\frac{2(-2a^2c+b^3x+ab(b-3cx))}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+x(b+cx))}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x + a*b*(b - 3*c*x)))/((b^2 - 4*a*c)*(a + x*(b + c*x)))
 + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c
)^(3/2) + Log[a + x*(b + c*x)])/(2*c^2)
```

Maple [A]
time = 0.05, size = 169, normalized size = 1.48

method	result
default	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a} + \frac{\frac{(4ac-b^2)\ln(cx^2+bx+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{c(4ac-b^2)}$
risch	$\frac{\frac{b(3ac-b^2)x}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{cx^2+bx+a} + \frac{8\ln\left(-24a^2bc^2+10ab^3c-b^5-2\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,cx-\sqrt{-b^2(4ac-b^2)(\right.}{(4ac-b^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^2+b*x+
a)+1/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^2+b*x+a)+2*(-a*b-1/2*(4*a*c-b^
2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(108) = 216.
time = 0.35, size = 635, normalized size = 5.57

$$\left[\frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + (ab^3 - 6\,a^2bc + (b^3c - 6\,abc^2)x^2 + (b^4 - 6\,ab^2c)x)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^2 + 2\,bcx + b^2 - 2\,ac + \sqrt{b^2 - 4\,ac}\,(2\,cx + b)}{cx^2 + bx + a}\right) + 2\,(b^5 - 7\,ab^3c + 12\,a^2bc^2)x + (ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x)\log\left(cx^2 + bx + a\right)}{2\,(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + (b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^2 + (b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x)}, \frac{2\,ab^4 - 12\,a^2b^2c + 16\,a^3c^2 + 2\,(ab^3 - 6\,a^2bc + (b^3c - 6\,abc^2)x^2 + (b^4 - 6\,ab^2c)x)\sqrt{-b^2 + 4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}\,(2\,cx + b)}{b^2 - 4\,ac}\right) + 2\,(b^5 - 7\,ab^3c + 12\,a^2bc^2)x + (ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^2 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x)\log\left(cx^2 + bx + a\right)}{2\,(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + (b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^2 + (b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b
^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*
log(c*x^2 + b*x + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x),
1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c -
6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*x
+ (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^
2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2 + b*x + a))/(a*b^4*c^2 -
8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2 + (b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(104) = 208$.

time = 0.76, size = 729, normalized size = 6.39

$$\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(-\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)\log\left(x+\frac{-16a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)+8a^2c+8ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)-ab^2-b^4c\left(\frac{b\sqrt{-(4ac-b^2)^3}\cdot(6ac-b^2)}{2c^2\cdot(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{2c^2}\right)}{6abc-b^3}\right)+\frac{2a^2c-ab^2+x(3abc-b^3)}{4a^2c^3-ab^2c^2+x^2\cdot(4ac^4-b^2c^3)+x(4abc^3-b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2
*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(-b*
sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(-b*sqrt(-(4*
a*c - b**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 +
12*a*b**4*c - b**6)) + 1/(2*c**2)))/(6*a*b*c - b**3)) + (b*sqrt(-(4*a*c - b
**2)**3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b*
*4*c - b**6)) + 1/(2*c**2))*log(x + (-16*a**2*c**3*(b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c
- b**6)) + 1/(2*c**2)) + 8*a**2*c + 8*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) + 1/(2*c**2)) - a*b**2 - b**4*c*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(2*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(2*c**2)))/(6*a*b*c - b**3)) + (2*a**2*c - a*b**2 + x*(3*a*b*c - b**3))/(
4*a**2*c**3 - a*b**2*c**2 + x**2*(4*a*c**4 - b**2*c**3) + x*(4*a*b*c**3 - b
**3*c**2))
```

Giac [A]

time = 3.57, size = 125, normalized size = 1.10

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}+\frac{\log\left(cx^2+bx+a\right)}{2\,c^2}+\frac{ab^2-2\,a^2c+(b^3-3\,abc)x}{(cx^2+bx+a)(b^2-4\,ac)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3
)*sqrt(-b^2 + 4*a*c)) + 1/2*log(c*x^2 + b*x + a)/c^2 + (a*b^2 - 2*a^2*c + (
b^3 - 3*a*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*c^2)
```

Mupad [B]

time = 2.49, size = 279, normalized size = 2.45

$$\frac{\frac{a\,(2\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}+\frac{b\,x\,(3\,a\,c-b^2)}{c^2\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{\ln\left(c\,x^2+b\,x+a\right)\,\left(-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6\right)}{2\,\left(64\,a^3\,c^5-48\,a^2\,b^2\,c^4+12\,a\,b^4\,c^3-b^6\,c^2\right)}+\frac{b\,\mathrm{atan}\left(\frac{c^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(\frac{2\,b\,x\,\left(6\,a\,c-b^2\right)}{c\,{\left(4\,a\,c-b^2\right)}^3}+\frac{b^2\,\left(4\,a\,c^2-b^2\,c\right)\,\left(6\,a\,c-b^2\right)}{c^3\,{\left(4\,a\,c-b^2\right)}^4}\right)}{b^3-6\,a\,b\,c}\right)\,\left(6\,a\,c-b^2\right)}{c^2\,{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^2,x)

[Out] ((a*(2*a*c - b^2))/(c^2*(4*a*c - b^2)) + (b*x*(3*a*c - b^2))/(c^2*(4*a*c - b^2)))/(a + b*x + c*x^2) - (log(a + b*x + c*x^2)*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))/(2*(64*a^3*c^5 - b^6*c^2 + 12*a*b^4*c^3 - 48*a^2*b^2*c^4)) + (b*atan((c^2*(4*a*c - b^2)^(5/2)*((2*b*x*(6*a*c - b^2))/(c*(4*a*c - b^2)^3) + (b^2*(4*a*c^2 - b^2*c)*(6*a*c - b^2))/(c^3*(4*a*c - b^2)^4)))/(b^3 - 6*a*b*c))*(6*a*c - b^2))/(c^2*(4*a*c - b^2)^(3/2))

3.21 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^2} \, dx$

Optimal. Leaf size=67

$$\frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*a*arctanh((2*c*x+b)/(-4*a*c+b^2)^(
1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1599, 736, 632, 212}

$$\frac{x(2a+bx)}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x))/((b^2 - 4*a*c)*(a + b*x + c*x^2)) + (4*a*ArcTanh[(b + 2*c*x
)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
```

```
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax^2+bx^3+cx^4)^2}\,dx &= \int \frac{x^2}{(a+bx+cx^2)^2}\,dx\\
&= \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} - \frac{(2a)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\
&= \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{(4a)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx\right)}{b^2-4ac}\\
&= \frac{x(2a+bx)}{(b^2-4ac)(a+bx+cx^2)} + \frac{4a\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 81, normalized size = 1.21

$$\frac{b^2x+a(b-2cx)}{c(-b^2+4ac)(a+x(b+cx))} + \frac{4a\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2*x + a*(b - 2*c*x))/(c*(-b^2 + 4*a*c)*(a + x*(b + c*x))) + (4*a*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.03, size = 97, normalized size = 1.45

method	result

default	$\frac{-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{4a\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{-\frac{(2ac-b^2)x}{c(4ac-b^2)}+\frac{ab}{c(4ac-b^2)}}{c\,x^2+bx+a}+\frac{2a\ln\left((-8c^2a+2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}-4abc+b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{2a\ln\left((8c^2a-2b^2c)x+(-4ac+b^2)^{\frac{3}{2}}+4abc-b^3\right)}{(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-(2*a*c-b^2)/c/(4*a*c-b^2)*x+a*b/c/(4*a*c-b^2))/(c*x^2+b*x+a)+4*a/(4*a*c-b
^2)^(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(63) = 126.
time = 0.35, size = 387, normalized size = 5.78

$$\left[-\frac{ab^3-4\,a^2bc+2\,(ac^2x^2+abcx+a^2c)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x},-\frac{ab^3-4\,a^2bc-4\,(ac^2x^2+abcx+a^2c)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+(b^4-6\,ab^2c+8\,a^2c^2)x}{ab^4c-8\,a^2b^2c^2+16\,a^3c^3+(b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^2+(b^5c-8\,ab^3c^2+16\,a^2bc^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a*b^3 - 4*a^2*b*c + 2*(a*c^2*x^2 + a*b*c*x + a^2*c)*sqrt(b^2 - 4*a*c)*lo
g((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^
2 + b*x + a)) + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 +
 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x), -(a*b^3 - 4*a^2*b*c - 4*(a*c^2*x^2 + a*b*c*x + a^2*c
)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
+ (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 +
(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(61) = 122$.
time = 0.32, size = 280, normalized size = 4.18

$$-2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{-32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+2a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x+\frac{32a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-16a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+2ab}{4ac}\right)+\frac{ab+x(-2ac+b^2)}{4a^2c^2-ab^2c+x^2\cdot(4ac^3-b^2c^2)+x(4abc^2-b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**3*c**2*sqrt(-1/(4*a*c - b**
2)**3) + 16*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - 2*a*b**4*sqrt(-1/(4*a*
c - b**2)**3) + 2*a*b)/(4*a*c)) + 2*a*sqrt(-1/(4*a*c - b**2)**3)*log(x + (3
2*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 16*a**2*b**2*c*sqrt(-1/(4*a*c - b*
*2)**3) + 2*a*b**4*sqrt(-1/(4*a*c - b**2)**3) + 2*a*b)/(4*a*c)) + (a*b + x*
(-2*a*c + b**2))/(4*a**2*c**2 - a*b**2*c + x**2*(4*a*c**3 - b**2*c**2) + x*
(4*a*b*c**2 - b**3*c))
```

Giac [A]
time = 3.69, size = 88, normalized size = 1.31

$$-\frac{4\,a\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x-2\,acx+ab}{(b^2c-4\,ac^2)(cx^2+bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (b^2*x - 2*a*c*x + a*b)/((b^2*c - 4*a*c^2)*(c*x^2 + b*x + a))
```

Mupad [B]
time = 2.13, size = 135, normalized size = 2.01

$$-\frac{\frac{x\,(2\,a\,c-b^2)}{c\,(4\,a\,c-b^2)}-\frac{a\,b}{c\,(4\,a\,c-b^2)}}{c\,x^2+b\,x+a}-\frac{4\,a\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,a\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,a}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((x*(2*a*c - b^2))/(c*(4*a*c - b^2)) - (a*b)/(c*(4*a*c - b^2)))/(a + b*x
+ c*x^2) - (4*a*atan((((2*a*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*a*c*x
)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/(2*a)))/(4*a*c - b^2)^(3/2)
```

3.22 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^2} \, dx$

Optimal. Leaf size=66

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (b*x+2*a)/(-4*a*c+b^2)/(c*x^2+b*x+a)-2*b*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1599, 652, 632, 212}

$$\frac{2a+bx}{(b^2-4ac)(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)) - (2*b*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(ax^2+bx^3+cx^4)^2}\,dx &= \int \frac{x}{(a+bx+cx^2)^2}\,dx\\ &= \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}+\frac{b\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{(2b)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= \frac{2a+bx}{(b^2-4ac)\,(a+bx+cx^2)}-\frac{2b\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 69, normalized size = 1.05

$$\frac{2a+bx}{(b^2-4ac)\,(a+x(b+cx))}-\frac{2b\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^2,x]
[Out] (2*a + b*x)/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*b*ArcTan[(b + 2*c*x)/Sqr
t[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.03, size = 70, normalized size = 1.06

method	result	size
default	$\frac{-bx-2a}{(4ac-b^2)(c\,x^2+bx+a)}-\frac{2b\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$	70

risch	$\frac{-\frac{bx}{4ac-b^2}-\frac{2a}{4ac-b^2}}{cx^2+bx+a}+\frac{b\ln\left(\left(-8c^2a+2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(8c^2a-2b^2c\right)x-\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	148

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-b*x-2*a)/(4*a*c-b^2)/(c*x^2+b*x+a)-2*b/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.
time = 0.36, size = 338, normalized size = 5.12

$$\left[\frac{2\,ab^2-8\,a^2c-\left(bcx^2+b^2x+ab\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x},\ \frac{2\,ab^2-8\,a^2c-2\,\left(bcx^2+b^2x+ab\right)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+\left(b^3-4\,abc\right)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2+\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [(2*a*b^2 - 8*a^2*c - (b*c*x^2 + b^2*x + a*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
 a)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), (2*a*b^2 -
 8*a^2*c - 2*(b*c*x^2 + b^2*x + a*b)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + (b^3 - 4*a*b*c)*x)/(a*b^4 - 8*a^2*b^2*
c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(60) = 120.
time = 0.34, size = 253, normalized size = 3.83

$$b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)-b\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{16a^2bc^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-8ab^3c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^5\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+b^2}{2bc}\right)+\frac{-2a-bx}{4a^2c-ab^2+x^2\cdot\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2
)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b**2)
**3) + b**2)/(2*b*c)) - b*sqrt(-1/(4*a*c - b**2)**3)*log(x + (16*a**2*b*c**
2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c)) + (-2*a - b*x)/(4*a**2*c - a*b
**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A]
time = 4.78, size = 76, normalized size = 1.15

$$\frac{2\,b\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx+2\,a}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + (b*x + 2*a)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 2.18, size = 110, normalized size = 1.67

$$-\frac{\frac{2\,a}{4\,a\,c-b^2}+\frac{b\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{2\,b\,\mathrm{atan}\left(\frac{\left(\frac{b^2}{{\left(4\,a\,c-b^2\right)}^{3/2}}+\frac{2\,b\,c\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{b}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] - ((2*a)/(4*a*c - b^2) + (b*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (2*b*atan
(((b^2/(4*a*c - b^2)^(3/2) + (2*b*c*x)/(4*a*c - b^2)^(3/2))*(4*a*c - b^2))/
b))/(4*a*c - b^2)^(3/2)
```

3.23 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^2} \, dx$

Optimal. Leaf size=66

$$-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}+\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (-2*c*x-b)/(-4*a*c+b^2)/(c*x^2+b*x+a)+4*c*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]

time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1599, 628, 632, 212}

$$\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*c*ArcTanh[(b + 2*c*x)
/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3+cx^4\right)^2}\,dx &= \int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}-\frac{(2c)\int \frac{1}{a+bx+cx^2}\,dx}{b^2-4ac}\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{(4c)\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{b^2-4ac}\\ &= -\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{4c\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 70, normalized size = 1.06

$$-\frac{\frac{b+2cx}{a+x(b+cx)}+\frac{4c\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{b^2-4ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -(((b + 2*c*x)/(a + x*(b + c*x)) + (4*c*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c))
```

Maple [A]
time = 0.03, size = 68, normalized size = 1.03

method	result	size
default	$\frac{2cx+b}{\left(4ac-b^2\right)\left(c\,x^2+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}$	68

risch	$\frac{\frac{2cx}{4ac-b^2}+\frac{b}{4ac-b^2}}{cx^2+bx+a}+\frac{2c\ln\left(\left(-8c^2a+2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}-4abc+b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{2c\ln\left(\left(8c^2a-2b^2c\right)x+\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)}{\left(-4ac+b^2\right)^{\frac{3}{2}}}$	144

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2)*arctan((2*c*x+b)/
(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(62) = 124$.
time = 0.36, size = 341, normalized size = 5.17

$$\left[-\frac{b^3-4\,abc+2\,(c^2x^2+bcx+ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x},\ -\frac{b^3-4\,abc-4\,(c^2x^2+bcx+ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(b^2c-4\,ac^2)x}{ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(b^3 - 4*a*b*c + 2*(c^2*x^2 + b*c*x + a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x
^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x +
a)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8
*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x), -(b^3 -
 4*a*b*c - 4*(c^2*x^2 + b*c*x + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 +
 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*x)/(a*b^4 - 8*a^2*
b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^
3*c + 16*a^2*b*c^2)*x)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(61) = 122$.
time = 0.32, size = 265, normalized size = 4.02

$$-2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{-32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\log\left(x+\frac{32a^2c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}-16ab^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2b^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}+2bc}{4c^2}\right)+\frac{b+2cx}{4a^2c-ab^2+x^2\cdot\left(4ac^2-b^2c\right)+x\left(4abc-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] -2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (-32*a**2*c**3*sqrt(-1/(4*a*c - b**
2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - 2*b**4*c*sqrt(-1/(4*a*
c - b**2)**3) + 2*b*c)/(4*c**2)) + 2*c*sqrt(-1/(4*a*c - b**2)**3)*log(x + (
32*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 2*b**4*c*sqrt(-1/(4*a*c - b**2)**3) + 2*b*c)/(4*c**2)) + (b + 2*
c*x)/(4*a**2*c - a*b**2 + x**2*(4*a*c**2 - b**2*c) + x*(4*a*b*c - b**3))
```

Giac [A]
time = 3.21, size = 76, normalized size = 1.15

$$-\frac{4\,c\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx+b}{(cx^2+bx+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*c*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*
c)) - (2*c*x + b)/((c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 0.08, size = 119, normalized size = 1.80

$$\frac{\frac{b}{4\,a\,c-b^2}+\frac{2\,c\,x}{4\,a\,c-b^2}}{c\,x^2+b\,x+a}-\frac{4\,c\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{4\,c^2\,x}{{\left(4\,a\,c-b^2\right)}^{3/2}}\right)\,\left(4\,a\,c-b^2\right)}{2\,c}\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (b/(4*a*c - b^2) + (2*c*x)/(4*a*c - b^2))/(a + b*x + c*x^2) - (4*c*atan((((
2*c*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(5/2) - (4*c^2*x)/(4*a*c - b^2)^(3/2))*(
4*a*c - b^2))/(2*c)))/(4*a*c - b^2)^(3/2)
```

3.24 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=108

$$\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}$$

```
[Out] (b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^2+b*x+a)+b*(-6*a*c+b^2)*arctanh((2*c*
x+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/2*ln(c*x^2+b*x+
a)/a^2
```

Rubi [A]
time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$,
Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx+cx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx}{a\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*(a + b*x + c*x^2)) + (b*(b^2 - 6*a*c
)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^2*(b^2 - 4*a*c)^(3/2)) + Log[x
]/a^2 - Log[a + b*x + c*x^2]/(2*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{1}{x\,(a+bx+cx^2)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx}{a\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} - \frac{\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)dx}{a\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx}{a^2\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2a^2} - \frac{(b(b^2-6ac))\int \frac{1}{a+bx+cx}}{2a^2\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a+bx+cx^2)}{2a^2} + \frac{(b(b^2-6ac))\,\mathrm{Subs}}{a}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,(a+bx+cx^2)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^2\,(b^2-4ac)^{3/2}} + \frac{\log(x)}{a^2} - \mathrm{lo}$$

Mathematica [A]
time = 0.13, size = 107, normalized size = 0.99

$$\frac{\frac{2a(b^2-2ac+bcx)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2\log(x) - \log(a+x(b+cx))}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(b^2
- 6*a*c)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*L
og[x] - Log[a + x*(b + c*x)])/(2*a^2)
```

Maple [A]
time = 0.05, size = 177, normalized size = 1.64

method	result	size
default	$-\frac{\frac{xabc}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(4c^2a-b^2c)\ln(c\,x^2+bx+a)}{2c}+\frac{2\left(5abc-b^3-\frac{(4c^2a-b^2c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^2}+\frac{\ln(x)}{a^2}$	177

risch	Expression too large to display	2292

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/(4*a*c-b^2)*x*a*b*c-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(
4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^2+b*x+a)+2*(5*a*b*c-b^3-1/2*(4*a*c
^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))+ln(x
)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.
time = 0.40, size = 781, normalized size = 7.23

$$\left[\frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^2+2\,bcx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,cx+b)}{cx^2+bx+a}\right)+2\,(ab^3c-4\,a^2bc^2)x-(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)+2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(x\right)}{2\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2+(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3)x^2+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x)}, \frac{2\,ab^4-12\,a^2b^2c+16\,a^3c^2+2\,(ab^3-6\,a^2bc+(b^3c-6\,abc^2)x^2+(b^4-6\,ab^2c)x)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,cx+b)}{b^2-4\,ac}\right)+2\,(ab^3c-4\,a^2bc^2)x-(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(cx^2+bx+a\right)+2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^2+(b^5-8\,ab^3c+16\,a^2bc^2)x)\log\left(x\right)}{2\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2+(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3)x^2+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + (a*b^3 - 6*a^2*b*c + (b^3*c - 6
*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b
*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) + 2*
(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*
a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(c*x^2
 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 +
16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4 - 8*
a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2 + (a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x), 1/2*(2*a*b^4 - 12*a^2*b^2*c + 16*a^
3*c^2 + 2*(a*b^3 - 6*a^2*b*c + (b^3*c - 6*a*b*c^2)*x^2 + (b^4 - 6*a*b^2*c)*
x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c))
 + 2*(a*b^3*c - 4*a^2*b*c^2)*x - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(
```

```
c*x^2 + b*x + a) + 2*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)*x^2 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x)*log(x))/(a^3*b^4
 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^2
+ (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.70, size = 126, normalized size = 1.17

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}}-\frac{\log\left(cx^2+bx+a\right)}{2\,a^2}+\frac{\log\left(|x|\right)}{a^2}+\frac{abcx+ab^2-2\,a^2c}{(cx^2+bx+a)(b^2-4\,ac)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(b^3 - 6*a*b*c)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c
)*sqrt(-b^2 + 4*a*c)) - 1/2*log(c*x^2 + b*x + a)/a^2 + log(abs(x))/a^2 + (a
*b*c*x + a*b^2 - 2*a^2*c)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^2)
```

Mupad [**B**]
time = 2.87, size = 620, normalized size = 5.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(a*(4*a*c - b^2)) - (b*c*x)/(a*(4*a*c - b^2)))/
(a + b*x + c*x^2) + (log(2*a*b^6 + 2*b^7*x - 96*a^4*c^3 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) - 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) + 84*a^3*
b^2*c^2 + 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) - 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 + b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c - 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3) + (log(96*a^4*c^3 - 2*b^7*x - 2*a*b^6 + 2*a*b^3*(-(4*a*c
- b^2)^3)^(1/2) + 23*a^2*b^4*c + 2*b^4*x*(-(4*a*c - b^2)^3)^(1/2) - 84*a^3*
```

```
b^2*c^2 - 94*a^2*b^3*c^2*x + 12*a^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2) + 24*a*b
^5*c*x - 9*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2) + 120*a^3*b*c^3*x - 12*a*b^2*c*
x*(-(4*a*c - b^2)^3)^(1/2))*(b^6 - 64*a^3*c^3 - b^3*(-(4*a*c - b^2)^3)^(1/2
) + 48*a^2*b^2*c^2 - 12*a*b^4*c + 6*a*b*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^2
*(4*a*c - b^2)^3)
```

3.25 $\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=148

$$-\frac{2(b^2-3ac)}{a^2(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x(a+bx+cx^2)}-\frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}-\frac{2b\log(x)}{a^3}+$$

```
[Out] -2*(-3*a*c+b^2)/a^2/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^
2+b*x+a)-2*(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+b*ln(c*x^2+b*x+a)/a^3
```

Rubi [A]
time = 0.13, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\frac{b\log(a+bx+cx^2)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{2(b^2-3ac)}{a^2x(b^2-4ac)}-\frac{2(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-2*(b^2 - 3*a*c))/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 -
4*a*c)*x*(a + b*x + c*x^2)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTanh[(b +
 2*c*x)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 +
(b*Log[a + b*x + c*x^2])/a^3
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^2}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{1}{x^2\,(a+bx+cx^2)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{\int \frac{-2(b^2-3ac)-2bcx}{x^2(a+bx+cx^2)}\,dx}{a\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{\int \left(\frac{2(-b^2+3ac)}{ax^2} - \frac{2b(-b^2+4ac)}{a^2x} + \frac{2(-b^4+5ab^2c-3a^2c^2-b}{a^2(a+bx+cx}\right.}{a\,(b^2-4ac)}$$

$$= -\frac{2(b^2-3ac)}{a^2\,(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} - \frac{2\int \frac{-b^4+5ab^2c-3a}{a+b}}{a^3\,(b^2}$$

$$= -\frac{2(b^2-3ac)}{a^2\,(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\int \frac{b+2cx}{a+bx+cx^2}\,dx}{a^3}$$

$$= -\frac{2(b^2-3ac)}{a^2\,(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx+}{a^3}$$

$$= -\frac{2(b^2-3ac)}{a^2\,(b^2-4ac)\,x} + \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x\,(a+bx+cx^2)} - \frac{2(b^4-6ab^2c+6a^2c^2)\tanh^{}}{a^3\,(b^2-4ac)}$$

Mathematica [A]
time = 0.18, size = 131, normalized size = 0.89

$$-\frac{\frac{a}{x} + \frac{a(b^3-3abc+b^2cx-2ac^2x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2(b^4-6ab^2c+6a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2b\log(x) - b\log(a+x(b+cx))}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -((a/x + (a*(b^3 - 3*a*b*c + b^2*c*x - 2*a*c^2*x))/((b^2 - 4*a*c)*(a + x*(b
 + c*x))) + (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2*b*Log[x] - b*Log[a + x*(b + c*x)])/a^3)
```

Maple [A]
time = 0.06, size = 205, normalized size = 1.39

method	result
default	$-\frac{\frac{\frac{ac(2ac-b^2)x}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(-4ab\,c^2+b^3c)\ln(c\,x^2+bx+a)}{c}+\frac{4\left(3a^2c^2-5a\,b^2c+b^4-\frac{(-4ab\,c^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^3} - \frac{}{a}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*x^2+
b*x+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^2+b*x+a)+2*(3*a^2*c^2
-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+
b)/(4*a*c-b^2)^(1/2))))-1/a^2/x-2*b*ln(x)/a^3
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(144) = 288.
time = 0.45, size = 975, normalized size = 6.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3
*c^3)*x^2 + ((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^3 + (b^5 - 6*a*b^3*c + 6*a
^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x)*sqrt(b^2 - 4*a*c)*log(
(2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2
+ b*x + a)) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3
*c^2 + 16*a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5
- 8*a^2*b^3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a
^5*c^3)*x^3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5
*b^2*c + 16*a^6*c^2)*x), -(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c
- 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^
3 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)
```

```
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)
) + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x - ((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(c*x^2 + b*x + a) + 2*((b^5*c - 8*a*b^3*c^2 + 16*
a^2*b*c^3)*x^3 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2 + (a*b^5 - 8*a^2*b^
3*c + 16*a^3*b*c^2)*x)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^
3 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2 + (a^4*b^4 - 8*a^5*b^2*c + 1
6*a^6*c^2)*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.59, size = 171, normalized size = 1.16

$$\frac{2\left(b^{4}-6\,ab^{2}c+6\,a^{2}c^{2}\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^{2}+4\,ac}}\right)}{\left(a^{3}b^{2}-4\,a^{4}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{2\,b^{2}cx^{2}-6\,ac^{2}x^{2}+2\,b^{3}x-7\,abcx+ab^{2}-4\,a^{2}c}{\left(a^{2}b^{2}-4\,a^{3}c\right)\left(cx^{3}+bx^{2}+ax\right)}+\frac{b\log\left(cx^{2}+bx+a\right)}{a^{3}}-\frac{2\,b\log\left(|x|\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - (2*b^2*c*x^2 - 6*a*c^2*x^2 + 2*b^3*x
 - 7*a*b*c*x + a*b^2 - 4*a^2*c)/((a^2*b^2 - 4*a^3*c)*(c*x^3 + b*x^2 + a*x))
 + b*log(c*x^2 + b*x + a)/a^3 - 2*b*log(abs(x))/a^3
```

Mupad [**B**]
time = 2.83, size = 775, normalized size = 5.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] log(2*a*b^7 + 2*b^8*x + 2*a*b^4*(-(4*a*c - b^2)^3)^(1/2) - 23*a^2*b^5*c - 1
08*a^4*b*c^3 + 24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) + 87*a^3*b^3
*c^2 + 3*a^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^
(1/2) + 97*a^2*b^4*c^2*x - 138*a^3*b^2*c^3*x - 24*a*b^6*c*x - 12*a*b^3*c*x*
(-(4*a*c - b^2)^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(
```

```
-(4*a*c - b^2)^3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-
(4*a*c - b^2)^3)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) + b/a^3) - (1/a - (x*(2*b^3 - 7*a*b*c))/(a^2*(4*a*c - b^2)) + (2*c*x^2*
(3*a*c - b^2))/(a^2*(4*a*c - b^2)))/(a*x + b*x^2 + c*x^3) - log(2*a*b^4*(-(
4*a*c - b^2)^3)^(1/2) - 2*b^8*x - 2*a*b^7 + 23*a^2*b^5*c + 108*a^4*b*c^3 -
24*a^4*c^4*x + 2*b^5*x*(-(4*a*c - b^2)^3)^(1/2) - 87*a^3*b^3*c^2 + 3*a^3*c^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a^2*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 97*a^2*
b^4*c^2*x + 138*a^3*b^2*c^3*x + 24*a*b^6*c*x - 12*a*b^3*c*x*(-(4*a*c - b^2)
^3)^(1/2) + 15*a^2*b*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*((b^4*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - b/a^3) -
(2*b*log(x))/a^3
```

3.26 $\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx$

Optimal. Leaf size=202

$$-\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2}+\frac{b(3b^2-11ac)}{a^3(b^2-4ac)x}+\frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)}+\frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\frac{\quad}{\sqrt{b}}\right.}{a^4(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^2+b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^2)/x
+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^2+b*x+a)+b*(30*a^2*c^2-20*a*b^2*
c+3*b^4)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(-2*a
*c+3*b^2)*ln(x)/a^4-1/2*(-2*a*c+3*b^2)*ln(c*x^2+b*x+a)/a^4
```

Rubi [A]
time = 0.18, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$-\frac{(3b^2-2ac)\log(a+bx+cx^2)}{2a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{a^3x(b^2-4ac)}-\frac{3b^2-8ac}{2a^2x^2(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^2(b^2-4ac)(a+bx+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] -1/2*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^2) + (b*(3*b^2 - 11*a*c))/(a^3*(b
^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^2*(a + b*x + c*x^
2)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4
*a*c]])/(a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/a^4 - ((3*b^2
- 2*a*c)*Log[a + b*x + c*x^2])/(2*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{1}{x^3(a+bx+cx^2)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} - \frac{\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx}{a(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} - \frac{\int \left(\frac{-3b^2+8ac}{ax^3} + \frac{3b^3-11abc}{a^2x^2} + \frac{(b^2-4ac)(-3b^2+2ac)}{a^3x} + \frac{b}{}\right.}{a(b^2-4a}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2 -}{}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2 -}{}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{(3b^2 -}{}$$

$$= -\frac{3b^2-8ac}{2a^2(b^2-4ac)x^2} + \frac{b(3b^2-11ac)}{a^3(b^2-4ac)x} + \frac{b^2-2ac+bcx}{a(b^2-4ac)x^2(a+bx+cx^2)} + \frac{b(3b^4 -}{}$$

Mathematica [A]

time = 0.23, size = 175, normalized size = 0.87

$$\frac{-\frac{a^2}{x^2} + \frac{4ab}{x} + \frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx-3abc^2x)}{(b^2-4ac)(a+x(b+cx))} + \frac{2b(3b^4-20ab^2c+30a^2c^2)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + 2(3b^2-2ac)\log(x) + (-3b^2+2ac)\log(a+x(b+cx))}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^2,x]
```

```
[Out] (-(a^2/x^2) + (4*a*b)/x + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x - 3*a
*b*c^2*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*b*(3*b^4 - 20*a*b^2*c + 3
0*a^2*c^2)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 2
*(3*b^2 - 2*a*c)*Log[x] + (-3*b^2 + 2*a*c)*Log[a + x*(b + c*x)])/(2*a^4)
```

Maple [A]

time = 0.06, size = 255, normalized size = 1.26

method	result
default	$\frac{\frac{\frac{acb(3ac-b^2)x}{4ac-b^2} - \frac{a(2a^2c^2-4ab^2c+b^4)}{4ac-b^2}}{cx^2+bx+a} + \frac{\frac{(8a^2c^3-14ab^2c^2+3b^4c)\ln(cx^2+bx+a)}{2c} + \frac{2\left(19a^2bc^2-17ab^3c+3b^5-\frac{(8a^2c^3-14ab^2c^2+3b^4c)b}{2c}\right)\text{arct}}{\sqrt{4ac-b^2}}}{4ac-b^2}}{a^4}$

risch	$\frac{\frac{bc(11ac-3b^2)x^3}{a^3(4ac-b^2)}-\frac{(8a^2c^2-25a\,b^2c+6b^4)x^2}{2a^3(4ac-b^2)}+\frac{3bx}{2a^2}-\frac{1}{2a}}{x^2(c\,x^2+bx+a)}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\left(\sum_{\_R=\mathrm{RootOf}((64a^7c^3-48a^6b^2c^2+12a^5b^4c-a^4b^6)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-
b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*c)/c*l
n(c*x^2+b*x+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a*b^2*c^2
+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))-1/2/
a^2/x^2+(-2*a*c+3*b^2)*ln(x)/a^4+2*b/a^3/x
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. $2(194) = 388$.
time = 0.52, size = 1226, normalized size = 6.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
)*x^2 + ((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a*b^4*c
+ 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)*sqrt(b
^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c
*x + b))/(c*x^2 + b*x + a)) - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x +
((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a
*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*
a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^
4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3
```

```
*c^2 - 32*a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^
5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
x^2), -1/2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*
c^2 + 44*a^3*b*c^3)*x^3 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^
4*c^3)*x^2 - 2*((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^4 + (3*b^6 - 20*a
*b^4*c + 30*a^2*b^2*c^2)*x^3 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^2)
*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) -
 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x + ((3*b^6*c - 26*a*b^4*c^2 + 64
*a^2*b^2*c^3 - 32*a^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*
a^3*b*c^3)*x^3 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2
)*log(c*x^2 + b*x + a) - 2*((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a
^3*c^4)*x^4 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^3 + (3
*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^2)*log(x))/((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^4 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.06, size = 229, normalized size = 1.13

$$-\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}}-\frac{(3\,b^2-2\,ac)\log\left(cx^2+bx+a\right)}{2\,a^4}+\frac{(3\,b^2-2\,ac)\log\left(|x|\right)}{a^4}-\frac{a^3b^2-4\,a^4c-2\,(3\,ab^3c-11\,a^2bc^2)x^3-(6\,ab^4-25\,a^2b^2c+8\,a^3c^2)x^2-3\,(a^2b^3-4\,a^3bc)x}{2\,(cx^2+bx+a)(b^2-4\,ac)a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))
/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(3*b^2 - 2*a*c)*log(c*x^2 +
 b*x + a)/a^4 + (3*b^2 - 2*a*c)*log(abs(x))/a^4 - 1/2*(a^3*b^2 - 4*a^4*c -
2*(3*a*b^3*c - 11*a^2*b*c^2)*x^3 - (6*a*b^4 - 25*a^2*b^2*c + 8*a^3*c^2)*x^2
 - 3*(a^2*b^3 - 4*a^3*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^4*x^2)
```

Mupad [**B**]
time = 2.96, size = 914, normalized size = 4.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 - 6*a*b^5*(-(4*a*c - b^2)^3)^(1/2) - 7
3*a^2*b^6*c - 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b^4*c^2 - 492*a^4*
b^2*c^3 + 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) - 27*a^3*b*c^2*(-(4*a*c - b
^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x + 24*a^3*c^3*x*(-(4*a*
c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x + 40*a*b^4*c*x*(-(4*a*c
- b^2)^3)^(1/2) - 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(3*b^8 + 128*a
^4*c^4 - 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2 - 288*a^3*b^2*c^3
 - 38*a*b^6*c - 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) + 20*a*b^3*c*(-(4*a*c
 - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3) - (log(x)*(2*a*c - 3*b^2))/a^4 -
 (1/(2*a) - (3*b*x)/(2*a^2) + (x^2*(6*b^4 + 8*a^2*c^2 - 25*a*b^2*c))/(2*a^3
*(4*a*c - b^2)) - (b*c*x^3*(11*a*c - 3*b^2))/(a^3*(4*a*c - b^2)))/(a*x^2 +
b*x^3 + c*x^4) + (log(6*a*b^8 + 6*b^9*x + 192*a^5*c^4 + 6*a*b^5*(-(4*a*c -
b^2)^3)^(1/2) - 73*a^2*b^6*c + 6*b^6*x*(-(4*a*c - b^2)^3)^(1/2) + 307*a^3*b
^4*c^2 - 492*a^4*b^2*c^3 - 31*a^2*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 27*a^3*b
*c^2*(-(4*a*c - b^2)^3)^(1/2) + 339*a^2*b^5*c^2*x - 602*a^3*b^3*c^3*x - 24*
a^3*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 76*a*b^7*c*x + 312*a^4*b*c^4*x - 40*a*
b^4*c*x*(-(4*a*c - b^2)^3)^(1/2) + 69*a^2*b^2*c^2*x*(-(4*a*c - b^2)^3)^(1/2
))*(3*b^8 + 128*a^4*c^4 + 3*b^5*(-(4*a*c - b^2)^3)^(1/2) + 168*a^2*b^4*c^2
- 288*a^3*b^2*c^3 - 38*a*b^6*c + 30*a^2*b*c^2*(-(4*a*c - b^2)^3)^(1/2) - 20
*a*b^3*c*(-(4*a*c - b^2)^3)^(1/2)))/(2*a^4*(4*a*c - b^2)^3)
```

3.27 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=252

$$-\frac{2(2b^2-5ac)}{3a^2\left(b^2-4ac\right)x^3}+\frac{b(2b^2-7ac)}{a^3\left(b^2-4ac\right)x^2}-\frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^3\left(a+bx+cx^2\right)}-\frac{2(2b^6-15a}{}$$

```
[Out] -2/3*(-5*a*c+2*b^2)/a^2/(-4*a*c+b^2)/x^3+b*(-7*a*c+2*b^2)/a^3/(-4*a*c+b^2)/
x^2-2*(5*a^2*c^2-9*a*b^2*c+2*b^4)/a^4/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-
4*a*c+b^2)/x^3/(c*x^2+b*x+a)-2*(-10*a^3*c^3+30*a^2*b^2*c^2-15*a*b^4*c+2*b^6
)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/a^5/(-4*a*c+b^2)^(3/2)-2*b*(-3*a*c+
2*b^2)*ln(x)/a^5+b*(-3*a*c+2*b^2)*ln(c*x^2+b*x+a)/a^5
```

Rubi [A]
time = 0.22, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {1608, 754, 814, 648, 632, 212, 642}

$$\frac{b(2b^2-3ac)\log\left(a+bx+cx^2\right)}{a^5}-\frac{2b\log(x)\left(2b^2-3ac\right)}{a^5}+\frac{b(2b^2-7ac)}{a^3x^2\left(b^2-4ac\right)}-\frac{2(2b^2-5ac)}{3a^2x^3\left(b^2-4ac\right)}-\frac{2(5a^2c^2-9ab^2c+2b^4)}{a^4x\left(b^2-4ac\right)}-\frac{2(-10a^3c^3+30a^2b^2c^2-15ab^4c+2b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^5\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^3\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-2*(2*b^2 - 5*a*c))/(3*a^2*(b^2 - 4*a*c)*x^3) + (b*(2*b^2 - 7*a*c))/(a^3*(
b^2 - 4*a*c)*x^2) - (2*(2*b^4 - 9*a*b^2*c + 5*a^2*c^2))/(a^4*(b^2 - 4*a*c)*
x) + (b^2 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^3*(a + b*x + c*x^2)) - (2*(2*
b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^
2 - 4*a*c]])/(a^5*(b^2 - 4*a*c)^(3/2)) - (2*b*(2*b^2 - 3*a*c)*Log[x])/a^5 +
 (b*(2*b^2 - 3*a*c)*Log[a + b*x + c*x^2])/a^5
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{(ax^2+bx^3+cx^4)^2}\,dx = \int \frac{1}{x^4\,(a+bx+cx^2)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x^3\,(a+bx+cx^2)} - \frac{\int \frac{-2(2b^2-5ac)-4bcx}{x^4(a+bx+cx^2)}\,dx}{a\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx}{a\,(b^2-4ac)\,x^3\,(a+bx+cx^2)} - \frac{\int \left(\frac{2(-2b^2+5ac)}{ax^4} - \frac{2(-2b^3+7abc)}{a^2x^3} - \frac{2(2b^4-9ab^2c+5a^2c^2}{a^3x^2}\right.}{}$$

$$= -\frac{2(2b^2-5ac)}{3a^2\,(b^2-4ac)\,x^3} + \frac{b(2b^2-7ac)}{a^3\,(b^2-4ac)\,x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\,(b^2-4ac)\,x} + \frac{b^2-}{a\,(b^2-4ac)}$$

$$= -\frac{2(2b^2-5ac)}{3a^2\,(b^2-4ac)\,x^3} + \frac{b(2b^2-7ac)}{a^3\,(b^2-4ac)\,x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\,(b^2-4ac)\,x} + \frac{b^2-}{a\,(b^2-4ac)}$$

$$= -\frac{2(2b^2-5ac)}{3a^2\,(b^2-4ac)\,x^3} + \frac{b(2b^2-7ac)}{a^3\,(b^2-4ac)\,x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\,(b^2-4ac)\,x} + \frac{b^2-}{a\,(b^2-4ac)}$$

$$= -\frac{2(2b^2-5ac)}{3a^2\,(b^2-4ac)\,x^3} + \frac{b(2b^2-7ac)}{a^3\,(b^2-4ac)\,x^2} - \frac{2(2b^4-9ab^2c+5a^2c^2)}{a^4\,(b^2-4ac)\,x} + \frac{b^2-}{a\,(b^2-4ac)}$$

<u>Mathematica</u> [A]
time = 0.21, size = 218, normalized size = 0.87

$$\frac{-\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3a(-3b^2+2ac)}{x}-\frac{3a(b^5-5ab^3c+5a^2bc^2+b^4cx-4ab^2c^2x+2a^2c^3x)}{(b^2-4ac)(a+x(b+cx))}-\frac{6(2b^6-15ab^4c+30a^2b^2c^2-10a^3c^3)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+6(-2b^3+3abc)\log(x)+3(2b^3-3abc)\log(a+x(b+cx))}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-2),x]
```

```
[Out] (-(a^3/x^3) + (3*a^2*b)/x^2 + (3*a*(-3*b^2 + 2*a*c))/x - (3*a*(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + b^4*c*x - 4*a*b^2*c^2*x + 2*a^2*c^3*x))/((b^2 - 4*a*c)*
(a + x*(b + c*x))) - (6*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*
ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 6*(-2*b^3 +
3*a*b*c)*Log[x] + 3*(2*b^3 - 3*a*b*c)*Log[a + x*(b + c*x)])/(3*a^5)
```

<u>Maple</u> [A]
time = 0.06, size = 295, normalized size = 1.17

method	result

default	$\frac{\frac{\frac{ac(2a^2c^2-4ab^2c+b^4)x}{4ac-b^2}+\frac{ab(5a^2c^2-5ab^2c+b^4)}{4ac-b^2}}{cx^2+bx+a}+\frac{\frac{(-12a^2bc^3+11ab^3c^2-2b^5c)\ln(cx^2+bx+a)}{c}+\frac{4\left(5a^3c^3-21a^2b^2c^2+13ab^4c-2b^6-\frac{(-12a^2bc^3}{}\right.}{\sqrt{4ac}}}{4ac-b^2}}{a^5}$
risch	$\frac{\frac{2c(5a^2c^2-9ab^2c+2b^4)x^4}{(4ac-b^2)a^4}+\frac{b(17a^2c^2-20ab^2c+4b^4)x^3}{a^4(4ac-b^2)}+\frac{(5ac-6b^2)x^2}{3a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(cx^2+bx+a)}+\frac{6b\ln(x)c}{a^4}-\frac{4b^3\ln(x)}{a^5}+2\left(\sum_{\_R=\text{RootOf}((64c^3a^8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*((a*c*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*a*c-b^2)*x+a*b*(5*a^2*c^2-5*a*b^2*
c+b^4)/(4*a*c-b^2))/(c*x^2+b*x+a)+2/(4*a*c-b^2)*(1/2*(-12*a^2*b*c^3+11*a*b^
3*c^2-2*b^5*c)/c*ln(c*x^2+b*x+a)+2*(5*a^3*c^3-21*a^2*b^2*c^2+13*a*b^4*c-2*b
^6-1/2*(-12*a^2*b*c^3+11*a*b^3*c^2-2*b^5*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((
2*c*x+b)/(4*a*c-b^2)^(1/2))))-1/3/a^2/x^3-(-2*a*c+3*b^2)/a^4/x+b/a^3/x^2+2*
b*(3*a*c-2*b^2)/a^5*ln(x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. $2(246) = 492$.
time = 0.62, size = 1407, normalized size = 5.58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 +
 41*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*
c^2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*
a^5*c^3)*x^2 - 3*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^
```

```
5 + (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 1
5*a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^
2*x^2 + 2*b*c*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*c*x + b))/(c*x^2 + b*x
 + a)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^
5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b
^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 4
8*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a) + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^
2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a
^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*
x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a
^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3), -
1/3*(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 6*(2*a*b^6*c - 17*a^2*b^4*c^2 + 4
1*a^3*b^2*c^3 - 20*a^4*c^4)*x^4 + 3*(4*a*b^7 - 36*a^2*b^5*c + 97*a^3*b^3*c^
2 - 68*a^4*b*c^3)*x^3 + (6*a^2*b^6 - 53*a^3*b^4*c + 136*a^4*b^2*c^2 - 80*a^
5*c^3)*x^2 + 6*((2*b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 10*a^3*c^4)*x^5
+ (2*b^7 - 15*a*b^5*c + 30*a^2*b^3*c^2 - 10*a^3*b*c^3)*x^4 + (2*a*b^6 - 15*
a^2*b^4*c + 30*a^3*b^2*c^2 - 10*a^4*c^3)*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 2*(a^3*b^5 - 8*a^4*b^3*c + 16
*a^5*b*c^2)*x - 3*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)
*x^5 + (2*b^8 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^
7 - 19*a^2*b^5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(c*x^2 + b*x + a)
 + 6*((2*b^7*c - 19*a*b^5*c^2 + 56*a^2*b^3*c^3 - 48*a^3*b*c^4)*x^5 + (2*b^8
 - 19*a*b^6*c + 56*a^2*b^4*c^2 - 48*a^3*b^2*c^3)*x^4 + (2*a*b^7 - 19*a^2*b^
5*c + 56*a^3*b^3*c^2 - 48*a^4*b*c^3)*x^3)*log(x))/((a^5*b^4*c - 8*a^6*b^2*c
^2 + 16*a^7*c^3)*x^5 + (a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^
4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.16, size = 282, normalized size = 1.12

$$\frac{2\,(2\,b^6-15\,ab^4c+30\,a^2b^2c^2-10\,a^3c^3)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^5b^2-4\,a^6c)\sqrt{-b^2+4\,ac}}+\frac{(2\,b^3-3\,abc)\log\left(cx^2+bx+a\right)}{a^5}-\frac{2\,(2\,b^3-3\,abc)\log\left(|x|\right)}{a^5}-\frac{a^4b^2-4\,a^5c+6\,(2\,ab^4c-9\,a^2b^2c^2+5\,a^3c^3)x^4+3\,(4\,ab^5-20\,a^2b^3c+17\,a^3bc^2)x^3+(6\,a^2b^4-29\,a^3b^2c+20\,a^4c^2)x^2-2\,(a^3b^3-4\,a^4bc)x}{3\,(cx^2+bx+a)(b^2-4\,ac)a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*(2*b^6 - 15*a*b^4*c + 30*a^2*b^2*c^2 - 10*a^3*c^3)*arctan((2*c*x + b)/sqr
t(-b^2 + 4*a*c))/((a^5*b^2 - 4*a^6*c)*sqrt(-b^2 + 4*a*c)) + (2*b^3 - 3*a*b*
c)*log(c*x^2 + b*x + a)/a^5 - 2*(2*b^3 - 3*a*b*c)*log(abs(x))/a^5 - 1/3*(a^
4*b^2 - 4*a^5*c + 6*(2*a*b^4*c - 9*a^2*b^2*c^2 + 5*a^3*c^3)*x^4 + 3*(4*a*b^
5 - 20*a^2*b^3*c + 17*a^3*b*c^2)*x^3 + (6*a^2*b^4 - 29*a^3*b^2*c + 20*a^4*c
^2)*x^2 - 2*(a^3*b^3 - 4*a^4*b*c)*x)/((c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^5*x
^3)
```

Mupad [B]
time = 3.06, size = 1120, normalized size = 4.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^2,x)
```

```
[Out] ((x^2*(5*a*c - 6*b^2))/(3*a^3) - 1/(3*a) + (2*b*x)/(3*a^2) + (x^3*(4*b^5 +
17*a^2*b*c^2 - 20*a*b^3*c))/(a^4*(4*a*c - b^2)) + (2*c*x^4*(2*b^4 + 5*a^2*c
^2 - 9*a*b^2*c))/(a^4*(4*a*c - b^2)))/(a*x^3 + b*x^4 + c*x^5) + (log(4*a*b^
9 + 4*b^10*x - 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) - 52*a^2*b^7*c + 308*a^5*b*
c^4 - 40*a^5*c^5*x - 4*b^7*x*(-(4*a*c - b^2)^3)^(1/2) + 243*a^3*b^5*c^2 - 4
73*a^4*b^3*c^3 + 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^4*c*(-(4*a*c
 - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4*c^3*x + 438*a^4*b^2*c^4*
x - 54*a*b^8*c*x - 33*a^3*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 30*a*b^5*c*x*(
-(4*a*c - b^2)^3)^(1/2) + 41*a^3*b*c^3*x*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*
b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*c^2 - 30*b^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 - 10*c^3*(-(4*a*c - b^2)^3)^(1/2)) + 2
*b^9 - 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*c - 15*b^4*c*(-(4*a*c - b
^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a
^7*b^2*c^2) + (log(4*a*b^9 + 4*b^10*x + 4*a*b^6*(-(4*a*c - b^2)^3)^(1/2) -
52*a^2*b^7*c + 308*a^5*b*c^4 - 40*a^5*c^5*x + 4*b^7*x*(-(4*a*c - b^2)^3)^(1
/2) + 243*a^3*b^5*c^2 - 473*a^4*b^3*c^3 - 5*a^4*c^3*(-(4*a*c - b^2)^3)^(1/2
) - 24*a^2*b^4*c*(-(4*a*c - b^2)^3)^(1/2) + 266*a^2*b^6*c^2*x - 563*a^3*b^4
*c^3*x + 438*a^4*b^2*c^4*x - 54*a*b^8*c*x + 33*a^3*b^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 30*a*b^5*c*x*(-(4*a*c - b^2)^3)^(1/2) - 41*a^3*b*c^3*x*(-(4*a*c
- b^2)^3)^(1/2) + 66*a^2*b^3*c^2*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*(132*b^5*
c^2 + 30*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2)) - a^3*(272*b^3*c^3 + 10*c^3*(-(4
*a*c - b^2)^3)^(1/2)) + 2*b^9 + 2*b^6*(-(4*a*c - b^2)^3)^(1/2) - a*(27*b^7*
c + 15*b^4*c*(-(4*a*c - b^2)^3)^(1/2)) + 192*a^4*b*c^4))/(a^5*b^6 - 64*a^8*
c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2) + (2*b*log(x)*(3*a*c - 2*b^2))/a^5
```

3.28 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx$

Optimal. Leaf size=318

$$-\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3}-\frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2}+\frac{b(5b^4-27ab^2c+29a^2c^2)}{a^5\left(b^2-4ac\right)x}+\frac{b^2-2ac}{a\left(b^2-4ac\right)x^4\left(\right.}$$

```
[Out] 1/4*(12*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^4+1/3*b*(-17*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x^3+1/2*(-12*a^2*c^2+22*a*b^2*c-5*b^4)/a^4/(-4*a*c+b^2)/x^2+b*(29*a^2*c
^2-27*a*b^2*c+5*b^4)/a^5/(-4*a*c+b^2)/x+(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^
4/(c*x^2+b*x+a)+b*(-70*a^3*c^3+105*a^2*b^2*c^2-42*a*b^4*c+5*b^6)*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))/a^6/(-4*a*c+b^2)^(3/2)+(3*a^2*c^2-12*a*b^2*c+5*
b^4)*ln(x)/a^6-1/2*(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(c*x^2+b*x+a)/a^6
```

Rubi [A]
time = 0.27, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1599, 754, 814, 648, 632, 212, 642}

$$\frac{b(5b^2-17ac)}{3a^3x^3\left(b^2-4ac\right)}-\frac{5b^2-12ac}{4a^2x^4\left(b^2-4ac\right)}-\frac{\left(3a^2c^2-12ab^2c+5b^4\right)\log\left(a+bx+cx^2\right)}{2a^6}+\frac{\log(x)\left(3a^2c^2-12ab^2c+5b^4\right)}{a^6}+\frac{b(29a^2c^2-27ab^2c+5b^4)}{a^5x\left(b^2-4ac\right)}-\frac{12a^2c^2-22ab^2c+5b^4}{2a^4x^2\left(b^2-4ac\right)}+\frac{b(-70a^3c^3+105a^2b^2c^2-42ab^4c+5b^6)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{a^6\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx}{ax^4\left(b^2-4ac\right)\left(a+bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] -1/4*(5*b^2 - 12*a*c)/(a^2*(b^2 - 4*a*c)*x^4) + (b*(5*b^2 - 17*a*c))/(3*a^3
*(b^2 - 4*a*c)*x^3) - (5*b^4 - 22*a*b^2*c + 12*a^2*c^2)/(2*a^4*(b^2 - 4*a*c
)*x^2) + (b*(5*b^4 - 27*a*b^2*c + 29*a^2*c^2))/(a^5*(b^2 - 4*a*c)*x) + (b^2
 - 2*a*c + b*c*x)/(a*(b^2 - 4*a*c)*x^4*(a + b*x + c*x^2)) + (b*(5*b^6 - 42*
a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*
c]])/(a^6*(b^2 - 4*a*c)^(3/2)) + ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[x])/
a^6 - ((5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + b*x + c*x^2])/(2*a^6)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^2}\,dx = \int \frac{1}{x^5\left(a+bx+cx^2\right)^2}\,dx$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \frac{-5b^2+12ac-5bcx}{x^5(a+bx+cx^2)}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx}{a\left(b^2-4ac\right)x^4\left(a+bx+cx^2\right)} - \frac{\int \left(\frac{-5b^2+12ac}{ax^5} + \frac{5b^3-17abc}{a^2x^4} + \frac{-5b^4+22ab^2c-12a^2c^2}{a^3x^3} \right.}{}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27}{a^5(b}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27}{a^5(b}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27}{a^5(b}$$

$$= -\frac{5b^2-12ac}{4a^2\left(b^2-4ac\right)x^4} + \frac{b(5b^2-17ac)}{3a^3\left(b^2-4ac\right)x^3} - \frac{5b^4-22ab^2c+12a^2c^2}{2a^4\left(b^2-4ac\right)x^2} + \frac{b(5b^4-27}{a^5(b}$$

Mathematica [A]
time = 0.25, size = 272, normalized size = 0.86

$$\frac{-\frac{3a^4}{x^4}+\frac{8a^3b}{x^3}+\frac{6a^2(-3b^2+2ac)}{x^2}-\frac{24ab(-2b^2+3ac)}{x}-\frac{12a(-b^6+6ab^4c-9a^2b^2c^2+2a^3c^3-b^5cx+5ab^3c^2x-5a^2bc^3x)}{(b^2-4ac)(a+x(b+cx))}+\frac{12b(5b^6-42ab^4c+105a^2b^2c^2-70a^3c^3)\tan^{-1}\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}+12(5b^4-12ab^2c+3a^2c^2)\log(x)-6(5b^4-12ab^2c+3a^2c^2)\log(a+x(b+cx))}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x]
```

```
[Out] ((-3*a^4)/x^4 + (8*a^3*b)/x^3 + (6*a^2*(-3*b^2 + 2*a*c))/x^2 - (24*a*b*(-2*
b^2 + 3*a*c))/x - (12*a*(-b^6 + 6*a*b^4*c - 9*a^2*b^2*c^2 + 2*a^3*c^3 - b^5
*c*x + 5*a*b^3*c^2*x - 5*a^2*b*c^3*x))/((b^2 - 4*a*c)*(a + x*(b + c*x))) +
(12*b*(5*b^6 - 42*a*b^4*c + 105*a^2*b^2*c^2 - 70*a^3*c^3)*ArcTan[(b + 2*c*x
)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + 12*(5*b^4 - 12*a*b^2*c + 3*a^
2*c^2)*Log[x] - 6*(5*b^4 - 12*a*b^2*c + 3*a^2*c^2)*Log[a + x*(b + c*x)])/(1
2*a^6)
```

Maple [A]
time = 0.06, size = 359, normalized size = 1.13

method	result

default	$-\frac{\frac{\frac{acb(5a^2c^2-5a\,b^2c+b^4)x}{4ac-b^2}-\frac{a(2a^3c^3-9a^2b^2c^2+6a\,b^4c-b^6)}{4ac-b^2}}{c\,x^2+bx+a}+\frac{\frac{(12a^3c^4-51a^2b^2c^3+32a\,b^4c^2-5b^6c)\ln(c\,x^2+bx+a)}{2c}+\frac{2(41a^3b\,c^3-78a^2b^3c^2+37\cdots}{\cdots}}{4ac-\cdots}}{a^6}$
risch	$\frac{-\frac{bc(29a^2c^2-27a\,b^2c+5b^4)x^5}{a^5(4ac-b^2)}+\frac{(12a^3c^3-80a^2b^2c^2+59a\,b^4c-10b^6)x^4}{2a^5(4ac-b^2)}-\frac{b(26ac-15b^2)x^3}{6a^4}+\frac{(9ac-10b^2)x^2}{12a^3}+\frac{5bx}{12a^2}-\frac{1}{4a}}{x^4(c\,x^2+bx+a)}+\frac{3\ln(x)c^2}{a^4}-\frac{12\ln(\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^6*((a*c*b*(5*a^2*c^2-5*a*b^2*c+b^4)/(4*a*c-b^2)*x-a*(2*a^3*c^3-9*a^2*b
^2*c^2+6*a*b^4*c-b^6)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-b^2)*(1/2*(12*a^3
*c^4-51*a^2*b^2*c^3+32*a*b^4*c^2-5*b^6*c)/c*ln(c*x^2+b*x+a)+2*(41*a^3*b*c^3
-78*a^2*b^3*c^2+37*a*b^5*c-5*b^7-1/2*(12*a^3*c^4-51*a^2*b^2*c^3+32*a*b^4*c^
2-5*b^6*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))-1/4
/a^2/x^4-1/2*(-2*a*c+3*b^2)/a^4/x^2+(3*a^2*c^2-12*a*b^2*c+5*b^4)*ln(x)/a^6+
2/3*b/a^3/x^3-2*b*(3*a*c-2*b^2)/a^5/x
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(306) = 612$.
time = 0.79, size = 1640, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*a^5*b^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*
c^2 + 137*a^3*b^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 3
16*a^3*b^4*c^2 - 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^
```

```
3*b^5*c + 448*a^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c
 + 232*a^5*b^2*c^2 - 144*a^6*c^3)*x^2 - 6*((5*b^7*c - 42*a*b^5*c^2 + 105*a^
2*b^3*c^3 - 70*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*
a^3*b^2*c^3)*x^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3
)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - 5*(a^4*b^5 - 8*a^5*b^3*c + 16*a^
6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4
 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^
3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4
*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) - 12*((5*b^8*c - 52*a*b^6*
c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b
^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 5
2*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(x))/
((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a^6*b^5 - 8*a^7*b^3*c + 16
*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*x^4), -1/12*(3*a^5*b
^4 - 24*a^6*b^2*c + 48*a^7*c^2 - 12*(5*a*b^7*c - 47*a^2*b^5*c^2 + 137*a^3*b
^3*c^3 - 116*a^4*b*c^4)*x^5 - 6*(10*a*b^8 - 99*a^2*b^6*c + 316*a^3*b^4*c^2
- 332*a^4*b^2*c^3 + 48*a^5*c^4)*x^4 - 2*(15*a^2*b^7 - 146*a^3*b^5*c + 448*a
^4*b^3*c^2 - 416*a^5*b*c^3)*x^3 + (10*a^3*b^6 - 89*a^4*b^4*c + 232*a^5*b^2*
c^2 - 144*a^6*c^3)*x^2 - 12*((5*b^7*c - 42*a*b^5*c^2 + 105*a^2*b^3*c^3 - 70
*a^3*b*c^4)*x^6 + (5*b^8 - 42*a*b^6*c + 105*a^2*b^4*c^2 - 70*a^3*b^2*c^3)*x
^5 + (5*a*b^7 - 42*a^2*b^5*c + 105*a^3*b^3*c^2 - 70*a^4*b*c^3)*x^4)*sqrt(-b
^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - 5*(a^4*
b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x + 6*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*
b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2
*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c +
179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 + 48*a^5*c^4)*x^4)*log(c*x^2 + b*x + a) -
 12*((5*b^8*c - 52*a*b^6*c^2 + 179*a^2*b^4*c^3 - 216*a^3*b^2*c^4 + 48*a^4*c
^5)*x^6 + (5*b^9 - 52*a*b^7*c + 179*a^2*b^5*c^2 - 216*a^3*b^3*c^3 + 48*a^4*
b*c^4)*x^5 + (5*a*b^8 - 52*a^2*b^6*c + 179*a^3*b^4*c^2 - 216*a^4*b^2*c^3 +
48*a^5*c^4)*x^4)*log(x))/((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*x^6 + (a
^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*x^5 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*
c^2)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]

time = 3.45, size = 347, normalized size = 1.09

$$-\frac{(5 b^7 - 42 a b^5 c + 105 a^2 b^3 c^2 - 70 a^3 b c^3) \arctan\left(\frac{2 c x + b}{\sqrt{-b^2 + 4 a c}}\right)}{(a^6 b^2 - 4 a^7 c)\sqrt{-b^2 + 4 a c}} - \frac{(5 b^4 - 12 a b^2 c + 3 a^2 c^2) \log(c x^2 + b x + a)}{2 a^6} + \frac{(5 b^4 - 12 a b^2 c + 3 a^2 c^2) \log(|x|)}{a^6} - \frac{3 a^5 b^2 - 12 a^6 c - 12 (5 a b^5 c - 27 a^2 b^3 c^2 + 29 a^3 b c^3) x^5 - 6 (10 a b^6 - 59 a^2 b^4 c + 80 a^3 b^2 c^2 - 12 a^4 c^3) x^4 - 2 (15 a^2 b^5 - 86 a^3 b^3 c + 104 a^4 b c^2) x^3 + (10 a^3 b^4 - 49 a^4 b^2 c + 36 a^5 c^2) x^2 - 5 (a^4 b^3 - 4 a^5 b c) x}{12 (c x^2 + b x + a)(b^2 - 4 a c) a^6 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -(5*b^7 - 42*a*b^5*c + 105*a^2*b^3*c^2 - 70*a^3*b*c^3)*arctan((2*c*x + b)/s
qrt(-b^2 + 4*a*c))/((a^6*b^2 - 4*a^7*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(5*b^4 -
12*a*b^2*c + 3*a^2*c^2)*log(c*x^2 + b*x + a)/a^6 + (5*b^4 - 12*a*b^2*c + 3*
a^2*c^2)*log(abs(x))/a^6 - 1/12*(3*a^5*b^2 - 12*a^6*c - 12*(5*a*b^5*c - 27*
a^2*b^3*c^2 + 29*a^3*b*c^3)*x^5 - 6*(10*a*b^6 - 59*a^2*b^4*c + 80*a^3*b^2*c
^2 - 12*a^4*c^3)*x^4 - 2*(15*a^2*b^5 - 86*a^3*b^3*c + 104*a^4*b*c^2)*x^3 +
(10*a^3*b^4 - 49*a^4*b^2*c + 36*a^5*c^2)*x^2 - 5*(a^4*b^3 - 4*a^5*b*c)*x)/(
(c*x^2 + b*x + a)*(b^2 - 4*a*c)*a^6*x^4)
```

Mupad [B]
time = 3.14, size = 1260, normalized size = 3.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^2),x)
```

```
[Out] (log(x)*(5*b^4 + 3*a^2*c^2 - 12*a*b^2*c))/a^6 - (1/(4*a) - (x^2*(9*a*c - 10
*b^2))/(12*a^3) - (5*b*x)/(12*a^2) + (x^4*(10*b^6 - 12*a^3*c^3 + 80*a^2*b^2
*c^2 - 59*a*b^4*c))/(2*a^5*(4*a*c - b^2)) + (b*x^3*(26*a*c - 15*b^2))/(6*a^
4) + (b*c*x^5*(5*b^4 + 29*a^2*c^2 - 27*a*b^2*c))/(a^5*(4*a*c - b^2)))/(a*x^
4 + b*x^5 + c*x^6) + (log(288*a^6*c^5 - 10*b^11*x - 10*a*b^10 + 10*a*b^7*(-
(4*a*c - b^2)^3)^(1/2) + 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2)
- 717*a^3*b^6*c^2 + 1643*a^4*b^4*c^3 - 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4
*a*c - b^2)^3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) - 779*a^2*b^7*
c^2*x + 1916*a^3*b^5*c^3*x - 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b
^2)^3)^(1/2) + 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) + 5
68*a^5*b*c^5*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*
(-(4*a*c - b^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^3
*(466*b^4*c^3 - 35*b*c^3*(-(4*a*c - b^2)^3)^(1/2)) - a^2*((387*b^6*c^2)/2 -
 (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - (5*b^10)/2 + 96*a^5*c^5 + (5*b
^7*(-(4*a*c - b^2)^3)^(1/2))/2 + a*(36*b^8*c - 21*b^5*c*(-(4*a*c - b^2)^3)^
(1/2)) - 456*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^
2*c^2) - (log(10*a*b^10 + 10*b^11*x - 288*a^6*c^5 + 10*a*b^7*(-(4*a*c - b^2
)^3)^(1/2) - 139*a^2*b^8*c + 10*b^8*x*(-(4*a*c - b^2)^3)^(1/2) + 717*a^3*b^
6*c^2 - 1643*a^4*b^4*c^3 + 1508*a^5*b^2*c^4 - 69*a^2*b^5*c*(-(4*a*c - b^2)^
3)^(1/2) - 53*a^4*b*c^3*(-(4*a*c - b^2)^3)^(1/2) + 779*a^2*b^7*c^2*x - 1916
*a^3*b^5*c^3*x + 1998*a^4*b^3*c^4*x + 36*a^4*c^4*x*(-(4*a*c - b^2)^3)^(1/2)
```

```
 - 144*a*b^9*c*x + 129*a^3*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2) - 568*a^5*b*c^5
*x - 84*a*b^6*c*x*(-(4*a*c - b^2)^3)^(1/2) + 225*a^2*b^4*c^2*x*(-(4*a*c - b
^2)^3)^(1/2) - 206*a^3*b^2*c^3*x*(-(4*a*c - b^2)^3)^(1/2))*(a^2*((387*b^6*c
^2)/2 + (105*b^3*c^2*(-(4*a*c - b^2)^3)^(1/2))/2) - a^3*(466*b^4*c^3 + 35*b
*c^3*(-(4*a*c - b^2)^3)^(1/2)) + (5*b^10)/2 - 96*a^5*c^5 + (5*b^7*(-(4*a*c
- b^2)^3)^(1/2))/2 - a*(36*b^8*c + 21*b^5*c*(-(4*a*c - b^2)^3)^(1/2)) + 456
*a^4*b^2*c^4))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
```

3.29 $\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=257

$$\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}-\frac{(7b^2-16ac)\,x\sqrt{ax^2}}{240c^2}$$

```
[Out] 1/256*b*(-12*a*c+7*b^2)*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2
+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/960*
b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/1920*(256*a^2*c^2-460*a
*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/240*(-16*a*c+7*b^2)*x*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/40*x^2*(8*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A]
time = 0.39, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1933, 1963, 12, 1928, 635, 212}

$$-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{1920c^4x}+\frac{bx(7b^2-12ac)(b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{9/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3}-\frac{x(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{240c^2}+\frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(960*c^3) - ((105*b^4 -
460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(1920*c^4*x) - ((7*
b^2 - 16*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(240*c^2) + (x^2*(b + 8*c*x)*S
qrt[a*x^2 + b*x^3 + c*x^4])/(40*c) + (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*x*Sq
rt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(256*c^(9/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x^2\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} + \frac{\int \frac{x^3\left(-3ab-\frac{1}{2}(7b^2-16ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{40c}$$

$$= -\frac{(7b^2-16ac)\,x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(b+8cx)\sqrt{ax^2+bx^3+cx^4}}{40c} - \frac{\int \frac{x^2(-}{}}{}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(7b^2-16ac)\,x\sqrt{ax^2+bx^3+cx^4}}{240c^2} + \frac{x^2(}{}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx}}{1920c^4x}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx}}{1920c^4x}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx}}{1920c^4x}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx}}{1920c^4x}$$

$$= \frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{960c^3} - \frac{(105b^4-460ab^2c+256a^2c^2)\sqrt{ax^2+bx}}{1920c^4x}$$

Mathematica [A]
time = 0.41, size = 184, normalized size = 0.72

$$\frac{2\sqrt{c}\,x(a+x(b+cx))\left(-105b^4+70b^3cx+4b^2c(115a-14cx^2)+8bc^2x(-29a+6cx^2)+128c^2(-2a^2+acx^2+3c^2x^4)\right)-15(7b^5-40ab^3c+48a^2bc^2)\,x\sqrt{a+x(b+cx)}\,\log\left(c^4\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{3840c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-105*b^4 + 70*b^3*c*x + 4*b^2*c*(115*a - 14
*c*x^2) + 8*b*c^2*x*(-29*a + 6*c*x^2) + 128*c^2*(-2*a^2 + a*c*x^2 + 3*c^2*x
^4)) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x*Sqrt[a + x*(b + c*x)]*Log[c
^4*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(3840*c^(9/2)*Sqrt[x^2*(
a + x*(b + c*x))])
```

Maple [A]
time = 0.04, size = 310, normalized size = 1.21

method	result
risch	$-\frac{\left(-384c^4x^4-48b\,c^3x^3-128a\,c^3x^2+56b^2c^2x^2+232ab\,c^2x-70b^3cx+256a^2c^2-460a\,b^2c+105b^4\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{1920c^4x}+$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(768x^2\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{9}{2}}-672c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}bx-512c^{\frac{7}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}a+720c^{\frac{7}{2}}\sqrt{c\,x^2+bx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3840*(c*x^4+b*x^3+a*x^2)^(1/2)*(768*x^2*(c*x^2+b*x+a)^(3/2)*c^(9/2)-672*c
^(7/2)*(c*x^2+b*x+a)^(3/2)*b*x-512*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a+720*c^(7/2
)*(c*x^2+b*x+a)^(1/2)*a*b*x+560*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^2-420*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*b^3*x+360*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^2-210*c^(3/2
)*(c*x^2+b*x+a)^(1/2)*b^4+720*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b
)/c^(1/2))*a^2*b*c^3-600*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*a*b^3*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*b^5*c)/x/(c*x^2+b*x+a)^(1/2)/c^(11/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2, x)
```

Fricas [A]
time = 0.36, size = 390, normalized size = 1.52

$$\left[\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)+4\left(384c^5x^4+48bc^4x^3-105b^4c+460ab^2c^2-256a^2c^3-8\left(7b^2c^3-16ac^4\right)x^2+2\left(35b^3c^2-116abc^3\right)x\right)\sqrt{cx^4+bx^3+ax^2}}{7680c^5x},-\frac{15\left(7b^5-40ab^3c+48a^2bc^2\right)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{-c}}{2\left(c^2x^3+bcx^2+acx\right)}\right)-2\left(384c^5x^4+48bc^4x^3-105b^4c+460ab^2c^2-256a^2c^3-8\left(7b^2c^3-16ac^4\right)x^2+2\left(35b^3c^2-116abc^3\right)x\right)\sqrt{cx^4+bx^3+ax^2}}{3840c^5x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 +
 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a
*c)*x)/x) + 4*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^4*c + 460*a*b^2*c^2 - 256
*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^3*c^2 - 116*a*b*c^3)*x)*s
```

```
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/3840*(15*(7*b^5 - 40*a*b^3*c + 48*a
^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqr
t(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(384*c^5*x^4 + 48*b*c^4*x^3 - 105*b^
4*c + 460*a*b^2*c^2 - 256*a^2*c^3 - 8*(7*b^2*c^3 - 16*a*c^4)*x^2 + 2*(35*b^
3*c^2 - 116*a*b*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x^2 (a + bx + cx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 3.74, size = 283, normalized size = 1.10

$$\frac{1}{1920}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\left(8\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{7\,b^2c^2\mathrm{sgn}(x)-16\,ac^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{35\,b^3c\mathrm{sgn}(x)-116\,abc^2\mathrm{sgn}(x)}{c^4}\right)x-\frac{105\,b^4\mathrm{sgn}(x)-460\,ab^2c\mathrm{sgn}(x)+256\,a^2c^2\mathrm{sgn}(x)}{c^4}\right)-\frac{(7\,b^5\mathrm{sgn}(x)-40\,ab^3c\mathrm{sgn}(x)+48\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{9}{2}}}+\frac{\left(105\,b^5\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+210\sqrt{a}\,b^4\sqrt{c}-920\,a^{\frac{3}{2}}b^2c^{\frac{3}{2}}+512\,a^{\frac{5}{2}}c^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{3840\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*sqrt(c*x^2 + b*x + a)*(2*(4*(6*(8*x*sgn(x) + b*sgn(x)/c)*x - (7*b^2*
c^2*sgn(x) - 16*a*c^3*sgn(x))/c^4)*x + (35*b^3*c*sgn(x) - 116*a*b*c^2*sgn(x
))/c^4)*x - (105*b^4*sgn(x) - 460*a*b^2*c*sgn(x) + 256*a^2*c^2*sgn(x))/c^4)
 - 1/256*(7*b^5*sgn(x) - 40*a*b^3*c*sgn(x) + 48*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2) + 1/3840*(105*b
^5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sq
rt(c))) + 720*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*
sqrt(c) - 920*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))*sgn(x)/c^(9/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.30 $\int x\sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=205

$$-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}-\frac{(b^2 \cdots}{\cdots}$$

```
[Out] -1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b
*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/96*(-1
2*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2+1/192*b*(-52*a*c+15*b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c^3/x+1/24*x*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c
```

Rubi [A]
time = 0.24, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1933, 1963, 12, 1928, 635, 212}

$$-\frac{x(b^2-4ac)(5b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2}+\frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -1/96*((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/c^2 + (b*(15*b^2 - 52*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*c^3*x) + (x*(b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(24*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*x*Sqrt[a + b*x + c
*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7/2)*
Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^3+cx^4}\,dx = \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} + \frac{\int \frac{x^2\left(-2ab-\frac{1}{2}(5b^2-12ac)x\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{24c}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{x(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{24c} - \frac{\int \frac{x(-\frac{1}{2}a}{}}{}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b}{}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b}{}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b}{}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b}{}$$

$$= -\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96c^2} + \frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192c^3x} + \frac{x(b}{}$$

Mathematica [A]
time = 0.30, size = 150, normalized size = 0.73

$$\frac{2\sqrt{c}\,x(a+x(b+cx))\left(15b^3-10b^2cx+24c^2x(a+2cx^2)+b(-52ac+8c^2x^2)\right)+3(5b^4-24ab^2c+16a^2c^2)\,x\sqrt{a+x(b+cx)}\,\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)}{384c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(15*b^3 - 10*b^2*c*x + 24*c^2*x*(a + 2*c*x^2
) + b*(-52*a*c + 8*c^2*x^2)) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x*Sqrt[a
 + x*(b + c*x)]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(384*c^(7
/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.04, size = 265, normalized size = 1.29

method	result

risch	$-\frac{\left(-48c^3x^3-8b\,c^2x^2-24a\,c^2x+10b^2cx+52abc-15b^3\right)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{192c^3x}+\frac{\left(-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)a^2}{8c^{\frac{3}{2}}}+\frac{3\ln}{}\right.}{}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(96x\left(c\,x^2+bx+a\right)^{\frac{3}{2}}c^{\frac{7}{2}}-48c^{\frac{7}{2}}\sqrt{c\,x^2+bx+a}\,ax-80c^{\frac{5}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}b+60c^{\frac{5}{2}}\sqrt{c\,x^2+bx+c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(96*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)-48*c^(7/2
)*(c*x^2+b*x+a)^(1/2)*a*x-80*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b+60*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*b^2*x-24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b+30*c^(3/2)*(c*x^2+b
*x+a)^(1/2)*b^3-48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*
a^2*c^3+72*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^
2-15*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x/(c*x^
2+b*x+a)^(1/2)/c^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)*x, x)
```

Fricas [A]
time = 0.35, size = 326, normalized size = 1.59

$$\left[\frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)+4\left(48c^4x^3+8bc^3x^2+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x\right)\sqrt{cx^4+bx^3+ax^2}}{768c^4x},\ \frac{3\left(5b^4-24ab^2c+16a^2c^2\right)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{-c}}{2\left(c^2x^3+bcx^2+acx\right)}\right)+2\left(48c^4x^3+8bc^3x^2+15b^3c-52abc^2-2\left(5b^2c^2-12ac^3\right)x\right)\sqrt{cx^4+bx^3+ax^2}}{384c^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b
*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) + 4*(48*c^4*x^3 + 8*b*c^3*x^2 + 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2
- 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x), 1/384*(3*(5*b^4 - 24*a
*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(48*c^4*x^3 + 8*b*c^3*x^2
```

```
+ 15*b^3*c - 52*a*b*c^2 - 2*(5*b^2*c^2 - 12*a*c^3)*x)*sqrt(c*x^4 + b*x^3 +
a*x^2))/(c^4*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(a+bx+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x*sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 5.00, size = 230, normalized size = 1.12

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6\,x\mathrm{sgn}(x)+\frac{b\mathrm{sgn}(x)}{c}\right)x-\frac{5\,b^2c\mathrm{sgn}(x)-12\,ac^2\mathrm{sgn}(x)}{c^3}\right)x+\frac{15\,b^3\mathrm{sgn}(x)-52\,abc\mathrm{sgn}(x)}{c^3}\right)+\frac{\left(5\,b^4\mathrm{sgn}(x)-24\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{7}{2}}}-\frac{\left(15\,b^4\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+30\sqrt{a}\,b^3\sqrt{c}-104\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{384\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*x*sgn(x) + b*sgn(x)/c)*x - (5*b^2*c*sg
n(x) - 12*a*c^2*sgn(x))/c^3)*x + (15*b^3*sgn(x) - 52*a*b*c*sgn(x))/c^3) + 1
/128*(5*b^4*sgn(x) - 24*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sqr
t(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/384*(15*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104
*a^(3/2)*b*c^(3/2))*sgn(x)/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.31 $\int \sqrt{ax^2+bx^3+cx^4}\,dx$

Optimal. Leaf size=163

$$-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx}+\frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\right.}{16c^{5/2}x\sqrt{a+bx+}}$$

```
[Out] -1/8*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x+1/3*(c*x^2+b*x+a)*(c*x^4+b
*x^3+a*x^2)^(1/2)/c/x+1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))*(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2)/x/(c*x^2+b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1917, 654, 626, 635, 212}

$$\frac{b(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}x\sqrt{a+bx+cx^2}}-\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x}+\frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -1/8*(b*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(c^2*x) + ((a + b*x + c*x^
2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*c*x) + (b*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x
^3 + c*x^4]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(
5/2)*x*Sqrt[a + b*x + c*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{ax^2+bx^3+cx^4}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}\int x\sqrt{a+bx+cx^2}\,dx}{x\sqrt{a+bx+cx^2}}$$

$$= \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} - \frac{\left(b\sqrt{ax^2+bx^3+cx^4}\right)\int \sqrt{a+bx+cx^2}\,d}{2cx\sqrt{a+bx+cx^2}}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b(b^2-\right.}{}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{\left(b(b^2-\right.}{}$$

$$= -\frac{b(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{8c^2x} + \frac{(a+bx+cx^2)\sqrt{ax^2+bx^3+cx^4}}{3cx} + \frac{b(b^2-4}{}$$

Mathematica [A]
time = 0.26, size = 121, normalized size = 0.74

$$\frac{2\sqrt{c}\,x(a+x(b+cx))\left(-3b^2+2bcx+8c(a+cx^2)\right)-3(b^3-4abc)\,x\sqrt{a+x(b+cx)}\,\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{48c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*x*(a + x*(b + c*x))*(-3*b^2 + 2*b*c*x + 8*c*(a + c*x^2)) - 3*(b^
3 - 4*a*b*c)*x*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])])/(48*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 167, normalized size = 1.02

method	result
risch	$\frac{\left(8c^{2}x^{2}+2bcx+8ac-3b^{2}\right)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{24c^{2}x}+\frac{\left(-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)a}{4c^{\frac{3}{2}}}+\frac{b^{3}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+\right.}{16c^{\frac{5}{2}}}}{x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\left(16\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,bx-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b^{2}-12\ln\left(\frac{2\sqrt{c\,x^{2}+\right.}{}\right.}{48x\sqrt{c\,x^{2}+bx+a}\,c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(
c*x^2+b*x+a)^(1/2)*b*x-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^2-12*ln(1/2*(2*(c*x^
2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^
(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x/(c*x^2+b*x+a)^(1/2)/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A]
time = 0.33, size = 260, normalized size = 1.60

$$\left[-\frac{3\left(b^{3}-4abc\right)\sqrt{c}\,x\log\left(-\frac{8c^{2}x^{3}+8bcx^{2}-4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{c}+(b^{2}+4ac)x}{x}\right)-4\left(8c^{3}x^{2}+2bc^{2}x-3b^{2}c+8ac^{2}\right)\sqrt{cx^{4}+bx^{3}+ax^{2}}}{96c^{3}x},\ -\frac{3\left(b^{3}-4abc\right)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{-c}}{2\left(c^{2}x^{3}+bcx^{2}+acx\right)}\right)-2\left(8c^{3}x^{2}+2bc^{2}x-3b^{2}c+8ac^{2}\right)\sqrt{cx^{4}+bx^{3}+ax^{2}}}{48c^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*
x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(8*c^3*x
^2 + 2*b*c^2*x - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -
1/48*(3*(b^3 - 4*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(8*c^3*x^2 + 2*b*c^2*x
 - 3*b^2*c + 8*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Giac [A]
time = 5.16, size = 166, normalized size = 1.02

$$\frac{1}{24}\sqrt{cx^2 + bx + a}\left(2\left(4\,x\mathrm{sgn}(x) + \frac{b\mathrm{sgn}(x)}{c}\right)x - \frac{3\,b^2\mathrm{sgn}(x) - 8\,ac\mathrm{sgn}(x)}{c^2}\right) - \frac{\left(b^3\mathrm{sgn}(x) - 4\,abc\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{c}\,x - \sqrt{cx^2 + bx + a}\right)\sqrt{c} - b\right|\right)}{16\,c^{\frac{5}{2}}} + \frac{\left(3\,b^3\log\left(\left|-b + 2\,\sqrt{a}\,\sqrt{c}\right|\right) - 12\,abc\log\left(\left|-b + 2\,\sqrt{a}\,\sqrt{c}\right|\right) + 6\,\sqrt{a}\,b^2\sqrt{c} - 16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{48\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(c*x^2 + b*x + a)*(2*(4*x*sgn(x) + b*sgn(x)/c)*x - (3*b^2*sgn(x) -
 8*a*c*sgn(x))/c^2) - 1/16*(b^3*sgn(x) - 4*a*b*c*sgn(x))*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/48*(3*b^3*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt
(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x^4 + b\,x^3 + a\,x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.32 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x} \, dx$

Optimal. Leaf size=119

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{(b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/8*(-4*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x
^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/4*(2*c*x+b)*(c*x^4+b*x^
3+a*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1932, 1928, 635, 212}

$$\frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx}-\frac{x(b^2-4ac)\,\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] ((b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c*x) - ((b^2 - 4*a*c)*x*Sqrt[a
 + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*
c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
```

```
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx = \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{8c}$$

$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left((b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx}}}{8c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{\left((b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,d\right.}{4c\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4cx} - \frac{(b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+}{2\sqrt{c}\sqrt{a+}}\right.}{8c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A]
time = 0.19, size = 101, normalized size = 0.85

$$\frac{x\left(2\sqrt{c}\,(b+2cx)(a+x(b+cx))+(b^2-4ac)\,\sqrt{a+x(b+cx)}\,\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{8c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x,x]
```

```
[Out] (x*(2*Sqrt[c]*(b + 2*c*x)*(a + x*(b + c*x)) + (b^2 - 4*a*c)*Sqrt[a + x*(b +
 c*x)]*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(8*c^(3/2)*Sq
rt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 146, normalized size = 1.23

method	result
risch	$\frac{(2cx+b)\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}{4cx}+\frac{\left(\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)a}{2\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)b^{2}}{8c^{\frac{3}{2}}}\right)\sqrt{x^{2}\left(c\,x\right.}}{x\sqrt{c\,x^{2}+bx+a}}$
default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\left(4c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x+2c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b+4\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c\right.}{8c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\,x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x+2*c^(3/2)*(c
*x^2+b*x+a)^(1/2)*b+4*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2
))*a*c^2-ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^2*c)/c^(
5/2)/(c*x^2+b*x+a)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x, x)
```

Fricas [A]
time = 0.36, size = 220, normalized size = 1.85

$$\left[-\frac{(b^2-4\,ac)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,c^2x+bc)}{16\,c^2x},\ \frac{(b^2-4\,ac)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}\,(2\,c^2x+bc)}{8\,c^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4
+ b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x), 1/8*((b^2 - 4*a*c)*sqrt(-c)*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2
 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x + b*c))/(c^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (a+bx+cx^2)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x, x)
```

Giac [A]
time = 6.29, size = 125, normalized size = 1.05

$$\frac{1}{8}\left(2\sqrt{cx^2+bx+a}\left(2x+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}\right)\mathrm{sgn}(x)-\frac{\left(b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\,b\sqrt{c}\right)\mathrm{sgn}(x)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(c*x^2 + b*x + a)*(2*x + b/c) + (b^2 - 4*a*c)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2))*sgn(x) - 1/8*(b^2*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sq
rt(a)*b*sqrt(c))*sgn(x)/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x, x)
```

3.33 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.}{2\sqrt{c}\,\sqrt{ax^2+bx^3\cdots}}$$

```
[Out] -x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)
^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x
^4+b*x^3+a*x^2)^(1/2)/x
```

Rubi [A]
time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1935, 1947, 857, 635, 212, 738}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{x}-\frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}+\frac{bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/x - (Sqrt[a]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2
*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] +
 (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{1}{2}\int \frac{2a+bx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} + \frac{\left(ax\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(bx\sqrt{a+b\right.}{}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\left(2ax\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+ca}}\right.}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{x} - \frac{\sqrt{a}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}} + \frac{b}{}$$

Mathematica [A]
time = 0.19, size = 133, normalized size = 0.77

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\,\sqrt{a+x(b+cx)}+4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-b\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{2\sqrt{c}\,\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)] + 4*Sqrt[a]*Sqrt[
c]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - b*Log[b + 2*c*x -
 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(2*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 126, normalized size = 0.73

method	result
default	$-\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\left(2\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\sqrt{c}-b\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\right.}{2x\sqrt{c\,x^2+bx+a}\,\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+
a)^(1/2))/x)*c^(1/2)-b*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/
2))-2*(c*x^2+b*x+a)^(1/2)*c^(1/2))/x/(c*x^2+b*x+a)^(1/2)/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^2, x)
```

Fricas [A]
time = 0.36, size = 638, normalized size = 3.69

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)+2\sqrt{a}\,cx\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}\,(bx+2a)\sqrt{a}}{x^3}\right)+4\sqrt{cx^4+bx^3+ax^2}\,c}{4cx},\ -\frac{b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)-\sqrt{a}\,cx\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}\,(bx+2a)\sqrt{a}}{x^3}\right)-2\sqrt{cx^4+bx^3+ax^2}\,c}{2cx},\ \frac{4\sqrt{-a}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)+b\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2+4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)+4\sqrt{cx^4+bx^3+ax^2}\,c}{4cx},\ \frac{2\sqrt{-a}\,cx\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)-b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)+2\sqrt{cx^4+bx^3+ax^2}\,c}{2cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(8*a*b*x^
2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)
*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), -1/2*(b*sqrt(-c)*x
*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c
*x^2 + a*c*x)) - sqrt(a)*c*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
- 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*c)/(c*x), 1/4*(4*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + b*sqrt(c)*x*lo
g(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(
c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*c)/(c*x), 1/2*(2*s
qrt(-a)*c*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*
c*x^3 + a*b*x^2 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*c)/(c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**2, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^2, x)
```

3.34 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$

Optimal. Leaf size=173

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{\cdot}{\cdot}\right.}{\sqrt{ax^2+bx^3+\cdots}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x
^2+b*x+a)^(1/2))*c^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(c*x
^4+b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A]

time = 0.08, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1934, 1947, 857, 635, 212, 738}

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}+\frac{\sqrt{c}\,x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/x^2) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3
+ c*x^4]) + (Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{1}{2}\int \frac{b+2cx}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} + \frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{ax^2+bx^3+cx^4}} + \frac{\left(cx\sqrt{a\right.}}{}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+}}\right.}{\sqrt{ax^2+bx^3+cx^4}}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{x^2} - \frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}} + \cdots$$

Mathematica [A]
time = 0.20, size = 125, normalized size = 0.72

$$\frac{\sqrt{a+x(b+cx)}\left(bx\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left(\sqrt{a+x(b+cx)}+\sqrt{c}\,x\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{\sqrt{a}\,\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(b*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqr
t[a]] - Sqrt[a]*(Sqrt[a + x*(b + c*x)] + Sqrt[c]*x*Log[b + 2*c*x - 2*Sqrt[c
]*Sqrt[a + x*(b + c*x)]])))/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 174, normalized size = 1.01

method	result
risch	$-\frac{\sqrt{x^2\left(c\,x^2+bx+a\right)}}{x^2}+\frac{\left(\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2\sqrt{a}}\right)\sqrt{\cdots}}{x\sqrt{c\,x^2+bx+a}}$

default	$\frac{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\left(2c^{\frac{5}{2}}\sqrt{c\,x^{2}+bx+a}\;x^{2}-c^{\frac{3}{2}}\sqrt{a}\;\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)bx-2\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}\right.}{2x^{2}\sqrt{c\,x^{2}+bx+a}\;a\,c^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^3+a*x^2)^(1/2)*(2*c^(5/2)*(c*x^2+b*x+a)^(1/2)*x^2-c^(3/2)*a^
(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b*x-2*(c*x^2+b*x+a)^(3/
2)*c^(3/2)+2*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b*x+2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2
)*c^(1/2)+2*c*x+b)/c^(1/2))*a*c^2*x)/x^2/(c*x^2+b*x+a)^(1/2)/a/c^(3/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^3, x)
```

<u>Fricas</u> [A]
time = 0.36, size = 653, normalized size = 3.77

$$\left[\frac{2\,a\sqrt{c}\,x^{2}\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)+\sqrt{a}\,bx^{2}\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{a}}{x^{3}}\right)-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,a}{4\,ax^{2}},\ -\frac{4\,a\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-\sqrt{a}\,bx^{2}\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{a}}{x^{3}}\right)+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,a}{4\,ax^{2}},\ \frac{\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)+a\sqrt{c}\,x^{2}\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)-2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,a}{2\,ax^{2}},\ \frac{\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)-2\,a\sqrt{-c}\,x^{2}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,a}{2\,ax^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + sqrt(a)*b*x^2*log(-(8*a*
b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), -1/4*(4*a*sq
rt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) - sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^
3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
a*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)
/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x +
 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*a*sqrt(-c)*x^2*arctan(1/2*s
qrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)
) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*a)/(a*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^3, x)
```

3.35 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/x^3-1/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/
a/x^2
```

Rubi [A]
time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1934, 1965, 12, 1918, 212}

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{3/2}}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^3 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a
*x^2) + ((b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}+\frac{1}{4}\int \frac{b+2cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{\int \frac{b^2-4ac}{2\sqrt{ax^2+bx^3+cx^4}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}-\frac{(b^2-4ac)\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}} \cdots}{8a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{(b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \cdots\right.}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{4ax^2}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{x(2\cdots}{2\sqrt{a}\,\sqrt{ax^2\cdots}}\right.}{8a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.26, size = 111, normalized size = 0.97

$$
-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\,(2a+bx)\sqrt{a+x(b+cx)}+(b^2-4ac)\,x^2\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+x(b+cx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4,x]
```

```
[Out] -1/4*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)
] + (b^2 - 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])
)/(a^(3/2)*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(96) = 192.
time = 0.05, size = 207, normalized size = 1.82

method	result
risch	$-\frac{(bx+2a)\sqrt{x^2\left(c\,x^2+bx+a\right)}}{4x^3a}+\frac{\left(-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)c}{2\sqrt{a}}+\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right.}{8a^{\frac{3}{2}}}\right.}{x\sqrt{c\,x^2+bx+a}}$
default	$-\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(4c\,a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^2+2c\sqrt{c\,x^2+bx+a}\,b\,x^3-4c\sqrt{c\,x^2+bx}\right.}{8x^3\sqrt{c\,x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(1/2)*(4*c*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*x^2+2*c*(c*x^2+b*x+a)^(1/2)*b*x^3-4*c*(c*x^2+b*x+a)^(1/2)*a*
x^2-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^2*x^2-2*(c*x^2+
b*x+a)^(3/2)*b*x+2*(c*x^2+b*x+a)^(1/2)*b^2*x^2+4*(c*x^2+b*x+a)^(3/2)*a)/x^3
/(c*x^2+b*x+a)^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^4, x)
```

Fricas [A]
time = 0.36, size = 226, normalized size = 1.98

$$\left[-\frac{(b^2-4\,ac)\sqrt{a}\,x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\sqrt{cx^4+bx^3+ax^2}\,(abx+2\,a^2)}{16\,a^2x^3},\ -\frac{(b^2-4\,ac)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\sqrt{cx^4+bx^3+ax^2}\,(abx+2\,a^2)}{8\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [-1/16*((b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a
^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/8*((b^2 - 4*a*c)*sqrt(-a
)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3
+ a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x
^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (a + bx + cx^2)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4 + b\,x^3 + a\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^4, x)
```

3.36 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$

Optimal. Leaf size=155

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*b*(-4*a*c+b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1
/2))/a^(5/2)-1/3*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4-1/12*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a/x^3+1/24*(-8*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1934, 1965, 12, 1918, 212}

$$-\frac{b(b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{5/2}}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] -1/3*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^4 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*
a*x^3) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*a^2*x^2) - (b*(b
^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])
])/(16*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}+\frac{1}{6}\int \frac{b+2cx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}-\frac{\int \frac{\frac{1}{2}(3b^2-8ac)+bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{12a} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2} \\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{3x^4}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{12ax^3}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{24a^2x^2}
\end{aligned}
$$

Mathematica [A]

time = 0.45, size = 130, normalized size = 0.84

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}\,(-8a^2+3b^2x^2-2ax(b+4cx))+3b(b^2-4ac)\,x^3\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{24a^{5/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(-8*a^2 + 3*b^2
*x^2 - 2*a*x*(b + 4*c*x)) + 3*b*(b^2 - 4*a*c)*x^3*ArcTanh[(Sqrt[c]*x - Sqrt
[a + x*(b + c*x)])/Sqrt[a]]))/(24*a^(5/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.04, size = 234, normalized size = 1.51

method	result
risch	$-\frac{(8acx^2-3b^2x^2+2abx+8a^2)\sqrt{x^2(cx^2+bx+a)}}{24x^4a^2}+\frac{\left(\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)c}{4a^{\frac{3}{2}}}-\frac{b^3\ln\left(\frac{2a+bx+2\sqrt{a}\cdots}{\cdots}\right.}{1\cdots}\right.}{x\sqrt{cx^2+bx+\cdots}}$
default	$\frac{\sqrt{cx^4+bx^3+ax^2}\left(12ca^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)bx^3+6c\sqrt{cx^2+bx+a}\,b^2x^4-12c\sqrt{cx^2+b\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(1/2)*(12*c*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b
*x+a)^(1/2))/x)*b*x^3+6*c*(c*x^2+b*x+a)^(1/2)*b^2*x^4-12*c*(c*x^2+b*x+a)^(1
/2)*a*b*x^3-3*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^3*x^3
-6*(c*x^2+b*x+a)^(3/2)*b^2*x^2+6*(c*x^2+b*x+a)^(1/2)*b^3*x^3+12*(c*x^2+b*x+
a)^(3/2)*a*b*x-16*(c*x^2+b*x+a)^(3/2)*a^2)/x^4/(c*x^2+b*x+a)^(1/2)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^5, x)
```

Fricas [A]
time = 0.35, size = 272, normalized size = 1.75

$$\left[-\frac{3\,(b^3-4\,abc)\sqrt{a}\,x^4\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,a^2bx+8\,a^3-(3\,ab^2-8\,a^2c)x^2)}{96\,a^3x^4},\frac{3\,(b^3-4\,abc)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)-2\,\sqrt{cx^4+bx^3+ax^2}\,(2\,a^2bx+8\,a^3-(3\,ab^2-8\,a^2c)x^2)}{48\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3 - 4*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 +
 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt
(c*x^4 + b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3
*x^4), 1/48*(3*(b^3 - 4*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*a^2*b*x + 8*a^3 - (3*a*b^2 - 8*a^2*c)*x^2))/(a^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a+bx+cx^2)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**5, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^5, x)
```

3.37 $\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$

Optimal. Leaf size=205

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+}}{192a^3x^2}$$

```
[Out] 1/128*(-4*a*c+b^2)*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*
x^3+a*x^2)^(1/2))/a^(7/2)-1/4*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5-1/24*b*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^4+1/96*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/
x^3-1/192*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.25, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1934, 1965, 12, 1918, 212}

$$\frac{(b^2-4ac)(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{7/2}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{192a^3x^2}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}-\frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4}-\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -1/4*Sqrt[a*x^2 + b*x^3 + c*x^4]/x^5 - (b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(24*
a*x^4) + ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(96*a^2*x^3) - (b*(
15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(192*a^3*x^2) + ((b^2 - 4*a*c
)*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

```
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} + \frac{1}{8}\int \frac{b+2cx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} - \frac{\int \frac{\frac{1}{2}(5b^2-12ac)+2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{24a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{4x^5} - \frac{b\sqrt{ax^2+bx^3+cx^4}}{24ax^4} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{96a^2x^3}$$

Mathematica [A]
time = 0.58, size = 159, normalized size = 0.78

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}\,(48a^3+15b^3x^3+8a^2x(b+3cx)-2abx^2(5b+26cx))+3(5b^4-24ab^2c+16a^2c^2)\,x^4\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{192a^{7/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3 + c*x^4]/x^6,x]
```

```
[Out] -1/192*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(48*a^3
+ 15*b^3*x^3 + 8*a^2*x*(b + 3*c*x) - 2*a*b*x^2*(5*b + 26*c*x)) + 3*(5*b^4 -
 24*a*b^2*c + 16*a^2*c^2)*x^4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/S
qrt[a]]))/(a^(7/2)*x^5*Sqrt[a + x*(b + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(179) = 358$.
time = 0.04, size = 387, normalized size = 1.89

method	result

risch	$-\frac{(-52abc\,x^3+15b^3x^3+24a^2c\,x^2-10a\,b^2x^2+8a^2bx+48a^3)\sqrt{x^2\,(c\,x^2+bx+a)}}{192x^5a^3}+\frac{\left(\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+}}{x}\right.}{8a^{\frac{3}{2}}}\right.}{}$
default	$\frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\left(48c^2a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2\sqrt{c\,x^2+bx+a}\,ab\,x^5-72c\,a^{\frac{3}{2}}\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/384*(c*x^4+b*x^3+a*x^2)^(1/2)*(48*c^2*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(1/2)*a*b*x^5-72*c*a^(3/2)*ln((
2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^2*x^4-48*c^2*(c*x^2+b*x+a)^(1/2
)*a^2*x^4-30*c*(c*x^2+b*x+a)^(1/2)*b^3*x^5-24*c*(c*x^2+b*x+a)^(3/2)*a*b*x^3
+84*c*(c*x^2+b*x+a)^(1/2)*a*b^2*x^4+15*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x)*b^4*x^4+48*c*(c*x^2+b*x+a)^(3/2)*a^2*x^2+30*(c*x^2+b*x+a)
^(3/2)*b^3*x^3-30*(c*x^2+b*x+a)^(1/2)*b^4*x^4-60*(c*x^2+b*x+a)^(3/2)*a*b^2*
x^2+80*(c*x^2+b*x+a)^(3/2)*a^2*b*x-96*(c*x^2+b*x+a)^(3/2)*a^3)/x^5/(c*x^2+b
*x+a)^(1/2)/a^4
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^3 + a*x^2)/x^6, x)
```

Fricas [**A**]
time = 0.37, size = 336, normalized size = 1.64

$$\left[\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,(8\,a^3bx+48\,a^4+(15\,ab^3-52\,a^2bc)x^3-2\,(5\,a^2b^2-12\,a^3c)x^2)\sqrt{cx^4+bx^3+ax^2}}{768\,a^4x^5},-\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^5\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(8\,a^3bx+48\,a^4+(15\,ab^3-52\,a^2bc)x^3-2\,(5\,a^2b^2-12\,a^3c)x^2)\sqrt{cx^4+bx^3+ax^2}}{384\,a^4x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (
b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt
(a))/x^3) - 4*(8*a^3*b*x + 48*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*
```

```
b^2 - 12*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^5), -1/384*(3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(8*a^3*b*x + 4
8*a^4 + (15*a*b^3 - 52*a^2*b*c)*x^3 - 2*(5*a^2*b^2 - 12*a^3*c)*x^2)*sqrt(c*
x^4 + b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\left(a+bx+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(1/2)/x**6,x)
[Out] Integral(sqrt(x**2*(a + b*x + c*x**2))/x**6, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(1/2)/x^6, x)
```

3.38 $\int x(ax^2 + bx^3 + cx^4)^{3/2}\, dx$

Optimal. Leaf size=422

$$\frac{(1155b^6 - 8988ab^4c + 18896a^2b^2c^2 - 6720a^3c^3)\sqrt{ax^2 + bx^3 + cx^4}}{286720c^5} - \frac{b(3465b^6 - 30660ab^4c + 81648a^2b^2c^2 -}{573440c^6}$$

```
[Out] 1/112*x*(14*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c+3/32768*(-4*a*c+b^2)^2*(16
*a^2*c^2-72*a*b^2*c+33*b^4)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))*(c*x^2+b*x+a)^(1/2)/c^(13/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/286720*(-672
0*a^3*c^3+18896*a^2*b^2*c^2-8988*a*b^4*c+1155*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^5-1/573440*b*(-58816*a^3*c^3+81648*a^2*b^2*c^2-30660*a*b^4*c+3465*b^6)*
(c*x^4+b*x^3+a*x^2)^(1/2)/c^6/x-1/71680*b*(2416*a^2*c^2-1560*a*b^2*c+231*b^
4)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(560*a^2*c^2-568*a*b^2*c+99*b^4)
*x^2*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/4480*x^3*(b*(68*a*c+11*b^2)+10*c*(-28*
a*c+11*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.75, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1933, 1959, 1963, 12, 1928, 635, 212}

$$\frac{3x(b^2-4ac)^2(16a^2c^2-72ab^2c+33b^4)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{32768c^{13/2}\sqrt{ax^2+bx^3+cx^4}} - \frac{bx(2416a^2c^2-1560ab^2c+231b^4)\sqrt{ax^2+bx^3+cx^4}}{71680c^4} + \frac{x^2(560a^2c^2-568ab^2c+99b^4)\sqrt{ax^2+bx^3+cx^4}}{35840c^3} - \frac{b(-58816a^3c^3+81648a^2b^2c^2-30660ab^4c+3465b^6)\sqrt{ax^2+bx^3+cx^4}}{573440c^6x} + \frac{(-6720a^3c^3+18896a^2b^2c^2-8988ab^4c+1155b^6)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{x^3(10cx(11b^2-28ac)+b(68ac+11b^2))\sqrt{ax^2+bx^3+cx^4}}{4480c^2} + \frac{x(3b+14cx)(ax^2+bx^3+cx^4)^{3/2}}{112c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] ((1155*b^6 - 8988*a*b^4*c + 18896*a^2*b^2*c^2 - 6720*a^3*c^3)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(286720*c^5) - (b*(3465*b^6 - 30660*a*b^4*c + 81648*a^2*b^2
*c^2 - 58816*a^3*c^3)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(573440*c^6*x) - (b*(231
*b^4 - 1560*a*b^2*c + 2416*a^2*c^2)*x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(71680*c
^4) + ((99*b^4 - 568*a*b^2*c + 560*a^2*c^2)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(35840*c^3) - (x^3*(b*(11*b^2 + 68*a*c) + 10*c*(11*b^2 - 28*a*c)*x)*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(4480*c^2) + (x*(3*b + 14*c*x)*(a*x^2 + b*x^3 + c*x
^4)^(3/2))/(112*c) + (3*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*
x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
]])])/(32768*c^(13/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
```

```
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x\left(ax^2+bx^3+cx^4\right)^{3/2}\,dx = \frac{x(3b+14cx)\left(ax^2+bx^3+cx^4\right)^{3/2}}{112c} + \frac{3\int x^2\left(-4ab-\frac{1}{2}\left(11b^2-28ac\right)x\right)\sqrt{ax^2}}{112c}$$

$$= -\frac{x^3\left(b\left(11b^2+68ac\right)+10c\left(11b^2-28ac\right)x\right)\sqrt{ax^2+bx^3+cx^4}}{4480c^2} + \frac{x(3b+14cx}{}$$

$$= \frac{\left(99b^4-568ab^2c+560a^2c^2\right)x^2\sqrt{ax^2+bx^3+cx^4}}{35840c^3} - \frac{x^3\left(b\left(11b^2+68ac\right)+10c\right.}{}$$

$$= -\frac{b\left(231b^4-1560ab^2c+2416a^2c^2\right)x\sqrt{ax^2+bx^3+cx^4}}{71680c^4} + \frac{\left(99b^4-568ab^2c+\right.}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(231b^4}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b}{}$$

$$= \frac{\left(1155b^6-8988ab^4c+18896a^2b^2c^2-6720a^3c^3\right)\sqrt{ax^2+bx^3+cx^4}}{286720c^5} - \frac{b(3465b}{}$$

Mathematica [A]
time = 0.90, size = 304, normalized size = 0.72

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(-3465b^7+2310b^6cx+84b^5c\left(365a-22cx^2\right)+24b^4c^2x\left(-749a+66cx^2\right)+32b^2c^3x\left(1181a^2-284acx^2+40c^2x^4\right)-16b^3c^2\left(5103a^2-780acx^2+88c^2x^4\right)+4480c^4x\left(-3a^3+2a^2cx^2+24ac^2x^4+16c^3x^6\right)+64bc^3\left(919a^3-302a^2cx^2+104ac^2x^4+1360c^3x^6\right)\right)-105\left(b^2-4ac\right)^2\left(33b^4-72ab^2c+16a^2c^2\right)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{1146880c^{13/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3465*b^7 + 2310
*b^6*c*x + 84*b^5*c*(365*a - 22*c*x^2) + 24*b^4*c^2*x*(-749*a + 66*c*x^2) +
 32*b^2*c^3*x*(1181*a^2 - 284*a*c*x^2 + 40*c^2*x^4) - 16*b^3*c^2*(5103*a^2
- 780*a*c*x^2 + 88*c^2*x^4) + 4480*c^4*x*(-3*a^3 + 2*a^2*c*x^2 + 24*a*c^2*x
```

```
^4 + 16*c^3*x^6) + 64*b*c^3*(919*a^3 - 302*a^2*c*x^2 + 104*a*c^2*x^4 + 1360
*c^3*x^6)) - 105*(b^2 - 4*a*c)^2*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*Log[b +
 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(1146880*c^(13/2)*Sqrt[x^2*(a +
 x*(b + c*x))])
```

Maple [A]
time = 0.04, size = 649, normalized size = 1.54

method	result
risch	$\frac{(71680c^7x^7+87040b\,c^6x^6+107520a\,c^6x^5+1280b^2c^5x^5+6656ab\,c^5x^4-1408b^3c^4x^4+8960a^2c^5x^3-9088a\,b^2c^4x^3+1584b^4c^3x^3-19328a}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-80640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}a\,b^2x-127680c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2b^2x+117600\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{}}{2\sqrt{c}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1146880*(c*x^4+b*x^3+a*x^2)^(3/2)*(-80640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*a*b
^2*x-127680*c^(9/2)*(c*x^2+b*x+a)^(1/2)*a^2*b^2*x+117600*ln(1/2*(2*(c*x^2+b
*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^4*c^3-35280*ln(1/2*(2*(c*x^2+b*
x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^6*c^2+18480*c^(5/2)*(c*x^2+b*x+a)^
(3/2)*b^5-134400*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^
3*b^2*c^4+143360*x^3*(c*x^2+b*x+a)^(5/2)*c^(13/2)-59136*c^(7/2)*(c*x^2+b*x+
a)^(5/2)*b^3+85680*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a*b^4*x-6930*c^(3/2)*(c*x^2+
b*x+a)^(1/2)*b^7+26880*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/
2))*a^4*c^5+3465*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^
8*c+26880*c^(11/2)*(c*x^2+b*x+a)^(1/2)*a^3*x-13860*c^(5/2)*(c*x^2+b*x+a)^(1
/2)*b^6*x+13440*c^(9/2)*(c*x^2+b*x+a)^(1/2)*a^3*b-63840*c^(7/2)*(c*x^2+b*x+
a)^(1/2)*a^2*b^3+42840*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^5+95232*c^(9/2)*(c*x
^2+b*x+a)^(5/2)*a*b+17920*c^(11/2)*(c*x^2+b*x+a)^(3/2)*a^2*x+36960*c^(7/2)*
(c*x^2+b*x+a)^(3/2)*b^4*x+8960*c^(9/2)*(c*x^2+b*x+a)^(3/2)*a^2*b-40320*c^(7
/2)*(c*x^2+b*x+a)^(3/2)*a*b^3-112640*c^(11/2)*(c*x^2+b*x+a)^(5/2)*b*x^2-716
80*c^(11/2)*(c*x^2+b*x+a)^(5/2)*a*x+84480*c^(9/2)*(c*x^2+b*x+a)^(5/2)*b^2*x
)/x^3/(c*x^2+b*x+a)^(3/2)/c^(15/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)*x, x)
```

Fricas [A]
time = 0.37, size = 664, normalized size = 1.57

$$\left[\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(71680\,c^8x^7+87040\,bc^7x^6-3465\,b^7c+30660\,ab^5c^2-81648\,a^2b^3c^3+58816\,a^3bc^4+1280\,(b^2c^6+84\,ac^7)x^5-128\,(11\,b^3c^5-52\,abc^6)x^4+16\,(99\,b^4c^4-568\,ab^2c^5+560\,a^2c^6)x^3-8\,(231\,b^5c^3-1560\,ab^3c^4+2416\,a^2bc^5)x^2+2\,(1155\,b^6c^2-8988\,ab^4c^3+18896\,a^2b^2c^4-6720\,a^3c^5)x)\sqrt{cx^4+bx^3+ax^2}}{2293760\,c^7x},\ -\frac{105\,(33\,b^8-336\,ab^6c+1120\,a^2b^4c^2-1280\,a^3b^2c^3+256\,a^4c^4)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(71680\,c^8x^7+87040\,bc^7x^6-3465\,b^7c+30660\,ab^5c^2-81648\,a^2b^3c^3+58816\,a^3bc^4+1280\,(b^2c^6+84\,ac^7)x^5-128\,(11\,b^3c^5-52\,abc^6)x^4+16\,(99\,b^4c^4-568\,ab^2c^5+560\,a^2c^6)x^3-8\,(231\,b^5c^3-1560\,ab^3c^4+2416\,a^2bc^5)x^2+2\,(1155\,b^6c^2-8988\,ab^4c^3+18896\,a^2b^2c^4-6720\,a^3c^5)x)\sqrt{cx^4+bx^3+ax^2}}{1146880\,c^7x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2293760*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3
 + 256*a^4*c^4)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(71680*c^8*x^7 + 8
7040*b*c^7*x^6 - 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a
^3*b*c^4 + 1280*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^
4 + 16*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 15
60*a*b^3*c^4 + 2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 188
96*a^2*b^2*c^4 - 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x), -1/
1146880*(105*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 +
256*a^4*c^4)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(71680*c^8*x^7 + 87040*b*c^7*x^6
- 3465*b^7*c + 30660*a*b^5*c^2 - 81648*a^2*b^3*c^3 + 58816*a^3*b*c^4 + 1280
*(b^2*c^6 + 84*a*c^7)*x^5 - 128*(11*b^3*c^5 - 52*a*b*c^6)*x^4 + 16*(99*b^4*
c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*x^3 - 8*(231*b^5*c^3 - 1560*a*b^3*c^4 +
2416*a^2*b*c^5)*x^2 + 2*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2*b^2*c^4
- 6720*a^3*c^5)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^7*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 5.40, size = 521, normalized size = 1.23

$$\frac{1}{573440}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\left(4\left(14\,cx\,\mathrm{sgn}(x)+17\,b\,\mathrm{sgn}(x)\right)x+\frac{b^2c^6\mathrm{sgn}(x)+84\,ac^7\mathrm{sgn}(x)}{c^7}\right)x-\frac{11\,b^3c^5\mathrm{sgn}(x)-52\,abc^6\mathrm{sgn}(x)}{c^7}\right)x+\frac{99\,b^4c^4\mathrm{sgn}(x)-568\,ab^2c^5\mathrm{sgn}(x)+560\,a^2c^6\mathrm{sgn}(x)}{c^7}\right)x-\frac{231\,b^5c^3\mathrm{sgn}(x)-1560\,ab^3c^4\mathrm{sgn}(x)+2416\,a^2bc^5\mathrm{sgn}(x)}{c^7}\right)x+\frac{1155\,b^6c^2\mathrm{sgn}(x)-8988\,ab^4c^3\mathrm{sgn}(x)+18896\,a^2b^2c^4\mathrm{sgn}(x)-6720\,a^3c^5\mathrm{sgn}(x)}{c^7}\right)x-\frac{3465\,b^7c\,\mathrm{sgn}(x)-30660\,ab^5c^2\mathrm{sgn}(x)+81648\,a^2b^3c^3\mathrm{sgn}(x)-58816\,a^3bc^4\mathrm{sgn}(x)}{c^7}\right)-\frac{3\,(33\,b^8\mathrm{sgn}(x)-336\,ab^6c\,\mathrm{sgn}(x)+1120\,a^2b^4c^2\mathrm{sgn}(x)-1280\,a^3b^2c^3\mathrm{sgn}(x)+256\,a^4c^4\mathrm{sgn}(x))\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{32768\,c^{\frac{15}{2}}}+\frac{\left(3465\,b^8\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-35280\,ab^6c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+117600\,a^2b^4c^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-134400\,a^3b^2c^3\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+26880\,a^4c^4\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+6930\sqrt{a}\,b^7\sqrt{c}-61320\,a^{\frac{3}{2}}b^5c^{\frac{3}{2}}+163296\,a^{\frac{5}{2}}b^3c^{\frac{5}{2}}-117632\,a^{\frac{7}{2}}bc^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{1146880\,c^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/573440*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*(14*c*x*sgn(x) + 17*b*sgn
(x))*x + (b^2*c^6*sgn(x) + 84*a*c^7*sgn(x))/c^7)*x - (11*b^3*c^5*sgn(x) - 5
2*a*b*c^6*sgn(x))/c^7)*x + (99*b^4*c^4*sgn(x) - 568*a*b^2*c^5*sgn(x) + 560*
a^2*c^6*sgn(x))/c^7)*x - (231*b^5*c^3*sgn(x) - 1560*a*b^3*c^4*sgn(x) + 2416
*a^2*b*c^5*sgn(x))/c^7)*x + (1155*b^6*c^2*sgn(x) - 8988*a*b^4*c^3*sgn(x) +
18896*a^2*b^2*c^4*sgn(x) - 6720*a^3*c^5*sgn(x))/c^7)*x - (3465*b^7*c*sgn(x)
 - 30660*a*b^5*c^2*sgn(x) + 81648*a^2*b^3*c^3*sgn(x) - 58816*a^3*b*c^4*sgn(
x))/c^7) - 3/32768*(33*b^8*sgn(x) - 336*a*b^6*c*sgn(x) + 1120*a^2*b^4*c^2*s
gn(x) - 1280*a^3*b^2*c^3*sgn(x) + 256*a^4*c^4*sgn(x))*log(abs(-2*(sqrt(c)*x
 - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2) + 1/1146880*(3465*b^8*log(
abs(-b + 2*sqrt(a)*sqrt(c))) - 35280*a*b^6*c*log(abs(-b + 2*sqrt(a)*sqrt(c)
)) + 117600*a^2*b^4*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 134400*a^3*b^2*c
^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 26880*a^4*c^4*log(abs(-b + 2*sqrt(a)*
sqrt(c))) + 6930*sqrt(a)*b^7*sqrt(c) - 61320*a^(3/2)*b^5*c^(3/2) + 163296*a
^(5/2)*b^3*c^(5/2) - 117632*a^(7/2)*b*c^(7/2))*sgn(x)/c^(13/2)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x*(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.39 $\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=364

$$-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+5488a^2b^2c^2-2048a^3c^3)\sqrt{ax^2+b}}{35840c^5x}$$

```
[Out] 1/7*x*(c*x^4+b*x^3+a*x^2)^(3/2)-3/2048*b*(-4*a*c+b^2)^2*(-4*a*c+3*b^2)*x*ar
ctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(11/
2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/17920*b*(1168*a^2*c^2-728*a*b^2*c+105*b^4)*(
c*x^4+b*x^3+a*x^2)^(1/2)/c^4+1/35840*(-2048*a^3*c^3+5488*a^2*b^2*c^2-2520*a
*b^4*c+315*b^6)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^5/x+1/4480*(-32*a*c+7*b^2)*(-4*
a*c+3*b^2)*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/2240*b*(-44*a*c+9*b^2)*x^2*(c*
x^4+b*x^3+a*x^2)^(1/2)/c^2+1/280*x^3*(10*b*c*x+24*a*c+b^2)*(c*x^4+b*x^3+a*x
^2)^(1/2)/c
```

Rubi [A]
time = 0.66, antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1920, 1959, 1963, 12, 1928, 635, 212}

$$-\frac{b(1168a^2c^2-728ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(-2048a^3c^3+5488a^2b^2c^2-2520ab^4c+315b^6)\sqrt{ax^2+bx^3+cx^4}}{35840c^5x}-\frac{3bx(b^2-4ac)^2(3b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2048c^{11/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{x(7b^2-32ac)(3b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{bx^2(9b^2-44ac)\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(24ac+b^2+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] -1/17920*(b*(105*b^4 - 728*a*b^2*c + 1168*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x
^4])/c^4 + ((315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(35840*c^5*x) + ((7*b^2 - 32*a*c)*(3*b^2 - 4*a*c)*
x*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4480*c^3) - (b*(9*b^2 - 44*a*c)*x^2*Sqrt[a*
x^2 + b*x^3 + c*x^4])/(2240*c^2) + (x^3*(b^2 + 24*a*c + 10*b*c*x)*Sqrt[a*x^
2 + b*x^3 + c*x^4])/(280*c) + (x*(a*x^2 + b*x^3 + c*x^4)^(3/2))/7 - (3*b*(b
^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2048*c^(11/2)*Sqrt[a*x^2 + b*x^3 + c*x^
4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1920

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[x*((a*x^q + b*x^n + c*x^(2*n - q))^p/(p*(2*n - q) + 1)), x] + Dis
t[(n - q)*(p/(p*(2*n - q) + 1)), Int[x^q*(2*a + b*x^(n - q))*(a*x^q + b*x^n
 + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*
n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0]
&& NeQ[p*(2*n - q) + 1, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
```

```
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \left(ax^2+bx^3+cx^4\right)^{3/2}\,dx=\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{3}{14}\int x^2(2a+bx)\sqrt{ax^2+bx^3+cx^4}\,dx$$

$$=\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+cx^4}}{280c}+\frac{1}{7}x\left(ax^2+bx^3+cx^4\right)^{3/2}+\frac{\int\frac{x^4(-4\cdots}{\cdots}}{\cdots}$$

$$=-\frac{b(9b^2-44ac)\,x^2\sqrt{ax^2+bx^3+cx^4}}{2240c^2}+\frac{x^3(b^2+24ac+10bcx)\sqrt{ax^2+bx^3+c\cdots}}{280c}$$

$$=\frac{(7b^2-32ac)\,(3b^2-4ac)\,x\sqrt{ax^2+bx^3+cx^4}}{4480c^3}-\frac{b(9b^2-44ac)\,x^2\sqrt{ax^2+bx^3+\cdots}}{2240c^2}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(7b^2-32ac)\,(3b^2-4a\cdots}{448\cdots}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+54\cdots}{\cdots}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+54\cdots}{\cdots}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+54\cdots}{\cdots}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+54\cdots}{\cdots}$$

$$=-\frac{b(105b^4-728ab^2c+1168a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{17920c^4}+\frac{(315b^6-2520ab^4c+54\cdots}{\cdots}$$

Mathematica [A]

time = 0.70, size = 239, normalized size = 0.66

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(315b^6-210b^5cx+16b^3c^2x(91a-9cx^2)+168b^4c(-15a+cx^2)+1024c^3(a+cx^2)^2(-2a+5cx^2)+16b^2c^2(343a^2-62acx^2+8c^2x^4)+32bc^3x(-73a^2+22acx^2+200c^2x^4)\right)+105b(b^2-4ac)^2(3b^2-4ac)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{71680c^{11/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(315*b^6 - 210*b^
5*c*x + 16*b^3*c^2*x*(91*a - 9*c*x^2) + 168*b^4*c*(-15*a + c*x^2) + 1024*c^
3*(a + c*x^2)^2*(-2*a + 5*c*x^2) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^2 + 8*c^2
*x^4) + 32*b*c^3*x*(-73*a^2 + 22*a*c*x^2 + 200*c^2*x^4)) + 105*b*(b^2 - 4*a
*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(7
1680*c^(11/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]

time = 0.04, size = 479, normalized size = 1.32

method	result
risch	$-\frac{\left(-5120c^6x^6-6400b\,c^5x^5-8192a\,c^5x^4-128b^2c^4x^4-704ab\,c^4x^3+144b^3c^3x^3-1024a^2c^4x^2+992a\,b^2c^3x^2-168b^4c^2x^2+2336a^2b\,c^3x-\right.}{35840c^5x}$
default	$\frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(10240x^2\left(c\,x^2+bx+a\right)^{\frac{5}{2}}c^{\frac{11}{2}}-7680c^{\frac{9}{2}}\left(c\,x^2+bx+a\right)^{\frac{5}{2}}bx-4096c^{\frac{9}{2}}\left(c\,x^2+bx+a\right)^{\frac{5}{2}}a+4480c^{\frac{9}{2}}\left(c\,x^2+bx+a\right)^{\frac{3}{2}}abx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/71680*(c*x^4+b*x^3+a*x^2)^(3/2)*(10240*x^2*(c*x^2+b*x+a)^(5/2)*c^(11/2)-7
680*c^(9/2)*(c*x^2+b*x+a)^(5/2)*b*x-4096*c^(9/2)*(c*x^2+b*x+a)^(5/2)*a+4480
*c^(9/2)*(c*x^2+b*x+a)^(3/2)*a*b*x+6720*c^(9/2)*(c*x^2+b*x+a)^(1/2)*a^2*b*x
+5376*c^(7/2)*(c*x^2+b*x+a)^(5/2)*b^2-3360*c^(7/2)*(c*x^2+b*x+a)^(3/2)*b^3*
x+2240*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b^2-6720*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a
*b^3*x+3360*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b^2-1680*c^(5/2)*(c*x^2+b*x+a)^
(3/2)*b^4+1260*c^(5/2)*(c*x^2+b*x+a)^(1/2)*b^5*x-3360*c^(5/2)*(c*x^2+b*x+a)
^(1/2)*a*b^4+630*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^6+6720*ln(1/2*(2*(c*x^2+b*x+
a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*b*c^4-8400*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^3*c^3+2940*ln(1/2*(2*(c*x^2+b*x+a)^(1/
2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^5*c^2-315*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^
(1/2)+2*c*x+b)/c^(1/2))*b^7*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(13/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.38, size = 558, normalized size = 1.53

$$\left[-\frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,(5120\,c^7x^6+6400\,bc^6x^5+315\,b^6c-2520\,ab^4c^2+5488\,a^2b^2c^3-2048\,a^3c^4+128\,(b^2c^5+64\,ac^6)x^4-16\,(9\,b^3c^4-44\,abc^5)x^3+8\,(21\,b^4c^3-124\,ab^2c^4+128\,a^2c^5)x^2-2\,(105\,b^5c^2-728\,ab^3c^3+1168\,a^2bc^4)x)\sqrt{cx^4+bx^3+ax^2}}{143360\,c^6x}, \frac{105\,(3\,b^7-28\,ab^5c+80\,a^2b^3c^2-64\,a^3bc^3)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,(5120\,c^7x^6+6400\,bc^6x^5+315\,b^6c-2520\,ab^4c^2+5488\,a^2b^2c^3-2048\,a^3c^4+128\,(b^2c^5+64\,ac^6)x^4-16\,(9\,b^3c^4-44\,abc^5)x^3+8\,(21\,b^4c^3-124\,ab^2c^4+128\,a^2c^5)x^2-2\,(105\,b^5c^2-728\,ab^3c^3+1168\,a^2bc^4)x)\sqrt{cx^4+bx^3+ax^2}}{71680\,c^6x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/143360*(105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(c
)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*(5120*c^7*x^6 + 6400*b*c^6*x^5 + 315*b^6
*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4 + 128*(b^2*c^5 + 64*a
*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b^4*c^3 - 124*a*b^2*c^4
 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3 + 1168*a^2*b*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x), 1/71680*(105*(3*b^7 - 28*a*b^5*c + 80*
a^2*b^3*c^2 - 64*a^3*b*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(5120*c^7*x^6 + 64
00*b*c^6*x^5 + 315*b^6*c - 2520*a*b^4*c^2 + 5488*a^2*b^2*c^3 - 2048*a^3*c^4
 + 128*(b^2*c^5 + 64*a*c^6)*x^4 - 16*(9*b^3*c^4 - 44*a*b*c^5)*x^3 + 8*(21*b
^4*c^3 - 124*a*b^2*c^4 + 128*a^2*c^5)*x^2 - 2*(105*b^5*c^2 - 728*a*b^3*c^3
+ 1168*a^2*b*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^6*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^2+bx^3+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(3/2), x)
```

Giac [A]
time = 4.67, size = 429, normalized size = 1.18

$$\frac{1}{35840}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,(4\,cx\mathrm{sgn}(x)+5\,b\mathrm{sgn}(x))x+\frac{b^2c^5\mathrm{sgn}(x)+64\,ac^6\mathrm{sgn}(x)}{c^6}\right)x-\frac{9\,b^3c^4\mathrm{sgn}(x)-44\,abc^5\mathrm{sgn}(x)}{c^6}\right)x+\frac{21\,b^4c^3\mathrm{sgn}(x)-124\,ab^2c^4\mathrm{sgn}(x)+128\,a^2c^5\mathrm{sgn}(x)}{c^6}\right)x-\frac{105\,b^5c^2\mathrm{sgn}(x)-728\,ab^3c^3\mathrm{sgn}(x)+1168\,a^2bc^4\mathrm{sgn}(x)}{c^6}\right)x+\frac{315\,b^6c\mathrm{sgn}(x)-2520\,ab^4c^2\mathrm{sgn}(x)+5488\,a^2b^2c^3\mathrm{sgn}(x)-2048\,a^3c^4\mathrm{sgn}(x)}{c^6}\right)+\frac{3\,(3\,b^7\mathrm{sgn}(x)-28\,ab^5c\mathrm{sgn}(x)+80\,a^2b^3c^2\mathrm{sgn}(x)-64\,a^3bc^3\mathrm{sgn}(x))\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2048\,c^{\frac{11}{2}}}-\frac{\left(315\,b^7\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-2940\,ab^5c\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+8400\,a^2b^3c^2\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-6720\,a^3bc^3\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+630\,\sqrt{a}\,b^6\sqrt{c}-5040\,a^{\frac{3}{2}}b^4c^{\frac{3}{2}}+10976\,a^{\frac{5}{2}}b^2c^{\frac{5}{2}}-4096\,a^{\frac{7}{2}}c^{\frac{7}{2}}\right)\mathrm{sgn}(x)}{71680\,c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/35840*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(4*c*x*sgn(x) + 5*b*sgn(x))*x
 + (b^2*c^5*sgn(x) + 64*a*c^6*sgn(x))/c^6)*x - (9*b^3*c^4*sgn(x) - 44*a*b*c
^5*sgn(x))/c^6)*x + (21*b^4*c^3*sgn(x) - 124*a*b^2*c^4*sgn(x) + 128*a^2*c^5
*sgn(x))/c^6)*x - (105*b^5*c^2*sgn(x) - 728*a*b^3*c^3*sgn(x) + 1168*a^2*b*c
^4*sgn(x))/c^6)*x + (315*b^6*c*sgn(x) - 2520*a*b^4*c^2*sgn(x) + 5488*a^2*b^
2*c^3*sgn(x) - 2048*a^3*c^4*sgn(x))/c^6) + 3/2048*(3*b^7*sgn(x) - 28*a*b^5*
c*sgn(x) + 80*a^2*b^3*c^2*sgn(x) - 64*a^3*b*c^3*sgn(x))*log(abs(-2*(sqrt(c)
*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2) - 1/71680*(315*b^7*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 2940*a*b^5*c*log(abs(-b + 2*sqrt(a)*sqrt(c)))
 + 8400*a^2*b^3*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 6720*a^3*b*c^3*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) + 630*sqrt(a)*b^6*sqrt(c) - 5040*a^(3/2)*b^4*c^
(3/2) + 10976*a^(5/2)*b^2*c^(5/2) - 4096*a^(7/2)*c^(7/2))*sgn(x)/c^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.40 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=288

$$\frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}-\frac{b(105b^4-760ab^2c+1296a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}-\frac{x(b(7b^2$$

```
[Out] 1/60*(10*c*x+3*b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x+1/1024*(-4*a*c+b^2)^2*(-4*a
*c+7*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a
)^(1/2)/c^(9/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/3840*(240*a^2*c^2-216*a*b^2*c+3
5*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3-1/7680*b*(1296*a^2*c^2-760*a*b^2*c+105
*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^4/x-1/960*x*(b*(12*a*c+7*b^2)+6*c*(-20*a*
c+7*b^2)*x)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2
```

Rubi [A]
time = 0.34, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1933, 1948, 1963, 12, 1928, 635, 212}

$$-\frac{b(1296a^2c^2-760ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{7680c^4x}+\frac{(240a^2c^2-216ab^2c+35b^4)\sqrt{ax^2+bx^3+cx^4}}{3840c^3}+\frac{x(7b^2-4ac)(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{1024c^{9/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{x(6cx(7b^2-20ac)+b(12ac+7b^2))\sqrt{ax^2+bx^3+cx^4}}{960c^2}+\frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((35*b^4 - 216*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3840*c^
3) - (b*(105*b^4 - 760*a*b^2*c + 1296*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(7680*c^4*x) - (x*(b*(7*b^2 + 12*a*c) + 6*c*(7*b^2 - 20*a*c)*x)*Sqrt[a*x^2
 + b*x^3 + c*x^4])/(960*c^2) + ((3*b + 10*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2
))/(60*c*x) + ((b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1948

```
Int[((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_)*((A_) + (
B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x*(b*B*(n - q)*p + A*c*(p*q + (n - q)*(
2*p + 1) + 1) + B*c*(p*(2*n - q) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n
 - q)*(p/(c*(p*(2*n - q) + 1)*(p*q + (n - q)*(2*p + 1) + 1))), Int[x^q*(2*a
*A*c*(p*q + (n - q)*(2*p + 1) + 1) - a*b*B*(p*q + 1) + (2*a*B*c*(p*(2*n - q
) + 1) + A*b*c*(p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(p*q + (n - q)*p + 1))
*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b,
 c, A, B, n, q}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] &&
 NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p*(2*n - q) + 1, 0] && NeQ[p*q + (
n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
```

```
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x}\,dx = \frac{(3b+10cx)(ax^2+bx^3+cx^4)^{3/2}}{60cx} + \frac{\int \left(-2ab+\frac{1}{2}(-7b^2+20ac)\,x\right)\sqrt{ax^2+bx^3}}{20c}$$

$$= -\frac{x(b(7b^2+12ac)+6c(7b^2-20ac)\,x)\sqrt{ax^2+bx^3+cx^4}}{960c^2} + \frac{(3b+10cx)(ax^2}{60cx}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{x(b(7b^2+12ac)+6c(7b^2-2}{960}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c}{7680c^4x}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c}{7680c^4x}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c}{7680c^4x}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c}{7680c^4x}$$

$$= \frac{(35b^4-216ab^2c+240a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{3840c^3} - \frac{b(105b^4-760ab^2c+1296a^2c}{7680c^4x}$$

Mathematica [A]
time = 0.56, size = 211, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(-105b^5+70b^4cx+8b^3c(95a-7cx^2)+48b^2c^2x(-9a+cx^2)+160c^3x(3a^2+14acx^2+8c^2x^4)+16bc^2(-81a^2+18acx^2+104c^2x^4))-15(b^2-4ac)^2(7b^2-4ac)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{15360c^{9/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x]
```

[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-105*b^5 + 70*b^4*c*x + 8*b^3*c*(95*a - 7*c*x^2) + 48*b^2*c^2*x*(-9*a + c*x^2) + 160*c^3*x*(3*a^2 + 14*a*c*x^2 + 8*c^2*x^4) + 16*b*c^2*(-81*a^2 + 18*a*c*x^2 + 104*c^2*x^4)) - 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]))/(15360*c^(9/2)*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A]
time = 0.04, size = 431, normalized size = 1.50

method	result
risch	$-\frac{(-1280c^5x^5-1664b\,c^4x^4-2240a\,c^4x^3-48b^2c^3x^3-288ab\,c^3x^2+56b^3c^2x^2-480a^2c^3x+432a\,b^2c^2x-70b^4cx+1296a^2b\,c^2-760a\,b^3c+}{7680c^4x}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(2560x(c\,x^2+bx+a)^{\frac{5}{2}}c^{\frac{9}{2}}-640c^{\frac{9}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}ax-960c^{\frac{9}{2}}\sqrt{c\,x^2+bx+a}\,a^2x-1792c^{\frac{7}{2}}(c\,x^2+bx+a)^{\frac{5}{2}}b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)

[Out] 1/15360*(c*x^4+b*x^3+a*x^2)^(3/2)*(2560*x*(c*x^2+b*x+a)^(5/2)*c^(9/2)-640*c^(9/2)*(c*x^2+b*x+a)^(3/2)*a*x-960*c^(9/2)*(c*x^2+b*x+a)^(1/2)*a^2*x-1792*c^(7/2)*(c*x^2+b*x+a)^(5/2)*b+1120*c^(7/2)*(c*x^2+b*x+a)^(3/2)*b^2*x-320*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*b+1920*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a*b^2*x-480*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a^2*b+560*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^3-420*c^(5/2)*(c*x^2+b*x+a)^(1/2)*b^4*x+960*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b^3-210*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^5-960*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^3*c^4+2160*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b^2*c^3-900*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^4*c^2+105*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^6*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(11/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x, x)

Fricas [A]
time = 0.37, size = 474, normalized size = 1.65

$$\left[-\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,(1280\,c^6x^5+1664\,bc^5x^4-105\,b^5c+760\,ab^3c^2-1296\,a^2bc^3+16\,(3\,b^2c^4+140\,ac^5)x^3-8\,(7\,b^3c^3-36\,abc^4)x^2+2\,(35\,b^4c^2-216\,ab^2c^3+240\,a^2c^4)x)\sqrt{cx^4+bx^3+ax^2}}{30720\,c^5x},\ -\frac{15\,(7\,b^6-60\,ab^4c+144\,a^2b^2c^2-64\,a^3c^3)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)-2\,(1280\,c^6x^5+1664\,bc^5x^4-105\,b^5c+760\,ab^3c^2-1296\,a^2bc^3+16\,(3\,b^2c^4+140\,ac^5)x^3-8\,(7\,b^3c^3-36\,abc^4)x^2+2\,(35\,b^4c^2-216\,ab^2c^3+240\,a^2c^4)x)\sqrt{cx^4+bx^3+ax^2}}{15360\,c^5x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(c)*x
*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sq
rt(c) + (b^2 + 4*a*c)*x)/x) - 4*(1280*c^6*x^5 + 1664*b*c^5*x^4 - 105*b^5*c
+ 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 + 140*a*c^5)*x^3 - 8*(7*b^
3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*x)*s
qrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x), -1/15360*(15*(7*b^6 - 60*a*b^4*c + 144
*a^2*b^2*c^2 - 64*a^3*c^3)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(1280*c^6*x^5 + 166
4*b*c^5*x^4 - 105*b^5*c + 760*a*b^3*c^2 - 1296*a^2*b*c^3 + 16*(3*b^2*c^4 +
140*a*c^5)*x^3 - 8*(7*b^3*c^3 - 36*a*b*c^4)*x^2 + 2*(35*b^4*c^2 - 216*a*b^2
*c^3 + 240*a^2*c^4)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^5*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(a+bx+cx^{2})\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x, x)
```

Giac [A]
time = 4.97, size = 365, normalized size = 1.27

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,cx\mathrm{sgn}(x)+13\,b\mathrm{sgn}(x)\right)x+\frac{3\,b^2c^4\mathrm{sgn}(x)+140\,ac^5\mathrm{sgn}(x)}{c^5}\right)x-\frac{7\,b^3c^3\mathrm{sgn}(x)-36\,abc^4\mathrm{sgn}(x)}{c^5}\right)x+\frac{35\,b^4c^2\mathrm{sgn}(x)-216\,ab^2c^3\mathrm{sgn}(x)+240\,a^2c^4\mathrm{sgn}(x)}{c^5}\right)x-\frac{105\,b^5c\mathrm{sgn}(x)-760\,ab^3c^2\mathrm{sgn}(x)+1296\,a^2bc^3\mathrm{sgn}(x)}{c^5}\right)-\frac{\left(7\,b^6\mathrm{sgn}(x)-60\,ab^4c\mathrm{sgn}(x)+144\,a^2b^2c^2\mathrm{sgn}(x)-64\,a^3c^3\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{1024\,c^{\frac{9}{2}}}+\frac{\left(105\,b^6\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-900\,ab^4c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2160\,a^2b^2c^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-960\,a^3c^3\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+210\sqrt{a}b^5\sqrt{c}-1520\,a^{\frac{3}{2}}b^3c^{\frac{3}{2}}+2592\,a^{\frac{5}{2}}bc^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{15360\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*x*sgn(x) + 13*b*sgn(x))*x +
(3*b^2*c^4*sgn(x) + 140*a*c^5*sgn(x))/c^5)*x - (7*b^3*c^3*sgn(x) - 36*a*b*c
^4*sgn(x))/c^5)*x + (35*b^4*c^2*sgn(x) - 216*a*b^2*c^3*sgn(x) + 240*a^2*c^4
*sgn(x))/c^5)*x - (105*b^5*c*sgn(x) - 760*a*b^3*c^2*sgn(x) + 1296*a^2*b*c^3
*sgn(x))/c^5) - 1/1024*(7*b^6*sgn(x) - 60*a*b^4*c*sgn(x) + 144*a^2*b^2*c^2*
sgn(x) - 64*a^3*c^3*sgn(x))*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) - b))/c^(9/2) + 1/15360*(105*b^6*log(abs(-b + 2*sqrt(a)*sqrt(c))) -
 900*a*b^4*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2160*a^2*b^2*c^2*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 960*a^3*c^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210
*sqrt(a)*b^5*sqrt(c) - 1520*a^(3/2)*b^3*c^(3/2) + 2592*a^(5/2)*b*c^(5/2))*s
gn(x)/c^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x, x)
```

3.41 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=198

$$\frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}+\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}-\frac{3b(b^2-4}{}$$

```
[Out] -1/16*b*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c^2/x^3+1/5*(c*x^4+b*x^3+a*x^2)
^(5/2)/c/x^5-3/256*b*(-4*a*c+b^2)^2*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+
b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/128*b
*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/x
```

Rubi [A]
time = 0.12, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1931, 1932, 1928, 635, 212}

$$-\frac{3bx(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x}-\frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3}+\frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(128*c^3*x) - (
b*(b + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(16*c^2*x^3) + (a*x^2 + b*x^3
+ c*x^4)^(5/2)/(5*c*x^5) - (3*b*(b^2 - 4*a*c)^2*x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*Sqrt[a*x^
2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
```

```
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1931

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^2}\,dx = \frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5} - \frac{b\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx}{2c}$$

$$= -\frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} + \frac{(ax^2+bx^3+cx^4)^{5/2}}{5cx^5} + \frac{(3b(b^2-4ac))\int$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} +$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} +$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} +$$

$$= \frac{3b(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{128c^3x} - \frac{b(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{16c^2x^3} +$$

Mathematica [A]
time = 0.41, size = 161, normalized size = 0.81

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\left(15b^4-10b^3cx+128c^2(a+cx^2)^2+4b^2c(-25a+2cx^2)+8bc^2x(7a+22cx^2)\right)+15b(b^2-4ac)^2\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{1280c^{7/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^4 - 10*b^3*
c*x + 128*c^2*(a + c*x^2)^2 + 4*b^2*c*(-25*a + 2*c*x^2) + 8*b*c^2*x*(7*a +
22*c*x^2)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b +
 c*x)]]))/(1280*c^(7/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 289, normalized size = 1.46

method	result
risch	$\frac{\left(128c^4x^4+176bc^3x^3+256ac^3x^2+8b^2c^2x^2+56abc^2x-10b^3cx+128a^2c^2-100ab^2c+15b^4\right)\sqrt{x^2\left(cx^2+bx+a\right)}}{640c^3x}+\frac{\left(-\frac{3b\ln\left(\frac{b}{2}\ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\left(cx^4+bx^3+ax^2\right)^{\frac{3}{2}}\left(256\left(cx^2+bx+a\right)^{\frac{5}{2}}c^{\frac{7}{2}}-160c^{\frac{7}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}bx-80c^{\frac{5}{2}}\left(cx^2+bx+a\right)^{\frac{3}{2}}b^2-240c^{\frac{7}{2}}\sqrt{cx^2+bx+a}\,abx+60c^{\frac{5}{2}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(256*(c*x^2+b*x+a)^(5/2)*c^(7/2)-160*c^(7/
2)*(c*x^2+b*x+a)^(3/2)*b*x-80*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^2-240*c^(7/2)*(
c*x^2+b*x+a)^(1/2)*a*b*x+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*b^3*x-120*c^(5/2)*(
c*x^2+b*x+a)^(1/2)*a*b^2+30*c^(3/2)*(c*x^2+b*x+a)^(1/2)*b^4-240*ln(1/2*(2*(
c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*b*c^3+120*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^3*c^2-15*ln(1/2*(2*(c*x^2+b*x+a
)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^5*c)/x^3/(c*x^2+b*x+a)^(3/2)/c^(9/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^2, x)
```

Fricas [**A**]
time = 0.36, size = 384, normalized size = 1.94

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^3+8\,bcx^2-4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)+4\,(128\,c^5x^4+176\,bc^4x^3+15\,b^4c-100\,ab^2c^2+128\,a^2c^3+8\,(b^2c^3+32\,ac^4)x^2-2\,(5\,b^3c^2-28\,abc^3)x)\sqrt{cx^4+bx^3+ax^2}}{2560\,c^4x}, \frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,(128\,c^5x^4+176\,bc^4x^3+15\,b^4c-100\,ab^2c^2+128\,a^2c^3+8\,(b^2c^3+32\,ac^4)x^2-2\,(5\,b^3c^2-28\,abc^3)x)\sqrt{cx^4+bx^3+ax^2}}{1280\,c^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*
b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)
*x)/x) + 4*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^
2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x)*sqrt(c*x
^4 + b*x^3 + a*x^2))/(c^4*x), 1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*s
qrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*
x^3 + b*c*x^2 + a*c*x)) + 2*(128*c^5*x^4 + 176*b*c^4*x^3 + 15*b^4*c - 100*a
*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^2 - 2*(5*b^3*c^2 - 28*a*b
*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^4*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**2, x)
```

Giac [**A**]
time = 3.05, size = 284, normalized size = 1.43

$$\frac{1}{640}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,(8\,cx\mathrm{sgn}(x)+11\,b\mathrm{sgn}(x))x+\frac{b^2c^3\mathrm{sgn}(x)+32\,ac^4\mathrm{sgn}(x)}{c^4}\right)x-\frac{5\,b^3c^2\mathrm{sgn}(x)-28\,abc^3\mathrm{sgn}(x)}{c^4}\right)x+\frac{15\,b^4c\mathrm{sgn}(x)-100\,ab^2c^2\mathrm{sgn}(x)+128\,a^2c^3\mathrm{sgn}(x)}{c^4}\right)+\frac{3\,(b^5\mathrm{sgn}(x)-8\,ab^3c\mathrm{sgn}(x)+16\,a^2bc^2\mathrm{sgn}(x))\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{7}{2}}}-\frac{\left(15\,b^5\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-120\,ab^3c\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+240\,a^2bc^2\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+30\,\sqrt{a}\,b^4\sqrt{c}-200\,a^{\frac{3}{2}}b^2c^{\frac{3}{2}}+256\,a^{\frac{5}{2}}c^{\frac{5}{2}}\right)\mathrm{sgn}(x)}{1280\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/640*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*c*x*sgn(x) + 11*b*sgn(x))*x + (b^2*
c^3*sgn(x) + 32*a*c^4*sgn(x))/c^4)*x - (5*b^3*c^2*sgn(x) - 28*a*b*c^3*sgn(x
))/c^4)*x + (15*b^4*c*sgn(x) - 100*a*b^2*c^2*sgn(x) + 128*a^2*c^3*sgn(x))/c
^4) + 3/256*(b^5*sgn(x) - 8*a*b^3*c*sgn(x) + 16*a^2*b*c^2*sgn(x))*log(abs(-
2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2) - 1/1280*(15*b^
5*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 120*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqr
```

```
t(c))) + 240*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^4*sq
rt(c) - 200*a^(3/2)*b^2*c^(3/2) + 256*a^(5/2)*c^(5/2))*sgn(x)/c^(7/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^2, x)
```

3.42 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=165

$$-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}+\frac{3(b^2-4ac)^2x\sqrt{a+bx+cx^2}\ \mathrm{t}}{128c^{5/2}\sqrt{ax^2}}$$

```
[Out] 1/8*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(3/2)/c/x^3+3/128*(-4*a*c+b^2)^2*x*arctan
h(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c
*x^4+b*x^3+a*x^2)^(1/2)-3/64*(-4*a*c+b^2)*(2*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/
2)/c^2/x
```

Rubi [A]
time = 0.08, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1932, 1928, 635, 212}

$$\frac{3x(b^2-4ac)^2\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3(b^2-4ac)(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x}+\frac{(b+2cx)(ax^2+bx^3+cx^4)^{3/2}}{8cx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*c^2*x) + ((b
 + 2*c*x)*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(8*c*x^3) + (3*(b^2 - 4*a*c)^2*x*S
qrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(128*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
```

```
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^3}\,dx = \frac{(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} - \frac{(3(b^2-4ac))\int \frac{\sqrt{ax^2+bx^3+cx^4}}{x}\,dx}{16c}$$

$$= -\frac{3(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{64c^2x} + \frac{(b+2cx)\,(ax^2+bx^3+cx^4)^{3/2}}{8cx^3} +$$

Mathematica [A]
time = 0.34, size = 130, normalized size = 0.79

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\,(b+2cx)\sqrt{a+x(b+cx)}\,(-3b^2+8bcx+4c(5a+2cx^2))-3(b^2-4ac)^2\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)}{128c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)]*(-3*b
^2 + 8*b*c*x + 4*c*(5*a + 2*c*x^2)) - 3*(b^2 - 4*a*c)^2*Log[b + 2*c*x - 2*S
qrt[c]*Sqrt[a + x*(b + c*x)]]))/(128*c^(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 265, normalized size = 1.61

method	result
risch	$\frac{\left(16 c^{3} x^{3}+24 b \,c^{2} x^{2}+40 a \,c^{2} x +2 b^{2} c x +20 a b c -3 b^{3}\right) \sqrt{x^{2} \left(c \,x^{2}+b x +a \right)}}{64 c^{2} x}+\frac{\left(\frac{3 a^{2} \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \,x^{2}+b x +a}\right)}{8 \sqrt{c}}-\frac{3 a \,b^{2} \ln}{}\right.}{}$
default	$\frac{\left(c \,x^{4}+b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(32 x \left(c \,x^{2}+b x +a \right)^{\frac{3}{2}} c^{\frac{7}{2}}+16 c^{\frac{5}{2}} \left(c \,x^{2}+b x +a \right)^{\frac{3}{2}} b +48 c^{\frac{7}{2}} \sqrt{c \,x^{2}+b x +a}\, a x -12 c^{\frac{5}{2}} \sqrt{c \,x^{2}+b x +a}\, b^{2} x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*x*(c*x^2+b*x+a)^(3/2)*c^(7/2)+16*c^(5/2
)*(c*x^2+b*x+a)^(3/2)*b+48*c^(7/2)*(c*x^2+b*x+a)^(1/2)*a*x-12*c^(5/2)*(c*x^
2+b*x+a)^(1/2)*b^2*x+24*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b-6*c^(3/2)*(c*x^2+b*
x+a)^(1/2)*b^3+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a
^2*c^3-24*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*c^2
+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^4*c)/x^3/(c*x^
2+b*x+a)^(3/2)/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^3, x)
```

Fricas [A]
time = 0.38, size = 320, normalized size = 1.94

$$\left[\frac{3 \,{\left(b^{4} - 8 \, a b^{2} c + 16 \, a^{2} c^{2}\right)} \sqrt{c} x \log\left(-\frac{8 \, c^{2} x^{3} + 8 \, b c x^{2} + 4 \, \sqrt{c x^{4} + b x^{3} + a x^{2}} {\left(2 \, c x + b\right)} \sqrt{c} + {\left(b^{2} + 4 \, a c\right)} x}{x}\right) + 4 \,{\left(16 \, c^{4} x^{3} + 24 \, b c^{3} x^{2} - 3 \, b^{3} c + 20 \, a b c^{2} + 2 \,{\left(b^{2} c^{2} + 20 \, a c^{3}\right)} x\right)} \sqrt{c x^{4} + b x^{3} + a x^{2}}}{256 \, c^{3} x}, -\frac{3 \,{\left(b^{4} - 8 \, a b^{2} c + 16 \, a^{2} c^{2}\right)} \sqrt{-c} x \arctan\left(\frac{\sqrt{c x^{4} + b x^{3} + a x^{2}} {\left(2 \, c x + b\right)} \sqrt{-c}}{2 \,{\left(c^{2} x^{3} + b c x^{2} + a c x\right)}}\right) - 2 \,{\left(16 \, c^{4} x^{3} + 24 \, b c^{3} x^{2} - 3 \, b^{3} c + 20 \, a b c^{2} + 2 \,{\left(b^{2} c^{2} + 20 \, a c^{3}\right)} x\right)} \sqrt{c x^{4} + b x^{3} + a x^{2}}}{128 \, c^{3} x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*
x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/
x) + 4*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*
```

```
a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^3*x), -1/128*(3*(b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 2*(16*c^4*x^3 + 24*b*c^3*x^2 - 3*b
^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/
(c^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**3, x)
```

Giac [A]
time = 3.13, size = 232, normalized size = 1.41

$$\frac{1}{64}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\,cx\mathrm{sgn}(x)+3\,b\mathrm{sgn}(x)\right)x+\frac{b^2c^2\mathrm{sgn}(x)+20\,ac^3\mathrm{sgn}(x)}{c^3}\right)x-\frac{3\,b^3c\mathrm{sgn}(x)-20\,abc^2\mathrm{sgn}(x)}{c^3}\right)-\frac{3\left(b^4\mathrm{sgn}(x)-8\,ab^2c\mathrm{sgn}(x)+16\,a^2c^2\mathrm{sgn}(x)\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{128\,c^{\frac{5}{2}}}+\frac{\left(3\,b^4\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-24\,ab^2c\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+6\,\sqrt{a}\,b^3\sqrt{c}-40\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{128\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/64*sqrt(c*x^2 + b*x + a)*(2*(4*(2*c*x*sgn(x) + 3*b*sgn(x))*x + (b^2*c^2*s
gn(x) + 20*a*c^3*sgn(x))/c^3)*x - (3*b^3*c*sgn(x) - 20*a*b*c^2*sgn(x))/c^3)
 - 3/128*(b^4*sgn(x) - 8*a*b^2*c*sgn(x) + 16*a^2*c^2*sgn(x))*log(abs(-2*(sq
rt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) + 1/128*(3*b^4*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) - 24*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^3*sqrt(c) - 40*a
^(3/2)*b*c^(3/2))*sgn(x)/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^3, x)
```

3.43 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=227

$$\frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^3-a^(3/2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(
c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-1/16*b*(-
12*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x
+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {1935, 1959, 1947, 857, 635, 212, 738}

$$-\frac{a^{3/2}x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx}+\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*c*x) + (a*x^2 + b*
x^3 + c*x^4)^(3/2)/(3*x^3) - (a^(3/2)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a
+ b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4] - (b
*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(16*c^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}\,dx = \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{1}{2}\int \frac{(2a+bx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\int \frac{8a^2c-\frac{1}{2}b(b^2\ldots}{\sqrt{ax^2+bx\ldots}}}{8c\ldots}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\left(x\sqrt{a+bx+\ldots}\right.}{8c\ldots}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} + \frac{\left(a^2x\sqrt{a+bx\ldots}\right.}{\ldots}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{\left(2a^2x\sqrt{a+b\ldots}\right.}{\ldots}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8cx} + \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^3} - \frac{a^{3/2}x\sqrt{a+bx\ldots}}{\ldots}$$

Mathematica [A]
time = 0.53, size = 166, normalized size = 0.73

$$\frac{x\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}(3b^2+14bcx+8c(4a+cx^2))+96a^{3/2}c^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+3(b^3-12abc)\log\left(c\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)\right)}{48c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2 + 14*b*c*x
 + 8*c*(4*a + c*x^2)) + 96*a^(3/2)*c^(3/2)*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*
(b + c*x)])/Sqrt[a]] + 3*(b^3 - 12*a*b*c)*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])]))/(48*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 222, normalized size = 0.98

method	result
default	$-\frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}\left(48c^{\frac{5}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)-16(cx^2+bx+a)^{\frac{3}{2}}c^{\frac{5}{2}}-12c^{\frac{5}{2}}\sqrt{cx^2+bx+a}\,bx-\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^(5/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(
c*x^2+b*x+a)^(1/2))/x)-16*(c*x^2+b*x+a)^(3/2)*c^(5/2)-12*c^(5/2)*(c*x^2+b*x
+a)^(1/2)*b*x-48*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a-6*c^(3/2)*(c*x^2+b*x+a)^(1/2
)*b^2-36*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a*b*c^2+3*
ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^3*c)/x^3/(c*x^2+b
*x+a)^(3/2)/c^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^4, x)
```

Fricas [A]
time = 0.42, size = 791, normalized size = 3.48

$$\left[\frac{48\,a^{\frac{3}{2}}c^{2}x\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{a}}{x^{3}}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,x\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)+4\,(8\,c^{3}x^{2}+14\,bc^{2}x+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{96\,c^{2}x},\ \frac{24\,a^{\frac{3}{2}}c^{2}x\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{a}}{x^{3}}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)+2\,(8\,c^{3}x^{2}+14\,bc^{2}x+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{48\,c^{2}x},\ \frac{96\,\sqrt{-a}\,ac^{2}x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)-3\,(b^{3}-12\,abc)\sqrt{c}\,x\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)+4\,(8\,c^{3}x^{2}+14\,bc^{2}x+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{96\,c^{2}x},\ \frac{48\,\sqrt{-a}\,ac^{2}x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)+3\,(b^{3}-12\,abc)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)+2\,(8\,c^{3}x^{2}+14\,bc^{2}x+3\,b^{2}c+32\,ac^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{48\,c^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^(3/2)*c^2*x*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*s
qrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 3*(b^3 - 12*a*b*c)*s
qrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x
 + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c +
 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(24*a^(3/2)*c^2*x*log
(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c
*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2
*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))
/(c^2*x), 1/96*(96*sqrt(-a)*a*c^2*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^3 - 12*a*b*c)*sqrt
(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*(8*c^3*x^2 + 14*b*c^2*x + 3*b^2*c + 32
*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x), 1/48*(48*sqrt(-a)*a*c^2*x*arc
tan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2
```

```
 + a^2*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 +
a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(8*c^3*x^2 + 1
4*b*c^2*x + 3*b^2*c + 32*a*c^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(c^2*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**4, x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^4, x)
```

3.44 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=219

$$\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}-\frac{3\sqrt{a}\,bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -(c*x^4+b*x^3+a*x^2)^(3/2)/x^4-3/2*b*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2
+b*x+a)^(1/2))*a^(1/2)*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(4
*a*c+b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a
)^(1/2)/c^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/4*(2*c*x+3*b)*(c*x^4+b*x^3+a*x^
2)^(1/2)/x
```

Rubi [A]
time = 0.16, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1934, 1959, 1947, 857, 635, 212, 738}

$$\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{3\sqrt{a}\,bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x) - (a*x^2 + b*x^3 + c*x^
4)^(3/2)/x^4 - (3*Sqrt[a]*b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*
Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (3*(b^2
+ 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(8*Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^5}\,dx = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{3}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^2}\,dx$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{3\int \frac{4abc+c(b^2+4ac)x}{\sqrt{ax^2+bx^3+cx}}}{8c}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{\left(3x\sqrt{a+bx+cx^2}\right)}{8c\sqrt{ax^2+}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}+\frac{\left(3abx\sqrt{a+bx+cx^2}\right.}{2\sqrt{ax^2}}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}-\frac{\left(3abx\sqrt{a+bx+cx^2}\right.}{}$$

$$= \frac{3(3b+2cx)\sqrt{ax^2+bx^3+cx^4}}{4x}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{x^4}-\frac{3\sqrt{a}\,bx\sqrt{a+bx+c}}{2\sqrt{}}$$

Mathematica [A]
time = 0.53, size = 156, normalized size = 0.71

$$\frac{\sqrt{a+x(b+cx)}\left(2\sqrt{c}\sqrt{a+x(b+cx)}\,(-4a+x(5b+2cx))+24\sqrt{a}\,b\sqrt{c}\,x\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-3(b^2+4ac)\,x\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{8\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-4*a + x*(5*b + 2*
c*x)) + 24*Sqrt[a]*b*Sqrt[c]*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/
Sqrt[a]] - 3*(b^2 + 4*a*c)*x*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)
]]))/(8*Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.05, size = 254, normalized size = 1.16

method	result

risch	$-\frac{a\sqrt{x^2(c\,x^2+bx+a)}}{x^2}+\frac{\left(\frac{cx\sqrt{c\,x^2+bx+a}}{2}+\frac{5b\sqrt{c\,x^2+bx+a}}{4}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}\right.}{}$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(8c^{\frac{5}{2}}(c\,x^2+bx+a)^{\frac{3}{2}}x^2+12c^{\frac{5}{2}}\sqrt{c\,x^2+bx+a}\,a\,x^2-12c^{\frac{3}{2}}a^{\frac{3}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)bx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(8*c^(5/2)*(c*x^2+b*x+a)^(3/2)*x^2+12*c^(5/2)
*(c*x^2+b*x+a)^(1/2)*a*x^2-12*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x)*b*x-8*(c*x^2+b*x+a)^(5/2)*c^(3/2)+8*c^(3/2)*(c*x^2+b*x+a)^
(3/2)*b*x+18*c^(3/2)*(c*x^2+b*x+a)^(1/2)*a*b*x+12*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*a^2*c^2*x+3*c*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*
c^(1/2)+2*c*x+b)/c^(1/2))*a*b^2*x)/x^4/(c*x^2+b*x+a)^(3/2)/a/c^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^5, x)
```

Fricas [A]
time = 0.43, size = 757, normalized size = 3.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/16*(12*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4
*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 3*(b^2 + 4*a*c)*sq
rt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*
x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2
*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8*(6*sqrt(a)*b*c*x^2*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
```

```
t(a))/x^3) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a
*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*
x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/16*(24*sqrt(-a)*b*c*
x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 +
a*b*x^2 + a^2*x)) + 3*(b^2 + 4*a*c)*sqrt(c)*x^2*log(-(8*c^2*x^3 + 8*b*c*x^2
 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x)
+ 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2), 1/8
*(12*sqrt(-a)*b*c*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 3*(b^2 + 4*a*c)*sqrt(-c)*x^2*arctan(1
/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*
c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x^2 + 5*b*c*x - 4*a*c))/(c*x^2
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}(a+bx+cx^{2})\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**5, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(sa
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^5, x)
```

3.45 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=219

$$-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}-\frac{3(b^2+4ac)\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+b}{2\sqrt{a}\,\sqrt{a+b}}\right.}{8\sqrt{a}\,\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*(c*x^4+b*x^3+a*x^2)^(3/2)/x^5-3/8*(4*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/
a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/a^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+3/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)*
(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/4*(-2*c*x+b)*(c*x^4+b*x^3+a
*x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.16, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1934, 1955, 1947, 857, 635, 212, 738}

$$-\frac{3x(4ac+b^2)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\,\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}\,\sqrt{ax^2+bx^3+cx^4}}-\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2}+\frac{3b\sqrt{c}\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ax^2+bx^3+cx^4}}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*x^2) - (a*x^2 + b*x^3 + c*x
^4)^(3/2)/(2*x^5) - (3*(b^2 + 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4]) + (3*b*Sqrt[c]*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^6}\,dx = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} + \frac{3}{4}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^3}\,dx$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3}{8}\int \frac{-b^2-4ac-}{\sqrt{ax^2+bx^3+}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{\left(3x\sqrt{a+bx+cx^2}\right.}{8\sqrt{ax^2}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} + \frac{\left(3bcx\sqrt{a+bx+c}\right.}{2\sqrt{ax^2}}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} + \frac{\left(3bcx\sqrt{a+bx+c}\right.}{}$$

$$= -\frac{3(b-2cx)\sqrt{ax^2+bx^3+cx^4}}{4x^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{2x^5} - \frac{3(b^2+4ac)\,x\sqrt{a+}}{8}$$

Mathematica [A]
time = 0.55, size = 160, normalized size = 0.73

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(3(b^2+4ac)\,x^2\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-\sqrt{a}\left((2a+x(5b-4cx))\sqrt{a+x(b+cx)}+6b\sqrt{c}\,x^2\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{4\sqrt{a}\,x^3\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(3*(b^2 + 4*a*c)*x^2*ArcTanh[(Sqrt[c]*x - Sqrt
[a + x*(b + c*x)])/Sqrt[a]] - Sqrt[a]*((2*a + x*(5*b - 4*c*x))*Sqrt[a + x*(
b + c*x)] + 6*b*Sqrt[c]*x^2*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]
])))/(4*Sqrt[a]*x^3*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.04, size = 338, normalized size = 1.54

method	result
risch	$-\frac{(5bx+2a)\sqrt{x^2\,(c\,x^2+bx+a)}}{4x^3}+\frac{\left(c\sqrt{c\,x^2+bx+a}+\frac{3b\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2}-\frac{3\sqrt{a}\,\ln\left(\frac{2a}{}\right.}{}\right.}{}$

default	$-\frac{\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(12 a^{\frac{5}{2}} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c\,x^{2}+b x+a}}{x}\right) c^{\frac{5}{2}} x^{2}-2 c^{\frac{5}{2}} \left(c\,x^{2}+b x+a\right)^{\frac{3}{2}} b\,x^{3}+3 a^{\frac{3}{2}} \ln \left(\frac{2 a+b x+2 \sqrt{a}\, \sqrt{c\,}}{x}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(c*x^4+b*x^3+a*x^2)^(3/2)*(12*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)*c^(5/2)*x^2-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b*x^3+3*a^(3/2)*ln(
(2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*c^(3/2)*b^2*x^2-4*c^(5/2)*(c*x^2
+b*x+a)^(3/2)*a*x^2-6*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a*b*x^3-12*c^(5/2)*(c*x^2
+b*x+a)^(1/2)*a^2*x^2+2*c^(3/2)*(c*x^2+b*x+a)^(5/2)*b*x-2*c^(3/2)*(c*x^2+b*
x+a)^(3/2)*b^2*x^2+4*(c*x^2+b*x+a)^(5/2)*a*c^(3/2)-6*c^(3/2)*(c*x^2+b*x+a)^
(1/2)*a*b^2*x^2-12*c^2*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/
2))*a^2*b*x^2)/x^5/(c*x^2+b*x+a)^(3/2)/a^2/c^(3/2)
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^6, x)
```

<u>Fricas</u> [A]
time = 0.39, size = 757, normalized size = 3.46

$$\left[\frac{12\,ab\sqrt{c}\,x^{3}\log\left(-\frac{8c^{2}x^{3}+8bcx^{2}+4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{c}+(b^{2}+4ac)x}{x}\right)+3\,(b^{2}+4\,ac)\sqrt{a}\,x^{3}\log\left(-\frac{8abx^{2}+(b^{2}+4ac)x^{3}+8a^{2}x-4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2a)\sqrt{a}}{x^{3}}\right)+4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(4\,acx^{2}-5\,abx-2\,a^{2})}{16\,ax^{3}},\ -\frac{24\,ab\sqrt{-c}\,x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-3\,(b^{2}+4\,ac)\sqrt{a}\,x^{3}\log\left(-\frac{8abx^{2}+(b^{2}+4ac)x^{3}+8a^{2}x-4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2a)\sqrt{a}}{x^{3}}\right)-4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(4\,acx^{2}-5\,abx-2\,a^{2})}{16\,ax^{3}},\ \frac{6\,ab\sqrt{c}\,x^{3}\log\left(-\frac{8c^{2}x^{3}+8bcx^{2}+4\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{c}+(b^{2}+4ac)x}{x}\right)+3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)+2\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(4\,acx^{2}-5\,abx-2\,a^{2})}{8\,ax^{3}},\ -\frac{12\,ab\sqrt{-c}\,x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)-3\,(b^{2}+4\,ac)\sqrt{-a}\,x^{3}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)-2\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(4\,acx^{2}-5\,abx-2\,a^{2})}{8\,ax^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b*sqrt(c)*x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt
(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c
*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1/16*(24*a*b*sqrt(-c)*x^3*arctan(1/2*sqr
t(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))
- 3*(b^2 + 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x
 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), 1/8*(6*a*b*sqrt(c)*
x^3*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan(1/2*sq
```

```
rt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))
 + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x^3), -1
/8*(12*a*b*sqrt(-c)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*
sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) - 3*(b^2 + 4*a*c)*sqrt(-a)*x^3*arctan
(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 +
a^2*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(4*a*c*x^2 - 5*a*b*x - 2*a^2))/(a*x
^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**6, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^6, x)
```

3.46 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=257

$$\frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5}+\frac{b(b^2-12ac)\,x\sqrt{a+bx}}{16a^3}$$

```
[Out] -1/3*(c*x^4+b*x^3+a*x^2)^(3/2)/x^6-1/4*b*(c*x^4+b*x^3+a*x^2)^(3/2)/a/x^5+1/
16*b*(-12*a*c+b^2)*x*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/a^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+c^(3/2)*x*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/(c*x^4+b*x^3+a*x^2
)^(1/2)+1/8*(2*b*c*x-8*a*c+b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A]
time = 0.23, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1934, 1965, 1955, 1947, 857, 635, 212, 738}

$$\frac{bx(b^2-12ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{16a^{3/2}\sqrt{ax^2+bx^3+cx^4}}+\frac{(-8ac+b^2+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2}+\frac{c^{3/2}x\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}-\frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{3x^6}-\frac{b\left(ax^2+bx^3+cx^4\right)^{3/2}}{4ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] ((b^2 - 8*a*c + 2*b*c*x)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a*x^2) - (a*x^2 +
b*x^3 + c*x^4)^(3/2)/(3*x^6) - (b*(a*x^2 + b*x^3 + c*x^4)^(3/2))/(4*a*x^5)
+ (b*(b^2 - 12*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/(16*a^(3/2)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + (c^(3/2
)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/Sqrt[a*x^2 + b*x^3 + c*x^4]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1947

```
Int[((A_) + (B_.)*(x_)^(j_.))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c
_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(
n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[(A + B*x^(n - q))/(x^(q/
2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; FreeQ[{a, b, c, A, B
, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ[n - q] && EqQ[n, 3]
 && EqQ[q, 2]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^7}\,dx = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} + \frac{1}{2}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^4}\,dx$$

$$= -\frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+cx^4)^{3/2}}{4ax^5} - \frac{\int \frac{\left(\frac{1}{2}(b^2-8ac)-bcx\right)\sqrt{ax^2+bx^3}}{x^3}}{4a}$$

$$= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+}{4ax^5}$$

$$= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+}{4ax^5}$$

$$= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+}{4ax^5}$$

$$= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+}{4ax^5}$$

$$= \frac{(b^2-8ac+2bcx)\sqrt{ax^2+bx^3+cx^4}}{8ax^2} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{3x^6} - \frac{b(ax^2+bx^3+}{4ax^5}$$

Mathematica [A]
time = 0.69, size = 172, normalized size = 0.67

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(3b(b^2-12ac)\,x^3\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+\sqrt{a}\left(\sqrt{a+x(b+cx)}\,(8a^2+3b^2x^2+2ax(7b+16cx))+24ac^{3/2}x^3\log\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{24a^{3/2}x^4\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -1/24*(Sqrt[x^2*(a + x*(b + c*x))]*(3*b*(b^2 - 12*a*c)*x^3*ArcTanh[(Sqrt[c]
*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Sqrt[a]*(Sqrt[a + x*(b + c*x)]*(8*a^
2 + 3*b^2*x^2 + 2*a*x*(7*b + 16*c*x)) + 24*a*c^(3/2)*x^3*Log[b + 2*c*x - 2*
Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(a^(3/2)*x^4*Sqrt[a + x*(b + c*x)])
```

Maple [A]
time = 0.04, size = 435, normalized size = 1.69

method	result
risch	$-\frac{(32acx^2+3b^2x^2+14abx+8a^2)\sqrt{x^2(cx^2+bx+a)}}{24x^4a}+\frac{\left(c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)-\frac{3b\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{}}{}\right)}{4\sqrt{}}\right.}{}$
default	$\frac{(cx^4+bx^3+ax^2)^{\frac{3}{2}}\left(32c^{\frac{7}{2}}(cx^2+bx+a)^{\frac{3}{2}}ax^4-36c^{\frac{5}{2}}a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)bx^3+48c^{\frac{7}{2}}\sqrt{cx^2+bx+a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*(c*x^4+b*x^3+a*x^2)^(3/2)*(32*c^(7/2)*(c*x^2+b*x+a)^(3/2)*a*x^4-36*c^(
5/2)*a^(5/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b*x^3+48*c^(7/2)
*(c*x^2+b*x+a)^(1/2)*a^2*x^4-2*c^(5/2)*(c*x^2+b*x+a)^(3/2)*b^2*x^4-32*c^(5/
2)*(c*x^2+b*x+a)^(5/2)*a*x^2+28*c^(5/2)*(c*x^2+b*x+a)^(3/2)*a*b*x^3-6*c^(5/
2)*(c*x^2+b*x+a)^(1/2)*a*b^2*x^4+3*c^(3/2)*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c
*x^2+b*x+a)^(1/2))/x)*b^3*x^3+60*c^(5/2)*(c*x^2+b*x+a)^(1/2)*a^2*b*x^3+2*c^
(3/2)*(c*x^2+b*x+a)^(5/2)*b^2*x^2-2*c^(3/2)*(c*x^2+b*x+a)^(3/2)*b^3*x^3+4*c
^(3/2)*(c*x^2+b*x+a)^(5/2)*a*b*x-6*c^(3/2)*(c*x^2+b*x+a)^(1/2)*a*b^3*x^3-16
*(c*x^2+b*x+a)^(5/2)*a^2*c^(3/2)+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2
*c*x+b)/c^(1/2))*a^3*c^3*x^3)/x^6/(c*x^2+b*x+a)^(3/2)/a^3/c^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^7, x)
```

Fricas [A]
time = 0.42, size = 815, normalized size = 3.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/96*(48*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^
3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 3*(b^3 - 12*a*b*c)*s
qrt(a)*x^4*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b
*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14
*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/96*(96*a^2*sqrt
(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2
*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(a)*x^4*log(-(8*a*b*x^2 +
 (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sq
rt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4), 1/48*(24*a^2*c^(3/2)*x^4*log(-(8*c^2*x^3 + 8*b
*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*
x)/x) - 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*sqrt(c*x^4 + b*x^
3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2 + 32*a^2*c)*x^2))/(a^2*x^4), -1/4
8*(48*a^2*sqrt(-c)*c*x^4*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 3*(b^3 - 12*a*b*c)*sqrt(-a)*x^4*ar
ctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^
2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(14*a^2*b*x + 8*a^3 + (3*a*b^2
+ 32*a^2*c)*x^2))/(a^2*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**7, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^7, x)
```

3.47 $\int \frac{\left(ax^2+bx^3+cx^4\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=197

$$-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}+\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{(ax^2+b}{}$$

```
[Out] -1/4*(c*x^4+b*x^3+a*x^2)^(3/2)/x^7-3/128*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*x+
2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)-1/32*(-12*a*c+b^2)*(c*x^4+b
*x^3+a*x^2)^(1/2)/a/x^3+1/64*b*(-20*a*c+3*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
2/x^2-1/8*(6*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^4
```

Rubi [A]
time = 0.25, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1934, 1955, 1965, 12, 1918, 212}

$$-\frac{3(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{5/2}}+\frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2}-\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3}-\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -1/32*((b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^3) + (b*(3*b^2 - 20
*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(64*a^2*x^2) - ((b + 6*c*x)*Sqrt[a*x^2 +
 b*x^3 + c*x^4])/(8*x^4) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(4*x^7) - (3*(b^2
- 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]
)/(128*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

```
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^8}\,dx = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{3}{8}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^5}\,dx$$

$$= -\frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{4x^7} + \frac{1}{16}\int \frac{b^2-12ac-}{x^2\sqrt{ax^2+bx^3}}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} - \frac{(b+6cx)\sqrt{ax^2+bx^3+cx^4}}{8x^4} - \frac{(ax^2+bx^3}{4x}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6}{}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6}{}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6}{}$$

$$= -\frac{(b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{32ax^3} + \frac{b(3b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{64a^2x^2} - \frac{(b+6}{}$$

Mathematica [A]
time = 0.75, size = 141, normalized size = 0.72

$$\frac{\sqrt{x^2(a+x(b+cx))}\left(-\sqrt{a}\,(2a+bx)\sqrt{a+x(b+cx)}\,(8a^2-3b^2x^2+4ax(2b+5cx))+3(b^2-4ac)^2x^4\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{64a^{5/2}x^5\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[x^2*(a + x*(b + c*x))]*(-(Sqrt[a]*(2*a + b*x)*Sqrt[a + x*(b + c*x)]*(
8*a^2 - 3*b^2*x^2 + 4*a*x*(2*b + 5*c*x))) + 3*(b^2 - 4*a*c)^2*x^4*ArcTanh[(
Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(64*a^(5/2)*x^5*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(171) = 342$.
time = 0.05, size = 501, normalized size = 2.54

method	result

risch	$-\frac{(20abc\,x^3-3b^3x^3+40a^2c\,x^2+2a\,b^2x^2+24a^2bx+16a^3)\sqrt{x^2\,(c\,x^2+bx+a)}}{64x^5a^2}+\frac{\left(-\frac{3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+}}{x}\right.}{8\sqrt{a}}\right.}{}$
default	$-\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(48c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x^4+24c^2(c\,x^2+bx+a)^{\frac{3}{2}}ab\,x^5-24c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/128*(c*x^4+b*x^3+a*x^2)^(3/2)*(48*c^2*a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*x
^2+b*x+a)^(1/2))/x)*x^4+24*c^2*(c*x^2+b*x+a)^(3/2)*a*b*x^5-24*c*a^(5/2)*ln(
(2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^2*x^4-16*c^2*(c*x^2+b*x+a)^(3/
2)*a^2*x^4+24*c^2*(c*x^2+b*x+a)^(1/2)*a^2*b*x^5-2*c*(c*x^2+b*x+a)^(3/2)*b^3
*x^5-48*c^2*(c*x^2+b*x+a)^(1/2)*a^3*x^4-24*c*(c*x^2+b*x+a)^(5/2)*a*b*x^3+20
*c*(c*x^2+b*x+a)^(3/2)*a*b^2*x^4-6*c*(c*x^2+b*x+a)^(1/2)*a*b^3*x^5+3*a^(3/2
)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^4*x^4+16*c*(c*x^2+b*x+a)^
(5/2)*a^2*x^2+36*c*(c*x^2+b*x+a)^(1/2)*a^2*b^2*x^4+2*(c*x^2+b*x+a)^(5/2)*b^
3*x^3-2*(c*x^2+b*x+a)^(3/2)*b^4*x^4+4*(c*x^2+b*x+a)^(5/2)*a*b^2*x^2-6*(c*x^
2+b*x+a)^(1/2)*a*b^4*x^4-16*(c*x^2+b*x+a)^(5/2)*a^2*b*x+32*(c*x^2+b*x+a)^(5
/2)*a^3)/x^7/(c*x^2+b*x+a)^(3/2)/a^4
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^8, x)
```

<u>Fricas</u> [A]
time = 0.39, size = 332, normalized size = 1.69

$$\left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{a}\,x^5\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,(24\,a^3bx+16\,a^4-(3\,ab^3-20\,a^2bc)x^3+2\,(a^2b^2+20\,a^3c)x^2)\sqrt{cx^4+bx^3+ax^2}}{256\,a^3x^5},\ \frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-a}\,x^5\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)-2\,(24\,a^3bx+16\,a^4-(3\,ab^3-20\,a^2bc)x^3+2\,(a^2b^2+20\,a^3c)x^2)\sqrt{cx^4+bx^3+ax^2}}{128\,a^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(a)*x^5*log(-(8*a*b*x^2 + (b^2
 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a)
```

```
)/x^3) - 4*(24*a^3*b*x + 16*a^4 - (3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 +
 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^3*x^5), 1/128*(3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*sqrt(-a)*x^5*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(24*a^3*b*x + 16*a^4 -
(3*a*b^3 - 20*a^2*b*c)*x^3 + 2*(a^2*b^2 + 20*a^3*c)*x^2)*sqrt(c*x^4 + b*x^3
 + a*x^2))/(a^3*x^5)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^2(a+bx+cx^2))^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**8,x)
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**8, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8,x)
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^8, x)
```

3.48 $\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=249

$$-\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{(15b^4-100ab^2c+128a^2c^2)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}$$

```
[Out] -1/5*(c*x^4+b*x^3+a*x^2)^(3/2)/x^8+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*x*(b*
x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(7/2)-1/80*(-8*a*c+b^2)*(c*x^4+
b*x^3+a*x^2)^(1/2)/a/x^4+1/320*b*(-28*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/
a^2/x^3-1/640*(128*a^2*c^2-100*a*b^2*c+15*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^
3/x^2-3/40*(4*c*x+b)*(c*x^4+b*x^3+a*x^2)^(1/2)/x^5
```

Rubi [A]
time = 0.34, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1934, 1955, 1965, 12, 1918, 212}

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{256a^{7/2}}+\frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3}-\frac{(128a^2c^2-100ab^2c+15b^4)\sqrt{ax^2+bx^3+cx^4}}{640a^3x^2}-\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4}-\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8}-\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/80*((b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a*x^4) + (b*(5*b^2 - 28*
a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(320*a^2*x^3) - ((15*b^4 - 100*a*b^2*c +
128*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(640*a^3*x^2) - (3*(b + 4*c*x)*Sq
rt[a*x^2 + b*x^3 + c*x^4])/(40*x^5) - (a*x^2 + b*x^3 + c*x^4)^(3/2)/(5*x^8)
 + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x
^3 + c*x^4])])/(256*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
```

```
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1934

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*q
+ 1)), x] - Dist[(n - q)*(p/(m + p*q + 1)), Int[x^(m + n)*(b + 2*c*x^(n - q
))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &
& EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] &&
 IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q + 1, -(n - q) +
1] && NeQ[m + p*q + 1, 0]
```

Rule 1955

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(A*(m + p*q + (n
- q)*(2*p + 1) + 1) + B*(m + p*q + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n
 - q))^p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n -
 q)*(p/((m + p*q + 1)*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(n + m)*Si
mp[2*a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(2*p + 1) + 1) + (b*B*(m +
p*q + 1) - 2*A*c*(m + p*q + (n - q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q +
b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[
r, n - q] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGt
Q[n, 0] && GtQ[p, 0] && RationalQ[m, q] && LeQ[m + p*q, -(n - q)] && NeQ[m
+ p*q + 1, 0] && NeQ[m + p*q + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3+cx^4)^{3/2}}{x^9}\,dx = -\frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} + \frac{3}{10}\int \frac{(b+2cx)\sqrt{ax^2+bx^3+cx^4}}{x^6}\,dx$$

$$= -\frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx^3+cx^4)^{3/2}}{5x^8} + \frac{3}{160}\int \frac{2(b^2-8ac}{x^3\sqrt{ax^2+b}}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} - \frac{3(b+4cx)\sqrt{ax^2+bx^3+cx^4}}{40x^5} - \frac{(ax^2+bx}{5}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{3(b+}{}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4}{}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4}{}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4}{}$$

$$= -\frac{(b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{80ax^4} + \frac{b(5b^2-28ac)\sqrt{ax^2+bx^3+cx^4}}{320a^2x^3} - \frac{(15b^4}{}$$

Mathematica [**A**]
time = 1.03, size = 176, normalized size = 0.71

$$-\frac{\sqrt{x^2(a+x(b+cx))}\left(\sqrt{a}\sqrt{a+x(b+cx)}\,(128a^4+15b^4x^4-10ab^2x^3(b+10cx)+16a^3x(11b+16cx)+8a^2x^2(b^2+7bcx+16c^2x^2))+15b(b^2-4ac)^2x^5\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{640a^{7/2}x^6\sqrt{a+x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] -1/640*(Sqrt[x^2*(a + x*(b + c*x))]*(Sqrt[a]*Sqrt[a + x*(b + c*x)]*(128*a^4
 + 15*b^4*x^4 - 10*a*b^2*x^3*(b + 10*c*x) + 16*a^3*x*(11*b + 16*c*x) + 8*a^
2*x^2*(b^2 + 7*b*c*x + 16*c^2*x^2)) + 15*b*(b^2 - 4*a*c)^2*x^5*ArcTanh[(Sqr
t[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(7/2)*x^6*Sqrt[a + x*(b + c*x
)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(219) = 438$.
time = 0.06, size = 534, normalized size = 2.14

method	result
risch	$-\frac{(128a^2c^2x^4-100a\,b^2c\,x^4+15b^4x^4+56a^2bc\,x^3-10a\,b^3x^3+256a^3c\,x^2+8a^2b^2x^2+176a^3bx+128a^4)\sqrt{x^2\,(c\,x^2+bx+a)}}{640x^6a^3}+$
default	$\frac{(c\,x^4+b\,x^3+a\,x^2)^{\frac{3}{2}}\left(240c^2a^{\frac{7}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)b\,x^5+120c^2(c\,x^2+bx+a)^{\frac{3}{2}}a\,b^2x^6-120c\,a^{\frac{5}{2}}\ln\left(\frac{2a+bx+2\sqrt{a}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(c*x^4+b*x^3+a*x^2)^(3/2)*(240*c^2*a^(7/2)*ln((2*a+b*x+2*a^(1/2)*(c*
x^2+b*x+a)^(1/2))/x)*b*x^5+120*c^2*(c*x^2+b*x+a)^(3/2)*a*b^2*x^6-120*c*a^(5
/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^3*x^5-80*c^2*(c*x^2+b*x
+a)^(3/2)*a^2*b*x^5+120*c^2*(c*x^2+b*x+a)^(1/2)*a^2*b^2*x^6-10*c*(c*x^2+b*x
+a)^(3/2)*b^4*x^6-240*c^2*(c*x^2+b*x+a)^(1/2)*a^3*b*x^5-120*c*(c*x^2+b*x+a)
^(5/2)*a*b^2*x^4+100*c*(c*x^2+b*x+a)^(3/2)*a*b^3*x^5-30*c*(c*x^2+b*x+a)^(1/
2)*a*b^4*x^6+15*a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*b^5*x
^5+80*c*(c*x^2+b*x+a)^(5/2)*a^2*b*x^3+180*c*(c*x^2+b*x+a)^(1/2)*a^2*b^3*x^5
+10*(c*x^2+b*x+a)^(5/2)*b^4*x^4-10*(c*x^2+b*x+a)^(3/2)*b^5*x^5+20*(c*x^2+b*
x+a)^(5/2)*a*b^3*x^3-30*(c*x^2+b*x+a)^(1/2)*a*b^5*x^5-80*(c*x^2+b*x+a)^(5/2
)*a^2*b^2*x^2+160*(c*x^2+b*x+a)^(5/2)*a^3*b*x-256*(c*x^2+b*x+a)^(5/2)*a^4)/
x^8/(c*x^2+b*x+a)^(3/2)/a^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(3/2)/x^9, x)
```

Fricas [A]
time = 0.43, size = 394, normalized size = 1.58

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{a}\,x^6\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,(176\,a^4bx+128\,a^5+(15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^4-2\,(5\,a^2b^3-28\,a^3bc)x^3+8\,(a^3b^2+32\,a^4c)x^2)\sqrt{cx^4+bx^3+ax^2}}{2560\,a^4x^6},\ -\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-a}\,x^6\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,(176\,a^4bx+128\,a^5+(15\,ab^4-100\,a^2b^2c+128\,a^3c^2)x^4-2\,(5\,a^2b^3-28\,a^3bc)x^3+8\,(a^3b^2+32\,a^4c)x^2)\sqrt{cx^4+bx^3+ax^2}}{1280\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(a)*x^6*log(-(8*a*b*x^2 +
(b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(a))/x^3) - 4*(176*a^4*b*x + 128*a^5 + (15*a*b^4 - 100*a^2*b^2*c + 128*a^3
*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3*b^2 + 32*a^4*c)*x^2)*sq
rt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6), -1/1280*(15*(b^5 - 8*a*b^3*c + 16*a^2
*b*c^2)*sqrt(-a)*x^6*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqr
t(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*(176*a^4*b*x + 128*a^5 + (15*a*b^4 -
 100*a^2*b^2*c + 128*a^3*c^2)*x^4 - 2*(5*a^2*b^3 - 28*a^3*b*c)*x^3 + 8*(a^3
*b^2 + 32*a^4*c)*x^2)*sqrt(c*x^4 + b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2(a+bx+cx^2)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x + c*x**2))**(3/2)/x**9, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9,x)
```

```
[Out] int((a*x^2 + b*x^3 + c*x^4)^(3/2)/x^9, x)
```

3.49 $\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{(3b^2-4ac)\,x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 1/8*(-4*a*c+3*b^2)*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*
x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(c*x^4+b*x^3+a*x^2)^
(1/2)/c-3/4*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/x
```

Rubi [A]
time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1942, 1963, 12, 1928, 635, 212}

$$\frac{x(3b^2-4ac)\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x}+\frac{\sqrt{ax^2+bx^3+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(2*c) - (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^
2*x) + ((3*b^2 - 4*a*c)*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1942

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - 2*n + q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(
p + 1)/(c*(m + p*q + 2*(n - q)*p + 1))), x] - Dist[1/(c*(m + p*q + 2*(n - q
)*p + 1)), Int[x^(m - 2*(n - q))*(a*(m + p*q - 2*(n - q) + 1) + b*(m + p*q
+ (n - q)*(p - 1) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
 /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
&& NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && Rational
Q[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{\int \frac{x\left(a+\frac{3bx}{2}\right)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\int \frac{(3b^2-4ac)x}{4\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c^2}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{(3b^2-4ac)\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}}{8c^2}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left((3b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\int}{8c^2\sqrt{ax^2+bx^3+}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{\left((3b^2-4ac)\,x\sqrt{a+bx+cx^2}\right)\mathrm{S}}{4c^2\sqrt{ax^2}}$$

$$= \frac{\sqrt{ax^2+bx^3+cx^4}}{2c} - \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4c^2x} + \frac{(3b^2-4ac)\,x\sqrt{a+bx+cx^2}\ \tanh}{8c^{5/2}\sqrt{ax^2+b}}$$

Mathematica [A]
time = 0.23, size = 107, normalized size = 0.75

$$\frac{x\left(2\sqrt{c}\,(-3b+2cx)(a+x(b+cx))+(-3b^2+4ac)\,\sqrt{a+x(b+cx)}\,\log\left(c^2\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{8c^{5/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*(2*Sqrt[c]*(-3*b + 2*c*x)*(a + x*(b + c*x)) + (-3*b^2 + 4*a*c)*Sqrt[a +
x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])]))/(8*c^
(5/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 144, normalized size = 1.01

method	result
risch	$-\frac{(-2cx+3b)(c\,x^2+bx+a)x}{4c^2\sqrt{x^2\,(c\,x^2+bx+a)}}+\frac{\left(-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{5}{2}}}\right)x\sqrt{c\,x^2 -}}{\sqrt{x^2\,(c\,x^2+bx+a)}}$

default	$\frac{x\sqrt{c\,x^{2}+bx+a}\,\left(4x\sqrt{c\,x^{2}+bx+a}\,c^{\frac{5}{2}}-6c^{\frac{3}{2}}\sqrt{c\,x^{2}+bx+a}\,b-4a\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)c^{2}\right.}{8\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x*(c*x^2+b*x+a)^(1/2)*(4*x*(c*x^2+b*x+a)^(1/2)*c^(5/2)-6*c^(3/2)*(c*x^2
+b*x+a)^(1/2)*b-4*a*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))
*c^2+3*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*b^2*c)/(c*x^
4+b*x^3+a*x^2)^(1/2)/c^(7/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A]
time = 0.36, size = 226, normalized size = 1.58

$$\left[-\frac{(3b^2-4ac)\sqrt{c}\,x\log\left(-\frac{8c^2x^3+8bcx^2-4\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{c}+(b^2+4ac)x}{x}\right)-4\sqrt{cx^4+bx^3+ax^2}\,(2c^2x-3bc)}{16c^3x},\ -\frac{(3b^2-4ac)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2cx+b)\sqrt{-c}}{2(c^2x^3+bcx^2+acx)}\right)-2\sqrt{cx^4+bx^3+ax^2}\,(2c^2x-3bc)}{8c^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^
4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4
 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x), -1/8*((3*b^2 - 4*a*c)*sqrt(-c
)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 +
b*c*x^2 + a*c*x)) - 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c^2*x - 3*b*c))/(c^3*x
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 6.04, size = 140, normalized size = 0.98

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2x}{c\mathrm{sgn}(x)}-\frac{3b}{c^2\mathrm{sgn}(x)}\right)+\frac{\left(3b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+6\sqrt{a}b\sqrt{c}\right)\mathrm{sgn}(x)}{8c^{\frac{5}{2}}}-\frac{(3b^2-4ac)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8c^{\frac{5}{2}}\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*x/(c*sgn(x)) - 3*b/(c^2*sgn(x))) + 1/8*(3*b^2*
log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) +
 6*sqrt(a)*b*sqrt(c))*sgn(x)/c^(5/2) - 1/8*(3*b^2 - 4*a*c)*log(abs(-2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/(c^(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.50 $\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -1/2*b*x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(
1/2)/c^(3/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+(c*x^4+b*x^3+a*x^2)^(1/2)/c/x
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1931, 1928, 635, 212}

$$\frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3 + c*x^4]/(c*x) - (b*x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(3/2)*Sqrt[a*x^2 + b*x^3 +
 c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
```

```
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1931

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n)*((a*x^(n - 1) + b*x^n + c*x^(n + 1))^(p + 1)
/(2*c*(p + 1))), x] - Dist[b/(2*c), Int[x^(m - 1)*(a*x^(n - 1) + b*x^n + c*
x^(n + 1))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n -
q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && RationalQ[m, p,
 q] && EqQ[m + p*(n - 1) - 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{b\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2c}\\ &= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\left(bx\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2c\sqrt{ax^2+bx^3+cx^4}}\\ &= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{\left(bx\sqrt{a+bx+cx^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c\sqrt{ax^2+bx^3+cx^4}}\\ &= \frac{\sqrt{ax^2+bx^3+cx^4}}{cx}-\frac{bx\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}\sqrt{ax^2+bx^3+cx^4}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.85

$$\frac{x\left(2\sqrt{c}\,(a+x(b+cx))+b\sqrt{a+x(b+cx)}\,\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{2c^{3/2}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
[Out] (x*(2*Sqrt[c]*(a + x*(b + c*x)) + b*Sqrt[a + x*(b + c*x)]*Log[c*(b + 2*c*x
- 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])))/(2*c^(3/2)*Sqrt[x^2*(a + x*(b + c*x))
])
```

Maple [A]
time = 0.03, size = 88, normalized size = 0.85

method	result	size
default	$\frac{x\sqrt{c\,x^{2}+b x+a}\,\left(2\sqrt{c\,x^{2}+b x+a}\,c^{\frac{3}{2}}-b\ln\left(\frac{2\sqrt{c\,x^{2}+b x+a}\,\sqrt{c}+2 c x+b}{2\sqrt{c}}\right) c\right)}{2\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,c^{\frac{5}{2}}}$	88
risch	$\frac{\left(c\,x^{2}+b x+a\right) x}{c\sqrt{x^{2}\left(c\,x^{2}+b x+a\right)}}-\frac{b\ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right) x\sqrt{c\,x^{2}+b x+a}}{2 c^{\frac{3}{2}}\sqrt{x^{2}\left(c\,x^{2}+b x+a\right)}}$	93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*c^(3/2)-b*ln(1/2*(2*(c*x^2
+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*c)/(c*x^4+b*x^3+a*x^2)^(1/2)/c^(5/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A]
time = 0.38, size = 188, normalized size = 1.83

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+(b^{2}+4\,ac)x}{x}\right)+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,c}{4\,c^{2}x},\ \frac{b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\,(c^{2}x^{3}+bcx^{2}+acx)}\right)+2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,c}{2\,c^{2}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*x*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2
)*c)/(c^2*x), 1/2*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c
*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^
2)*c)/(c^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{2}}{\sqrt{x^{2}\left(a+bx+cx^{2}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 3.65, size = 94, normalized size = 0.91

$$-\frac{\left(b\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\sqrt{c}\right)\mathrm{sgn}(x)}{2c^{\frac{3}{2}}}+\frac{b\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2c^{\frac{3}{2}}\mathrm{sgn}(x)}+\frac{\sqrt{cx^2+bx+a}}{c\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] -1/2*(b*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(a)*sqrt(c))*sgn(x)/c^(3/2
) + 1/2*b*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/(c^(
3/2)*sgn(x)) + sqrt(c*x^2 + b*x + a)/(c*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.51 $\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=71

$$\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] x*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^
(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1928, 635, 212}

$$\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\,\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^
2])])/(Sqrt[c]*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
```

```
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}\,dx = \frac{\left(x\sqrt{a+bx+cx^2}\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{\left(2x\sqrt{a+bx+cx^2}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{ax^2+bx^3+cx^4}}$$
$$= \frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [**A**]
time = 0.08, size = 67, normalized size = 0.94

$$-\frac{x\sqrt{a+bx+cx^2}\,\log\left(b+2cx-2\sqrt{c}\sqrt{a+bx+cx^2}\right)}{\sqrt{c}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
[Out] -((x*Sqrt[a + b*x + c*x^2]*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + b*x + c*x^2]]
)/(Sqrt[c]*Sqrt[x^2*(a + x*(b + c*x))]))
```

Maple [**A**]
time = 0.02, size = 65, normalized size = 0.92

method	result	size
default	$\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2\sqrt{c\,x^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)}{\sqrt{c\,x^4+b\,x^3+a\,x^2}\,\sqrt{c}}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)*ln(1/2*(2*(c*x^2+b*x+a)^(
1/2)*c^(1/2)+2*c*x+b)/c^(1/2))/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^4 + b*x^3 + a*x^2), x)
```

Fricas [A]
time = 0.36, size = 129, normalized size = 1.82

$$\left[\frac{\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b
)*sqrt(c) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -sqrt(-c)*arctan(1/2*sqrt(c*x^4 +
b*x^3 + a*x^2)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 4.66, size = 61, normalized size = 0.86

$$\frac{\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)\operatorname{sgn}(x)}{\sqrt{c}} - \frac{\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{\sqrt{c}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-b + 2*sqrt(a)*sqrt(c)))*sgn(x)/sqrt(c) - log(abs(-2*(sqrt(c)*x - s
qrt(c*x^2 + b*x + a))*sqrt(c) - b))/(sqrt(c)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.52 $\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]
```

```
[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 68, normalized size = 1.51

$$\frac{2x\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^2 + b*x^3 + c*x^4],x]

[Out] (2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A]
time = 0.02, size = 66, normalized size = 1.47

method	result	size
default	$-\frac{x\sqrt{c\,x^{2}+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	66

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/(c*x^4+b*x^3+a*x^2)^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(c*x^4 + b*x^3 + a*x^2), x)

Fricas [A]
time = 0.37, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8\,abx^{2}+(b^{2}+4\,ac)x^{3}+8\,a^{2}x-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{a}}{x^{3}}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^{3}+abx^{2}+a^{2}x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^3 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3 + c*x**4), x)
```

Giac [**A**]
time = 4.76, size = 59, normalized size = 1.31

$$-\frac{2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(1/2), x)
```

3.53 $\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=77

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)-(c
*x^4+b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1941, 1918, 212}

$$\frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}-\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^2)) + (b*ArcTanh[(x*(2*a + b*x))/(2*Sqrt
[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1941

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*((a*x^q + b*x^n + c*x^(2*n - q))^(p +
 1)/(2*a*(n - q)*(p + 1))), x] - Dist[b/(2*a), Int[x^(m + n - q)*(a*x^q + b
*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[
```

```
p, -1] && LtQ[p, 0] && RationalQ[m, q] && EqQ[m + p*q + 1, -2*(n - q)*(p +
1)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^3+cx^4}}\,dx &= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} - \frac{b\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}\\
&= -\frac{\sqrt{ax^2+bx^3+cx^4}}{ax^2} + \frac{b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.15, size = 87, normalized size = 1.13

$$
\frac{-\sqrt{a}\,(a+x(b+cx)) - bx\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+x(b+cx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] (-(Sqrt[a]*(a + x*(b + c*x))) - b*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*
x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 88, normalized size = 1.14

method	result	size
default	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(2\sqrt{c\,x^2+bx+a}\,a^{\frac{3}{2}}-b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)ax\right)}{2\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{5}{2}}}$	88
risch	$-\frac{c\,x^2+bx+a}{a\sqrt{x^2\,(c\,x^2+bx+a)}}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)x\sqrt{c\,x^2+bx+a}}{2a^{\frac{3}{2}}\sqrt{x^2\,(c\,x^2+bx+a)}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+b*x+a)^(1/2)*(2*(c*x^2+b*x+a)^(1/2)*a^(3/2)-b*ln((2*a+b*x+2*a^(
1/2)*(c*x^2+b*x+a)^(1/2))/x)*a*x)/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(5/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x), x)
```

Fricas [**A**]
time = 0.35, size = 194, normalized size = 2.52

$$\left[\frac{\sqrt{a}\, b x^{2} \log\left(-\frac{8\, a b x^{2} + (b^{2} + 4\, a c) x^{3} + 8\, a^{2} x + 4\, \sqrt{c x^{4} + b x^{3} + a x^{2}}\, (b x + 2\, a) \sqrt{a}}{x^{3}}\right) - 4\, \sqrt{c x^{4} + b x^{3} + a x^{2}}\, a}{4\, a^{2} x^{2}}, -\frac{\sqrt{-a}\, b x^{2} \arctan\left(\frac{\sqrt{c x^{4} + b x^{3} + a x^{2}}\, (b x + 2\, a) \sqrt{-a}}{2\, (a c x^{3} + a b x^{2} + a^{2} x)}\right) + 2\, \sqrt{c x^{4} + b x^{3} + a x^{2}}\, a}{2\, a^{2} x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*b*x^2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(
c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a
*x^2)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x
^3 + a*x^2)*a)/(a^2*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{x^{2} \left(a + b x + c x^{2}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.54 $\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(5/2)-1/2*(c*x^4+b*x^3+a*x^2)^(1/2)/a/x^3+3/4*b*(c*x^4+b*x^3+a*x^2)^(
1/2)/a^2/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1943, 1965, 12, 1918, 212}

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a*x^2 + b*x^3 + c*x^4]/(a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3 + c*x^4]
)/(4*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*
x^2 + b*x^3 + c*x^4])])/(8*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1943

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx = -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{\int \frac{-\frac{3b}{2}-cx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{\int \frac{-\frac{3b^2}{4}+ac}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{2a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} + \frac{(3b^2-4ac)\int \frac{1}{\sqrt{ax^2+bx^3+}}}{8a^2}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\,\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\right.}{4a}$$

$$= -\frac{\sqrt{ax^2+bx^3+cx^4}}{2ax^3} + \frac{3b\sqrt{ax^2+bx^3+cx^4}}{4a^2x^2} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{}{2\sqrt{a}\,\sqrt{}}\right.}{8a^{5/2}}$$

Mathematica [A]

time = 0.25, size = 111, normalized size = 0.93

$$\frac{-\sqrt{a}\,(2a-3bx)(a+x(b+cx))+(3b^2-4ac)\,x^2\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3 + c*x^4]),x]

[Out] (-(Sqrt[a]*(2*a - 3*b*x)*(a + x*(b + c*x))) + (3*b^2 - 4*a*c)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a + x*(b + c*x))])

Maple [A]
time = 0.04, size = 152, normalized size = 1.28

method	result
risch	$-\frac{(c\,x^2+bx+a)(-3bx+2a)}{4a^2x\sqrt{x^2\,(c\,x^2+bx+a)}}+\frac{\left(\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{8a^{\frac{5}{2}}}\right)}{\sqrt{x^2\,(c\,x^2+bx+a)}}$
default	$-\frac{\sqrt{c\,x^2+bx+a}\,\left(-6a^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,bx-4c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)a^2x^2+3\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{\cdots}}{\cdots}\right)\cdots\right)}{8x\sqrt{c\,x^4+b\,x^3+a\,x^2}\,a^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/8/x*(c*x^2+b*x+a)^(1/2)*(-6*a^(3/2)*(c*x^2+b*x+a)^(1/2)*b*x-4*c*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a^2*x^2+3*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*a*b^2*x^2+4*(c*x^2+b*x+a)^(1/2)*a^(5/2))/(c*x^4+b*x^3+a*x^2)^(1/2)/a^(7/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^3 + a*x^2)*x^2), x)

Fricas [A]
time = 0.39, size = 232, normalized size = 1.95

$$\left[-\frac{(3\,b^2-4\,ac)\sqrt{a}\,x^3\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\,(3\,abx-2\,a^2)}{16\,a^3x^3},\ \frac{(3\,b^2-4\,ac)\sqrt{-a}\,x^3\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}\,(3\,abx-2\,a^2)}{8\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((3*b^2 - 4*a*c)*sqrt(a)*x^3*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8
*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c
*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/8*((3*b^2 - 4*a*c)*sq
rt(-a)*x^3*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c
*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))
/(a^3*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x + c*x**2))), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(1/2)), x)
```

3.55 $\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3(b^2-4ac)\,x}-\frac{2bx\sqrt{c}}{c}$$

```
[Out] 2*x^4*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+3/8*(-4*a*c+5*b^2)*x
*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(
7/2)/(c*x^4+b*x^3+a*x^2)^(1/2)+1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2
)/c^2/(-4*a*c+b^2)-1/4*b*(-52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^3/(-4
*a*c+b^2)/x-2*b*x*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)
```

Rubi [A]
time = 0.33, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\frac{3x(5b^2-4ac)\sqrt{a+bx+cx^2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{7/2}\sqrt{ax^2+bx^3+cx^4}}-\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4c^3x(b^2-4ac)}+\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2(b^2-4ac)}+\frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^4*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) + ((5*b^2 -
 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*c^2*(b^2 - 4*a*c)) - (b*(15*b^2 -
52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*c^3*(b^2 - 4*a*c)*x) - (2*b*x*Sqrt[
a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*x*Sqrt[a + b
*x + c*x^2]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7
/2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^7}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx &= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^3(6a+3bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2bx\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)} + \frac{2\int \frac{x^2\left(6ab+\frac{3}{2}(5b^2-12ac\right.}{\sqrt{ax^2+bx^3+c}}}{3c\,(b^2-4ac)} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{2bx\sqrt{ax^2}}{c\,(b^2} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{b(15b^2-5}{4} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{b(15b^2-5}{4} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{b(15b^2-5}{4} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{b(15b^2-5}{4} \\
&= \frac{2x^4(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} + \frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2c^2\,(b^2-4ac)} - \frac{b(15b^2-5}{4}
\end{aligned}
$$

Mathematica [A]
time = 0.65, size = 185, normalized size = 0.71

$$
\frac{x\left(2\sqrt{c}\,(4a^2c(-13b+6cx)+b^2x(15b^2+5bcx-2c^2x^2)+a(15b^3-62b^2cx-20bc^2x^2+8c^3x^3))+3(5b^4-24ab^2c+16a^2c^2)\sqrt{a+x(b+cx)}\,\log\left(c^3\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{8c^{7/2}\,(-b^2+4ac)\,\sqrt{x^2(a+x(b+cx))}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(4*a^2*c*(-13*b + 6*c*x) + b^2*x*(15*b^2 + 5*b*c*x - 2*c^2*x^
2) + a*(15*b^3 - 62*b^2*c*x - 20*b*c^2*x^2 + 8*c^3*x^3)) + 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Sqrt[a + x*(b + c*x)]*Log[c^3*(b + 2*c*x - 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])]))/(8*c^(7/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x)
)])
```

Maple [A]
time = 0.06, size = 283, normalized size = 1.08

method	result
default	$-\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(-16c^{\frac{9}{2}}a\,x^{3}+4c^{\frac{7}{2}}b^{2}x^{3}+40c^{\frac{7}{2}}ab\,x^{2}-10c^{\frac{5}{2}}b^{3}x^{2}-48c^{\frac{7}{2}}a^{2}x+124c^{\frac{5}{2}}a\,b^{2}x-30c^{\frac{3}{2}}b^{4}x+48\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}}{2\sqrt{c}}\right.\right.}{}$
risch	$-\frac{\left(-2cx+7b\right)\left(c\,x^{2}+bx+a\right)x}{4c^{3}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{\left(\frac{3xa}{2c^{2}\sqrt{c\,x^{2}+bx+a}}-\frac{15x\,b^{2}}{8c^{3}\sqrt{c\,x^{2}+bx+a}}-\frac{5ba}{4c^{3}\sqrt{c\,x^{2}+bx+a}}+\frac{}{16c^{4}\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x^3*(c*x^2+b*x+a)/c^(9/2)*(-16*c^(9/2)*a*x^3+4*c^(7/2)*b^2*x^3+40*c^(7
/2)*a*b*x^2-10*c^(5/2)*b^3*x^2-48*c^(7/2)*a^2*x+124*c^(5/2)*a*b^2*x-30*c^(3
/2)*b^4*x+48*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2
+b*x+a)^(1/2)*a^2*c^3-72*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(
1/2))*(c*x^2+b*x+a)^(1/2)*a*b^2*c^2+15*ln(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2
)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b^4*c+104*c^(5/2)*a^2*b-30*c^(3/2)*
a*b^3)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.43, size = 616, normalized size = 2.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(c)*log(
-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)*sqrt(c)
 + (b^2 + 4*a*c)*x)/x) + 4*(15*a*b^3*c - 52*a^2*b*c^2 - 2*(b^2*c^3 - 4*a*c^
4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^
```

```
3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 - 4*a*c^6)*x^3 + (b^3*c^4 - 4*
a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x), -1/8*(3*((5*b^4*c - 24*a*b^2*c^2
 + 16*a^2*c^3)*x^3 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2 + (5*a*b^4 - 2
4*a^2*b^2*c + 16*a^3*c^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2
)*(2*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*(15*a*b^3*c - 52*a^
2*b*c^2 - 2*(b^2*c^3 - 4*a*c^4)*x^3 + 5*(b^3*c^2 - 4*a*b*c^3)*x^2 + (15*b^4
*c - 62*a*b^2*c^2 + 24*a^2*c^3)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((b^2*c^5 -
 4*a*c^6)*x^3 + (b^3*c^4 - 4*a*b*c^5)*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.75, size = 326, normalized size = 1.24

$$\frac{\left(15\,b^4\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+30\,\sqrt{a}\,b^3\sqrt{c}-104\,a^{\frac{3}{2}}bc^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{8\left(b^2c^{\frac{7}{2}}-4\,ac^{\frac{9}{2}}\right)}+\frac{\left(\left(\frac{2\left(b^2c^2-4\,ac^3\right)x}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}-\frac{5\left(b^3c-4\,abc^2\right)}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,b^4-62\,ab^2c+24\,a^2c^2}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}\right)x-\frac{15\,ab^3-52\,a^2bc}{b^2c^3\mathrm{sgn}(x)-4\,ac^4\mathrm{sgn}(x)}}{4\,\sqrt{cx^2+bx+a}}-\frac{3\left(5\,b^2-4\,ac\right)\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{7}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(15*b^4*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sq
rt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*
b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))*sgn(x)/(b^2*c^(7/2) - 4*a*c^(9/2)) + 1
/4*(((2*(b^2*c^2 - 4*a*c^3)*x/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)) - 5*(b^3*c
- 4*a*b*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*b^4 - 62*a*b^2*c +
24*a^2*c^2)/(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))*x - (15*a*b^3 - 52*a^2*b*c)/
(b^2*c^3*sgn(x) - 4*a*c^4*sgn(x)))/sqrt(c*x^2 + b*x + a) - 3/8*(5*b^2 - 4*a
*c)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/(c^(7/2)*s
gn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^7/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.56 $\int \frac{x^6}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=201

$$\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}+\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2\,(b^2-4ac)\,x}-\frac{3bx\sqrt{a+bx+cx^2}}{2c^{5/2}}$$

```
[Out] 2*x^3*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-3/2*b*x*arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(5/2)/(c*x^4+b
*x^3+a*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)+(-8*a*c+3*b^
2)*(c*x^4+b*x^3+a*x^2)^(1/2)/c^2/(-4*a*c+b^2)/x
```

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{c^2x\,(b^2-4ac)}+\frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)}-\frac{3bx\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{2c^{5/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^3*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)) + ((3*b^2 - 8*a*c)*Sqrt[a*x^2 +
b*x^3 + c*x^4])/(c^2*(b^2 - 4*a*c)*x) - (3*b*x*Sqrt[a + b*x + c*x^2]*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*Sqrt[a*x^2 + b
*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^6}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x^2(4a+2bx)}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{\int \frac{x(2ab+(3b^2-8ac)x)}{\sqrt{ax^2+bx^3+cx^4}}}{c(b^2-4ac)}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+}}{c^2(b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+}}{c^2(b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+}}{c^2(b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+}}{c^2(b^2-4ac}$$

$$= \frac{2x^3(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)} + \frac{(3b^2-8ac)\sqrt{ax^2+}}{c^2(b^2-4ac}$$

Mathematica [A]
time = 0.46, size = 143, normalized size = 0.71

$$\frac{x\left(2\sqrt{c}\,(8a^2c-b^2x(3b+cx)+a(-3b^2+10bcx+4c^2x^2))-3b(b^2-4ac)\sqrt{a+x(b+cx)}\,\log\left(c^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)\right)}{2c^{5/2}\,(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(8*a^2*c - b^2*x*(3*b + c*x) + a*(-3*b^2 + 10*b*c*x + 4*c^2*x
^2)) - 3*b*(b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*Log[c^2*(b + 2*c*x - 2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/(2*c^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b +
 c*x))])
```

Maple [A]
time = 0.06, size = 199, normalized size = 0.99

method	result

default	$\frac{x^{3}\left(c\,x^{2}+bx+a\right)\left(8c^{\frac{7}{2}}a\,x^{2}-2c^{\frac{5}{2}}b^{2}x^{2}+20c^{\frac{5}{2}}abx-6c^{\frac{3}{2}}b^{3}x+16c^{\frac{5}{2}}a^{2}-6c^{\frac{3}{2}}a\,b^{2}-12\ln\left(\frac{2\sqrt{c\,x^{2}+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{c\,x^{2}+}\right.}{2c^{\frac{7}{2}}\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(4ac-b^{2}\right)}$
risch	$\frac{\left(c\,x^{2}+bx+a\right)x}{c^{2}\sqrt{x^{2}\left(c\,x^{2}+bx+a\right)}}+\frac{\left(\frac{3bx}{2c^{2}\sqrt{c\,x^{2}+bx+a}}-\frac{b^{2}}{4c^{3}\sqrt{c\,x^{2}+bx+a}}-\frac{b^{3}x}{2c^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}-\frac{}{4c^{3}(}\right.}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^3*(c*x^2+b*x+a)/c^(7/2)*(8*c^(7/2)*a*x^2-2*c^(5/2)*b^2*x^2+20*c^(5/2)
*a*b*x-6*c^(3/2)*b^3*x+16*c^(5/2)*a^2-6*c^(3/2)*a*b^2-12*ln(1/2*(2*(c*x^2+b
*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a*b*c^2+3*ln(1/2*
(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b^3*c)
/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.41, size = 486, normalized size = 2.42

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{3}+\left(b^{4}-4\,ab^{2}c\right)x^{2}+\left(ab^{3}-4\,a^{2}bc\right)x\right)\sqrt{c}\log\left(-\frac{8\,c^{2}x^{3}+8\,bcx^{2}-4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{c}+\left(b^{2}+4\,ac\right)x}{x}\right)+4\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(3\,ab^{2}c-8\,a^{2}c^{2}+\left(b^{2}c^{2}-4\,ac^{3}\right)x^{2}+\left(3\,b^{3}c-10\,abc^{2}\right)x\right)}{4\left(\left(b^{2}c^{4}-4\,ac^{5}\right)x^{3}+\left(b^{3}c^{3}-4\,abc^{4}\right)x^{2}+\left(ab^{2}c^{3}-4\,a^{2}c^{4}\right)x\right)},\ \frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{3}+\left(b^{4}-4\,ab^{2}c\right)x^{2}+\left(ab^{3}-4\,a^{2}bc\right)x\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)\sqrt{-c}}{2\left(c^{2}x^{3}+bcx^{2}+acx\right)}\right)+2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(3\,ab^{2}c-8\,a^{2}c^{2}+\left(b^{2}c^{2}-4\,ac^{3}\right)x^{2}+\left(3\,b^{3}c-10\,abc^{2}\right)x\right)}{2\left(\left(b^{2}c^{4}-4\,ac^{5}\right)x^{3}+\left(b^{3}c^{3}-4\,abc^{4}\right)x^{2}+\left(ab^{2}c^{3}-4\,a^{2}c^{4}\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3 - 4*a^2*b
*c)*x)*sqrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*
(2*c*x + b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(
3*a*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x)
)/((b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2
*c^4)*x), 1/2*(3*((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 4*a*b^2*c)*x^2 + (a*b^3
- 4*a^2*b*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)
*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(3*a
```

```
*b^2*c - 8*a^2*c^2 + (b^2*c^2 - 4*a*c^3)*x^2 + (3*b^3*c - 10*a*b*c^2)*x))/(
(b^2*c^4 - 4*a*c^5)*x^3 + (b^3*c^3 - 4*a*b*c^4)*x^2 + (a*b^2*c^3 - 4*a^2*c^
4)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 5.20, size = 237, normalized size = 1.18

$$-\frac{\left(3\,b^3\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)-12\,abc\log\left(\left|-b+2\,\sqrt{a}\,\sqrt{c}\right|\right)+6\,\sqrt{a}\,b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{2\left(b^2c^{\frac{5}{2}}-4\,ac^{\frac{7}{2}}\right)}+\frac{\left(\frac{(b^2c-4\,ac^2)x}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}+\frac{3\,b^3-10\,abc}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}\right)x+\frac{3\,ab^2-8\,a^2c}{b^2c^2\mathrm{sgn}(x)-4\,ac^3\mathrm{sgn}(x)}}{\sqrt{cx^2+bx+a}}+\frac{3\,b\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{2\,c^{\frac{5}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt
(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2))*sgn(x)/(b^2*c^(
5/2) - 4*a*c^(7/2)) + (((b^2*c - 4*a*c^2)*x/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x
)) + (3*b^3 - 10*a*b*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))*x + (3*a*b^2 - 8
*a^2*c)/(b^2*c^2*sgn(x) - 4*a*c^3*sgn(x)))/sqrt(c*x^2 + b*x + a) + 3/2*b*lo
g(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/(c^(5/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^6/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.57 $\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{c\,(b^2-4ac)\,x}+\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x^2*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)+x*arctanh(1/2*(2*c*x
+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*(c*x^2+b*x+a)^(1/2)/c^(3/2)/(c*x^4+b*x^3+a
*x^2)^(1/2)-2*b*(c*x^4+b*x^3+a*x^2)^(1/2)/c/(-4*a*c+b^2)/x
```

Rubi [A]
time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1937, 1963, 12, 1928, 635, 212}

$$\frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2b\sqrt{ax^2+bx^3+cx^4}}{cx\,(b^2-4ac)}+\frac{x\sqrt{a+bx+cx^2}\,\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\,\sqrt{a+bx+cx^2}}\right)}{c^{3/2}\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^2*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (2*b*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(c*(b^2 - 4*a*c)*x) + (x*Sqrt[a + b*x + c*x^2]*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(c^(3/2)*Sqrt[a*x^2 +
b*x^3 + c*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1937

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - 2*n + q + 1))*(2*a + b*x^(n - q))*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/((n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1
/((n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - 2*n + q)*(2*a*(m + p*q - 2*(n
- q) + 1) + b*(m + p*q + (n - q)*(2*p + 1) + 1)*x^(n - q))*(a*x^q + b*x^n +
 c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &
& PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[
p, -1] && RationalQ[m, q] && GtQ[m + p*q + 1, 2*(n - q)]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int \frac{x^5}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}dx}{b^2-4ac}$$

$$= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{2\int \frac{(b^2-4ac)x}{2\sqrt{ax^2+bx^3+cx^4}}}{c(b^2-4ac)}$$

$$= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{\int \frac{x}{\sqrt{ax^2+bx^3+cx^4}}}{c}$$

$$= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{\left(x\sqrt{a+bx+cx^2}\right)\int}{c\sqrt{ax^2+b}}$$

$$= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{\left(2x\sqrt{a+bx+cx^2}\right)}{c}$$

$$= \frac{2x^2(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2b\sqrt{ax^2+bx^3+cx^4}}{c(b^2-4ac)x} + \frac{x\sqrt{a+bx+cx^2}\tanh}{c^{3/2}\sqrt{ax}}$$

Mathematica [A]
time = 0.40, size = 109, normalized size = 0.71

$$\frac{x\left(2\sqrt{c}\,(b^2x+a(b-2cx))+(b^2-4ac)\sqrt{a+x(b+cx)}\,\log\left(c\left(b+2cx-2\sqrt{c}\,\sqrt{a+x(b+cx)}\right)\right)\right)}{c^{3/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*Sqrt[c]*(b^2*x + a*(b - 2*c*x)) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]
*Log[c*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/(c^(3/2)*(-b^2 + 4*
a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 166, normalized size = 1.08

method	result
default	$-\frac{x^3(cx^2+bx+a)\left(4c^{\frac{5}{2}}ax-2c^{\frac{3}{2}}b^2x-2c^{\frac{3}{2}}ab-4\ln\left(\frac{2\sqrt{cx^2+bx+a}\,\sqrt{c}+2cx+b}{2\sqrt{c}}\right)\sqrt{cx^2+bx+a}\,ac^2+\ln\left(\frac{2\sqrt{cx^2-}}{}\right.\right.}{c^{\frac{5}{2}}(cx^4+bx^3+ax^2)^{\frac{3}{2}}(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^3*(c*x^2+b*x+a)*(4*c^(5/2)*a*x-2*c^(3/2)*b^2*x-2*c^(3/2)*a*b-4*ln(1/2*(2
*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*a*c^2+ln
(1/2*(2*(c*x^2+b*x+a)^(1/2)*c^(1/2)+2*c*x+b)/c^(1/2))*(c*x^2+b*x+a)^(1/2)*b
^2*c)/c^(5/2)/(c*x^4+b*x^3+a*x^2)^(3/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.38, size = 414, normalized size = 2.71

$$\left[\frac{\left((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x\right)\sqrt{c}\,\log\left(-\frac{8\,c^2x^3+8\,bcx^2+4\,\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{c}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\,\left(abc+(b^2c-2\,ac^2)x\right)}{2\left((b^2c^3-4\,ac^4)x^3+(b^3c^2-4\,abc^3)x^2+(ab^2c^2-4\,a^2c^3)x\right)},-\frac{\left((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x\right)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(2\,cx+b)\sqrt{-c}}{2\,(c^2x^3+bcx^2+acx)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}\,\left(abc+(b^2c-2\,ac^2)x\right)}{(b^2c^3-4\,ac^4)x^3+(b^3c^2-4\,abc^3)x^2+(ab^2c^2-4\,a^2c^3)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(c)*log(-(8*c^2*x^3 + 8*b*c*x^2 + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x +
 b)*sqrt(c) + (b^2 + 4*a*c)*x)/x) - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c +
(b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4*a*b*c^3)*x^2
+ (a*b^2*c^2 - 4*a^2*c^3)*x), -(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^
2 + (a*b^2 - 4*a^2*c)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2
*c*x + b)*sqrt(-c)/(c^2*x^3 + b*c*x^2 + a*c*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a*b*c + (b^2*c - 2*a*c^2)*x))/((b^2*c^3 - 4*a*c^4)*x^3 + (b^3*c^2 - 4
*a*b*c^3)*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.42, size = 170, normalized size = 1.11

$$\frac{\left(b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4\,ac\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\,b\sqrt{c}\right)\mathrm{sgn}(x)}{b^2c^{\frac{3}{2}}-4\,ac^{\frac{5}{2}}}-\frac{2\left(\frac{ab}{b^2c\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}+\frac{(b^2-2\,ac)x}{b^2c\mathrm{sgn}(x)-4\,ac^2\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}-\frac{\log\left(\left|-2\left(\sqrt{c}\,x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] (b^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c
))) + 2*sqrt(a)*b*sqrt(c))*sgn(x)/(b^2*c^(3/2) - 4*a*c^(5/2)) - 2*(a*b/(b^2
*c*sgn(x) - 4*a*c^2*sgn(x)) + (b^2 - 2*a*c)*x/(b^2*c*sgn(x) - 4*a*c^2*sgn(x
)))/sqrt(c*x^2 + b*x + a) - log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*
sqrt(c) - b))/(c^(3/2)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^5/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.58 $\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] 2*x*(b*x+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1930}

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1930

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[x^((n - 1)/2)*((4*a + 2*b*x)/((b^2 - 4*a*c)*Sqrt[a*x^
(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, (3
*n - 1)/2] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^4}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2x(2a+bx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A]
time = 0.24, size = 37, normalized size = 0.92

$$\frac{2x(2a+bx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(2*a + b*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 53, normalized size = 1.32

method	result	size
gosper	$-\frac{2\left(c\,x^{2}+b x+a\right)\left(b x+2 a\right) x^{3}}{\left(4 a c-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	53
default	$-\frac{2\left(c\,x^{2}+b x+a\right)\left(b x+2 a\right) x^{3}}{\left(4 a c-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	53
trager	$-\frac{2\left(b x+2 a\right)\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{\left(c\,x^{2}+b x+a\right) x\left(4 a c-b^{2}\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -2*(c*x^2+b*x+a)*(b*x+2*a)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.35, size = 73, normalized size = 1.82

$$\frac{2\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)}{(b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)/((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4
*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left(x^2\,(a+bx+cx^2)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 4.24, size = 69, normalized size = 1.72

$$\frac{2\left(\frac{bx}{b^2\text{sgn}(x)-4\,ac\text{sgn}(x)}+\frac{2\,a}{b^2\text{sgn}(x)-4\,ac\text{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}-\frac{4\,\sqrt{a}\,\text{sgn}(x)}{b^2-4\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b*x/(b^2*sgn(x) - 4*a*c*sgn(x)) + 2*a/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(
c*x^2 + b*x + a) - 4*sqrt(a)*sgn(x)/(b^2 - 4*a*c)
```

Mupad [B]
time = 2.12, size = 75, normalized size = 1.88

$$-\frac{\left(\frac{4\,a\,c}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c\,x}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] -(((4*a*c)/(4*a*c^2 - b^2*c) + (2*b*c*x)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3
+ c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.59 $\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx$

Optimal. Leaf size=39

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

```
[Out] -2*x*(2*c*x+b)/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {1929}

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])
```

Rule 1929

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3+cx^4)^{3/2}} dx = -\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Mathematica [A]
time = 0.24, size = 36, normalized size = 0.92

$$-\frac{2x(b+2cx)}{(b^2-4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 52, normalized size = 1.33

method	result	size
gosper	$\frac{2\left(c\,x^{2}+b x+a\right)\left(2 c x+b\right)x^{3}}{\left(4 a c-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	52
default	$\frac{2\left(c\,x^{2}+b x+a\right)\left(2 c x+b\right)x^{3}}{\left(4 a c-b^{2}\right)\left(c\,x^{4}+b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	52
trager	$\frac{2\left(2 c x+b\right)\sqrt{c\,x^{4}+b\,x^{3}+a\,x^{2}}}{\left(c\,x^{2}+b x+a\right)x\left(4 a c-b^{2}\right)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 2*(c*x^2+b*x+a)*(2*c*x+b)*x^3/(4*a*c-b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.85

$$-\frac{2\,\sqrt{cx^{4}+bx^{3}+ax^{2}}\,(2\,cx+b)}{(b^{2}c-4\,ac^{2})x^{3}+(b^{3}-4\,abc)x^{2}+(ab^{2}-4\,a^{2}c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(c*x^4 + b*x^3 + a*x^2)*(2*c*x + b)/((b^2*c - 4*a*c^2)*x^3 + (b^3 -
4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{3}}{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [A]
time = 3.80, size = 74, normalized size = 1.90

$$\frac{2\sqrt{a}\,b\mathrm{sgn}(x)}{ab^2-4\,a^2c}-\frac{2\left(\frac{2\,cx}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}+\frac{b}{b^2\mathrm{sgn}(x)-4\,ac\mathrm{sgn}(x)}\right)}{\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(a)*b*sgn(x)/(a*b^2 - 4*a^2*c) - 2*(2*c*x/(b^2*sgn(x) - 4*a*c*sgn(x))
 + b/(b^2*sgn(x) - 4*a*c*sgn(x)))/sqrt(c*x^2 + b*x + a)
```

Mupad [B]
time = 2.03, size = 75, normalized size = 1.92

$$\frac{\left(\frac{4\,c^2\,x}{4\,a\,c^2-b^2\,c}+\frac{2\,b\,c}{4\,a\,c^2-b^2\,c}\right)\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}{x\,\left(c\,x^2+b\,x+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] (((4*c^2*x)/(4*a*c^2 - b^2*c) + (2*b*c)/(4*a*c^2 - b^2*c))*(a*x^2 + b*x^3 +
 c*x^4)^(1/2))/(x*(a + b*x + c*x^2))
```

3.60 $\int \frac{x^2}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2x(b^2-2ac+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

```
[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x*(b*
c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1936, 1918, 212}

$$\frac{2x(-2ac+b^2+bcx)}{a\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/a^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1936

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q
```

```
+ b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] && !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0
] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p +
 3)]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(ax^2+bx^3+cx^4)^{3/2}}\,dx &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}+\frac{\int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx}{a}\\ &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{2\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)}{a}\\ &= \frac{2x(b^2-2ac+bcx)}{a\,(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 108, normalized size = 1.15

$$-\frac{2x\left(\sqrt{a}\,(b^2-2ac+bcx)+(b^2-4ac)\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)\right)}{a^{3/2}\,(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (-2*x*(Sqrt[a]*(b^2 - 2*a*c + b*c*x) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*
ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]]))/(a^(3/2)*(-b^2 + 4*a
*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 164, normalized size = 1.74

method	result
default	$\frac{x^3(cx^2+bx+a)\left(-2a^{\frac{3}{2}}bcx+4a^{\frac{5}{2}}c-2a^{\frac{3}{2}}b^2-4\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2+bx+a}\,a^2c+\ln\left(\frac{2a+bx+2\sqrt{a}\cdots}{\cdots}\right)\cdots\right)}{(cx^4+bx^3+ax^2)^{\frac{3}{2}}a^{\frac{5}{2}}(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] x^3*(c*x^2+b*x+a)*(-2*a^(3/2)*b*c*x+4*a^(5/2)*c-2*a^(3/2)*b^2-4*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^2*c+ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a*b^2)/(c*x^4+b*x^3+a*x
^2)^(3/2)/a^(5/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(84) = 168.
time = 0.40, size = 411, normalized size = 4.37

$$\left[\frac{\left((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x\right)\sqrt{a}\,\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)+4\,\sqrt{cx^4+bx^3+ax^2}\,(abcx+ab^2-2\,a^2c)}{2\left((a^2b^2c-4\,a^3c^2)x^3+(a^2b^3-4\,a^3bc)x^2+(a^3b^2-4\,a^4c)x\right)},\ \frac{\left((b^2c-4\,ac^2)x^3+(b^3-4\,abc)x^2+(ab^2-4\,a^2c)x\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}\,(abcx+ab^2-2\,a^2c)}{(a^2b^2c-4\,a^3c^2)x^3+(a^2b^3-4\,a^3bc)x^2+(a^3b^2-4\,a^4c)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3
 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(a*b*c*
x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 - 4*a^3*b*c)*
x^2 + (a^3*b^2 - 4*a^4*c)*x), (((b^2*c - 4*a*c^2)*x^3 + (b^3 - 4*a*b*c)*x^2
 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*
x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*x
^2)*(a*b*c*x + a*b^2 - 2*a^2*c))/((a^2*b^2*c - 4*a^3*c^2)*x^3 + (a^2*b^3 -
4*a^3*b*c)*x^2 + (a^3*b^2 - 4*a^4*c)*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\left(a+bx+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(84) = 168.
time = 5.28, size = 199, normalized size = 2.12

$$-\frac{2\left(ab^{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^{2}c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\,b^{2}-2\,\sqrt{-a}\,a^{\frac{3}{2}}c\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^{2}b^{2}-4\,\sqrt{-a}\,a^{3}c}+\frac{2\left(\frac{abcx\mathrm{sgn}(x)}{a^{2}b^{2}-4\,a^{3}c}+\frac{ab^{2}\mathrm{sgn}(x)-2\,a^{2}c\mathrm{sgn}(x)}{a^{2}b^{2}-4\,a^{3}c}\right)}{\sqrt{cx^{2}+bx+a}}+\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^{2}+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqrt(a)/sqrt(-a)) + sqr
t(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)*sgn(x)/(sqrt(-a)*a^2*b^2 - 4*sqrt
(-a)*a^3*c) + 2*(a*b*c*x*sgn(x)/(a^2*b^2 - 4*a^3*c) + (a*b^2*sgn(x) - 2*a^2
*c*sgn(x))/(a^2*b^2 - 4*a^3*c))/sqrt(c*x^2 + b*x + a) + 2*arctan(-(sqrt(c)*
x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*a*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.61 $\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=144

$$\frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2}+\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*b*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(5/2)+2*
(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^3+a*x^2)^(1/2)-(-8*a*c+3*b^2)*(
c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^2
```

Rubi [A]
time = 0.11, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1938, 1965, 12, 1918, 212}

$$\frac{3b\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{2a^{5/2}}-\frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2x^2(b^2-4ac)}+\frac{2(-2ac+b^2+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4]) - (
(3*b^2 - 8*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(a^2*(b^2 - 4*a*c)*x^2) + (3*b
*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(2*a^(5/
2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

```
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{x}{(ax^2+bx^3+cx^4)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{3b^2}{2}+4ac-bcx}{x\sqrt{ax^2+bx^3+cx^4}}\,dx}{a(b^2-4ac)}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{2\int -\frac{}{4\sqrt{}}}{a}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} - \frac{(3b)\int \frac{}{\sqrt{}}}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{(3b)\text{Subs}}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\sqrt{ax^2+bx^3+cx^4}} - \frac{(3b^2-8ac)\sqrt{ax^2+bx^3+cx^4}}{a^2(b^2-4ac)x^2} + \frac{3b\tanh^{-}}{}$$

Mathematica [A]
time = 0.52, size = 134, normalized size = 0.93

$$\frac{\sqrt{a}\left(-4a^2c+3b^2x(b+cx)+a(b^2-10bcx-8c^2x^2)\right)+3b(b^2-4ac)\,x\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}(-b^2+4ac)\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*(-4*a^2*c + 3*b^2*x*(b + c*x) + a*(b^2 - 10*b*c*x - 8*c^2*x^2)) +
3*b*(b^2 - 4*a*c)*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(
b + c*x)])/Sqrt[a]])/(a^(5/2)*(-b^2 + 4*a*c)*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.05, size = 201, normalized size = 1.40

method	result
default	$-\frac{x^2(cx^2+bx+a)\left(16a^{\frac{5}{2}}c^2x^2-6a^{\frac{3}{2}}b^2cx^2+20a^{\frac{5}{2}}bcx-6a^{\frac{3}{2}}b^3x-12\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)\sqrt{cx^2+bx+a}\right.}{2(cx^4+bx^3+ax^2)^{\frac{3}{2}}a^{\frac{7}{2}}(4ac-b^2}$

risch	$-\frac{c\,x^2+bx+a}{a^2\sqrt{x^2\left(c\,x^2+bx+a\right)}}+\frac{\left(-\frac{b}{a^2\sqrt{c\,x^2+bx+a}}+\frac{2b^2cx}{a^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+\frac{b^3}{a^2\left(4ac-b^2\right)\sqrt{c\,x^2+bx-}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2*(c*x^2+b*x+a)*(16*a^(5/2)*c^2*x^2-6*a^(3/2)*b^2*c*x^2+20*a^(5/2)*b
*c*x-6*a^(3/2)*b^3*x-12*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^
2+b*x+a)^(1/2)*a^2*b*c*x+3*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c
*x^2+b*x+a)^(1/2)*a*b^3*x+8*a^(7/2)*c-2*a^(5/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3
/2)/a^(7/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^4 + b*x^3 + a*x^2)^(3/2), x)
```

Fricas [A]
time = 0.41, size = 496, normalized size = 3.44

$$\left[\frac{3\left((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2\right)\sqrt{a}\,\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x+4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)-4\,\sqrt{cx^4+bx^3+ax^2}\,(a^2b^2-4\,a^3c+(3\,ab^2c-8\,a^2c^2)x^2+(3\,ab^3-10\,a^2bc)x)}{4\left((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2-4\,a^5c)x^2\right)},-\frac{3\left((b^3c-4\,abc^2)x^4+(b^4-4\,ab^2c)x^3+(ab^3-4\,a^2bc)x^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)+2\,\sqrt{cx^4+bx^3+ax^2}\,(a^2b^2-4\,a^3c+(3\,ab^2c-8\,a^2c^2)x^2+(3\,ab^3-10\,a^2bc)x)}{2\left((a^3b^2c-4\,a^4c^2)x^4+(a^3b^3-4\,a^4bc)x^3+(a^4b^2-4\,a^5c)x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x
^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) - 4*sqrt(c*x^4 + b*x^3 + a*x^
2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b*c
)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2 -
 4*a^5*c)*x^2), -1/2*(3*((b^3*c - 4*a*b*c^2)*x^4 + (b^4 - 4*a*b^2*c)*x^3 +
(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b
*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) + 2*sqrt(c*x^4 + b*x^3 + a*
x^2)*(a^2*b^2 - 4*a^3*c + (3*a*b^2*c - 8*a^2*c^2)*x^2 + (3*a*b^3 - 10*a^2*b
*c)*x))/((a^3*b^2*c - 4*a^4*c^2)*x^4 + (a^3*b^3 - 4*a^4*b*c)*x^3 + (a^4*b^2
 - 4*a^5*c)*x^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x + c*x**2))**(3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.62 $\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3\left(b^2-4ac\right)x^2}-\frac{3(5}{}$$

```
[Out] -3/8*(-4*a*c+5*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/
2))/a^(7/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^3+a*x^2)^(1/2)-
1/2*(-12*a*c+5*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^3+1/4*b*(-
52*a*c+15*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^2
```

Rubi [A]
time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1921, 1965, 12, 1918, 212}

$$-\frac{3(5b^2-4ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{8a^{7/2}}+\frac{b(15b^2-52ac)\sqrt{ax^2+bx^3+cx^4}}{4a^3x^2\left(b^2-4ac\right)}-\frac{(5b^2-12ac)\sqrt{ax^2+bx^3+cx^4}}{2a^2x^3\left(b^2-4ac\right)}+\frac{2(-2ac+b^2+bcx)}{ax\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x*Sqrt[a*x^2 + b*x^3 + c*x^4]) -
 ((5*b^2 - 12*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*x^3) +
 (b*(15*b^2 - 52*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^3*(b^2 - 4*a*c)*x^2
) - (3*(5*b^2 - 4*a*c)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^
3 + c*x^4])])/(8*a^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
```

```
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1921

```
Int[((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_), x_Symbol
] :> Simp[(-x^(-q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q + b*x^n + c*x
^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*(n -
 q)*(p + 1)*(b^2 - 4*a*c)), Int[(((p*q + 1)*(b^2 - 2*a*c) + (n - q)*(p + 1)
*(b^2 - 4*a*c) + b*c*(p*q + (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^
n + c*x^(2*n - q))^(p + 1))/x^q, x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-2(b^2-2ac)+\frac{1}{2}\left(-b^2+4ac\right)-2bcx}{x^2\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{\int \frac{-\frac{1}{4}b(15\cdots}{x\sqrt{\cdots}}}{\cdots}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b(15b^2\cdots}{\cdots}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b(15b^2\cdots}{\cdots}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b(15b^2\cdots}{\cdots}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(5b^2-12ac\right)\sqrt{ax^2+bx^3+cx^4}}{2a^2\left(b^2-4ac\right)x^3} + \frac{b(15b^2\cdots}{\cdots}$$

Mathematica [A]
time = 0.72, size = 180, normalized size = 0.86

$$-\frac{\sqrt{a}\left(-8a^3c-15b^3x^2(b+cx)+2a^2(b^2+10bcx-12c^2x^2)+abx(-5b^2+62bcx+52c^2x^2)\right)-3(5b^4-24ab^2c+16a^2c^2)\,x^2\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{4a^{7/2}\left(b^2-4ac\right)x\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3 + c*x^4)^(-3/2),x]
```

```
[Out] -1/4*(Sqrt[a]*(-8*a^3*c - 15*b^3*x^2*(b + c*x) + 2*a^2*(b^2 + 10*b*c*x - 12
*c^2*x^2) + a*b*x*(-5*b^2 + 62*b*c*x + 52*c^2*x^2)) - 3*(5*b^4 - 24*a*b^2*c
 + 16*a^2*c^2)*x^2*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
 + c*x)])/Sqrt[a]])/(a^(7/2)*(b^2 - 4*a*c)*x*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.06, size = 292, normalized size = 1.40

method	result
default	$-\frac{x\left(c\,x^2+bx+a\right)\left(-104a^{\frac{5}{2}}\,b\,c^2x^3+30a^{\frac{3}{2}}\,b^3c\,x^3+48a^{\frac{7}{2}}\,c^2x^2-124a^{\frac{5}{2}}\,b^2c\,x^2+30a^{\frac{3}{2}}\,b^4x^2-48\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\cdots\right.}{\cdots}$

risch	$-\frac{(c x^2+bx+a)(-7bx+2a)}{4a^3x\sqrt{x^2(c x^2+bx+a)}}+\frac{\left(-\frac{c}{a^2\sqrt{c x^2+bx+a}}+\frac{6c^2bx}{a^2(4ac-b^2)\sqrt{c x^2+bx+a}}+\frac{3c b^2}{a^2(4ac-b^2)\sqrt{c x^2+b}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*x*(c*x^2+b*x+a)*(-104*a^(5/2)*b*c^2*x^3+30*a^(3/2)*b^3*c*x^3+48*a^(7/2
)*c^2*x^2-124*a^(5/2)*b^2*c*x^2+30*a^(3/2)*b^4*x^2-48*ln((2*a+b*x+2*a^(1/2)
*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^3*c^2*x^2+72*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^2*b^2*c*x^2-15*ln((2*a
+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a*b^4*x^2-40*a^(
7/2)*b*c*x+10*a^(5/2)*b^3*x+16*a^(9/2)*c-4*a^(7/2)*b^2)/(c*x^4+b*x^3+a*x^2)
^(3/2)/a^(9/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^3 + a*x^2)^(-3/2), x)
```

Fricas [A]
time = 0.45, size = 630, normalized size = 3.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c
+ 16*a^2*b*c^2)*x^4 + (5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(a)*lo
g(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x + 4*sqrt(c*x^4 + b*x^3 + a*x^2)
*(b*x + 2*a)*sqrt(a))/x^3) + 4*(2*a^3*b^2 - 8*a^4*c - (15*a*b^3*c - 52*a^2*
b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^
3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*
b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*x^3), 1/8*(3*((5*b^4*c - 24*a*b^
2*c^2 + 16*a^2*c^3)*x^5 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^4 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3
```

```
+ a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(2*a^3*b^2 -
 8*a^4*c - (15*a*b^3*c - 52*a^2*b*c^2)*x^3 - (15*a*b^4 - 62*a^2*b^2*c + 24*
a^3*c^2)*x^2 - 5*(a^2*b^3 - 4*a^3*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^
4*b^2*c - 4*a^5*c^2)*x^5 + (a^4*b^3 - 4*a^5*b*c)*x^4 + (a^5*b^2 - 4*a^6*c)*
x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2 + bx^3 + cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3 + c*x**4)**(-3/2), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3 + c*x^4)^(3/2), x)
```

3.63 $\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=271

$$\frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3\left(b^2-4ac\right)x^3}-$$

```
[Out] 5/16*b*(-12*a*c+7*b^2)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^
(1/2))/a^(9/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^3+a*x^2)^(
1/2)-1/3*(-16*a*c+7*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4*a*c+b^2)/x^4+1/1
2*b*(-116*a*c+35*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+b^2)/x^3-1/24*(
256*a^2*c^2-460*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^4/(-4*a*c+b^2)
/x^2
```

Rubi [A]
time = 0.30, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1938, 1965, 12, 1918, 212}

$$\frac{5b(7b^2-12ac)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{16a^{9/2}}+\frac{b(35b^2-116ac)\sqrt{ax^2+bx^3+cx^4}}{12a^3x^3\left(b^2-4ac\right)}-\frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2x^4\left(b^2-4ac\right)}-\frac{(256a^2c^2-460ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{24a^4x^2\left(b^2-4ac\right)}+\frac{2(-2ac+b^2+bcx)}{ax^2\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^2*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((7*b^2 - 16*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(3*a^2*(b^2 - 4*a*c)*x^4)
 + (b*(35*b^2 - 116*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(12*a^3*(b^2 - 4*a*c)
*x^3) - ((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])
/(24*a^4*(b^2 - 4*a*c)*x^2) + (5*b*(7*b^2 - 12*a*c)*ArcTanh[(x*(2*a + b*x))
/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])])/(16*a^(9/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{7b^2}{2}+8ac-3bcx}{x^3\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{2\int}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(3}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(3}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(3}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(3}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^2\sqrt{ax^2+bx^3+cx^4}} - \frac{(7b^2-16ac)\sqrt{ax^2+bx^3+cx^4}}{3a^2\left(b^2-4ac\right)x^4} + \frac{b(3}{}$$

Mathematica [A]
time = 0.98, size = 224, normalized size = 0.83

$$\frac{\sqrt{a}\left(-32a^4c+105b^4x^3(b+cx)+5ab^2x^2(7b^2-106bcx-92c^2x^2)+8a^3(b^2+7bcx+16c^2x^2)+2a^2x(-7b^3-86b^2cx+244bc^2x^2+128c^3x^3)\right)+15b(7b^4-40ab^2c+48a^2c^2)x^3\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{24a^{9/2}\left(-b^2+4ac\right)x^2\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-32*a^4*c + 105*b^4*x^3*(b + c*x) + 5*a*b^2*x^2*(7*b^2 - 106*b*c*
x - 92*c^2*x^2) + 8*a^3*(b^2 + 7*b*c*x + 16*c^2*x^2) + 2*a^2*x*(-7*b^3 - 86
*b^2*c*x + 244*b*c^2*x^2 + 128*c^3*x^3)) + 15*b*(7*b^4 - 40*a*b^2*c + 48*a^
2*c^2)*x^3*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(24*a^(9/2)*(-b^2 + 4*a*c)*x^2*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.06, size = 340, normalized size = 1.25

method	result

default	$-\frac{(c\,x^2+bx+a)\left(-512a^{\frac{7}{2}}c^3x^4+920a^{\frac{5}{2}}b^2c^2x^4-210a^{\frac{3}{2}}b^4c\,x^4-976a^{\frac{7}{2}}b\,c^2x^3+1060a^{\frac{5}{2}}b^3c\,x^3-210a^{\frac{3}{2}}b^5x^3+720\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2}}{x}\right.\right.}{}$
risch	$-\frac{(c\,x^2+bx+a)\left(-40ac\,x^2+57b^2x^2-22abx+8a^2\right)}{24a^4x^2\sqrt{x^2\left(c\,x^2+bx+a\right)}}+\frac{\left(\frac{2bc}{a^3\sqrt{c\,x^2+bx+a}}-\frac{8b^2c^2x}{a^3\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{}{a^3\left(4ac-b^2\right)\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/48*(c*x^2+b*x+a)*(-512*a^(7/2)*c^3*x^4+920*a^(5/2)*b^2*c^2*x^4-210*a^(3/
2)*b^4*c*x^4-976*a^(7/2)*b*c^2*x^3+1060*a^(5/2)*b^3*c*x^3-210*a^(3/2)*b^5*x
^3+720*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^
3*b*c^2*x^3-600*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)
^(1/2)*a^2*b^3*c*x^3+105*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x
^2+b*x+a)^(1/2)*a*b^5*x^3-256*a^(9/2)*c^2*x^2+344*a^(7/2)*b^2*c*x^2-70*a^(5
/2)*b^4*x^2-112*a^(9/2)*b*c*x+28*a^(7/2)*b^3*x+64*a^(11/2)*c-16*a^(9/2)*b^2
)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(11/2)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x), x)
```

Fricas [A]
time = 0.54, size = 716, normalized size = 2.64

$$\left[-\frac{15\left((7b^5c-40ab^3c^2+48a^2bc^3)x^6+(7b^6-40ab^4c+48a^2b^2c^2)x^5+(7ab^5-40a^2b^3c+48a^3bc^2)x^4\right)\sqrt{a}\log\left(-\frac{8abx^2+(b^2+4ac)x^3+8a^2x-4\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{a}}{x^3}\right)+4\left(8a^4b^2-32a^5c+(105ab^4c-460a^2b^2c^2+256a^3c^3)x^4+(105ab^5-530a^2b^3c+488a^3bc^2)x^3+(35a^2b^4-172a^3b^2c+128a^4c^2)x^2-14(a^3b^3-4a^4bc)x\right)\sqrt{cx^4+bx^3+ax^2}}{96\left((a^5b^2c-4a^6c^2)x^6+(a^5b^3-4a^6bc)x^5+(a^6b^2-4a^7c)x^4\right)}, -\frac{15\left((7b^5c-40ab^3c^2+48a^2bc^3)x^6+(7b^6-40ab^4c+48a^2b^2c^2)x^5+(7ab^5-40a^2b^3c+48a^3bc^2)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}(bx+2a)\sqrt{-a}}{2(acx^3+abx^2+a^2x)}\right)+2\left(8a^4b^2-32a^5c+(105ab^4c-460a^2b^2c^2+256a^3c^3)x^4+(105ab^5-530a^2b^3c+488a^3bc^2)x^3+(35a^2b^4-172a^3b^2c+128a^4c^2)x^2-14(a^3b^3-4a^4bc)x\right)\sqrt{cx^4+bx^3+ax^2}}{48\left((a^5b^2c-4a^6c^2)x^6+(a^5b^3-4a^6bc)x^5+(a^6b^2-4a^7c)x^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*((7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4
*c + 48*a^2*b^2*c^2)*x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqr
t(a)*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3) + 4*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c
```

```
- 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3
*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3
- 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 +
 (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b^2 - 4*a^7*c)*x^4), -1/48*(15*((7*b^5*c
- 40*a*b^3*c^2 + 48*a^2*b*c^3)*x^6 + (7*b^6 - 40*a*b^4*c + 48*a^2*b^2*c^2)*
x^5 + (7*a*b^5 - 40*a^2*b^3*c + 48*a^3*b*c^2)*x^4)*sqrt(-a)*arctan(1/2*sqrt
(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) +
 2*(8*a^4*b^2 - 32*a^5*c + (105*a*b^4*c - 460*a^2*b^2*c^2 + 256*a^3*c^3)*x^
4 + (105*a*b^5 - 530*a^2*b^3*c + 488*a^3*b*c^2)*x^3 + (35*a^2*b^4 - 172*a^3
*b^2*c + 128*a^4*c^2)*x^2 - 14*(a^3*b^3 - 4*a^4*b*c)*x)*sqrt(c*x^4 + b*x^3
+ a*x^2))/((a^5*b^2*c - 4*a^6*c^2)*x^6 + (a^5*b^3 - 4*a^6*b*c)*x^5 + (a^6*b
^2 - 4*a^7*c)*x^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^{2}\left(a+bx+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**3+a*x**2)**(3/2),x)
[Out] Integral(1/(x*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\left(c\,x^4+b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
[Out] int(1/(x*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.64 $\int \frac{1}{x^2\left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx$

Optimal. Leaf size=343

$$\frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5}+\frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3\left(b^2-4ac\right)x^4}-\frac{(1$$

```
[Out] -15/128*(16*a^2*c^2-56*a*b^2*c+21*b^4)*arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x
^4+b*x^3+a*x^2)^(1/2))/a^(11/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x
^4+b*x^3+a*x^2)^(1/2)-1/4*(-20*a*c+9*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^2/(-4
*a*c+b^2)/x^5+1/8*b*(-68*a*c+21*b^2)*(c*x^4+b*x^3+a*x^2)^(1/2)/a^3/(-4*a*c+
b^2)/x^4-1/32*(240*a^2*c^2-448*a*b^2*c+105*b^4)*(c*x^4+b*x^3+a*x^2)^(1/2)/a
^4/(-4*a*c+b^2)/x^3+1/64*b*(1808*a^2*c^2-1680*a*b^2*c+315*b^4)*(c*x^4+b*x^3
+a*x^2)^(1/2)/a^5/(-4*a*c+b^2)/x^2
```

Rubi [A]
time = 0.40, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1938, 1965, 12, 1918, 212}

$$\frac{b(21b^2-68ac)\sqrt{ax^2+bx^3+cx^4}}{8a^3x^4\left(b^2-4ac\right)}-\frac{(9b^2-20ac)\sqrt{ax^2+bx^3+cx^4}}{4a^2x^5\left(b^2-4ac\right)}-\frac{15(16a^2c^2-56ab^2c+21b^4)\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{128a^{11/2}}+\frac{b(1808a^2c^2-1680ab^2c+315b^4)\sqrt{ax^2+bx^3+cx^4}}{64a^5x^2\left(b^2-4ac\right)}-\frac{(240a^2c^2-448ab^2c+105b^4)\sqrt{ax^2+bx^3+cx^4}}{32a^4x^3\left(b^2-4ac\right)}+\frac{2(-2ac+b^2+bcx)}{ax^3\left(b^2-4ac\right)\sqrt{ax^2+bx^3+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*x^3*Sqrt[a*x^2 + b*x^3 + c*x^4])
 - ((9*b^2 - 20*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(4*a^2*(b^2 - 4*a*c)*x^5)
 + (b*(21*b^2 - 68*a*c)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(8*a^3*(b^2 - 4*a*c)*x
^4) - ((105*b^4 - 448*a*b^2*c + 240*a^2*c^2)*Sqrt[a*x^2 + b*x^3 + c*x^4])/(
32*a^4*(b^2 - 4*a*c)*x^3) + (b*(315*b^4 - 1680*a*b^2*c + 1808*a^2*c^2)*Sqrt
[a*x^2 + b*x^3 + c*x^4])/(64*a^5*(b^2 - 4*a*c)*x^2) - (15*(21*b^4 - 56*a*b^
2*c + 16*a^2*c^2)*ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c
*x^4])])/(128*a^(11/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2 \left(ax^2+bx^3+cx^4\right)^{3/2}}\,dx = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{2\int \frac{-\frac{9b^2}{2}+10ac-4bcx}{x^4\sqrt{ax^2+bx^3+cx^4}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{\int -}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)x^3\sqrt{ax^2+bx^3+cx^4}} - \frac{\left(9b^2-20ac\right)\sqrt{ax^2+bx^3+cx^4}}{4a^2\left(b^2-4ac\right)x^5} + \frac{b(21}{}$$

Mathematica [A]
time = 1.42, size = 271, normalized size = 0.79

$$\frac{\sqrt{a}\left(-64a^5c-315b^5x^4(b+cx)-105ab^3x^3(b^2-18bcx-16c^2x^2)+16a^4(b^2+6bcx+10c^2x^2)+2a^2bx^2(21b^3+308b^2cx-1352bc^2x^2-904c^3x^3)-8a^3x(3b^3+26b^2cx+98bc^2x^2-60c^3x^3)\right)-15(21b^6-140ab^4c+240a^2b^2c^2-64a^3c^3)x^4\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{64a^{11/2}\left(-b^2+4ac\right)x^3\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-64*a^5*c - 315*b^5*x^4*(b + c*x) - 105*a*b^3*x^3*(b^2 - 18*b*c*x
 - 16*c^2*x^2) + 16*a^4*(b^2 + 6*b*c*x + 10*c^2*x^2) + 2*a^2*b*x^2*(21*b^3
+ 308*b^2*c*x - 1352*b*c^2*x^2 - 904*c^3*x^3) - 8*a^3*x*(3*b^3 + 26*b^2*c*x
 + 98*b*c^2*x^2 - 60*c^3*x^3)) - 15*(21*b^6 - 140*a*b^4*c + 240*a^2*b^2*c^2
 - 64*a^3*c^3)*x^4*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b
 + c*x)])/Sqrt[a]])/(64*a^(11/2)*(-b^2 + 4*a*c)*x^3*Sqrt[x^2*(a + x*(b + c*
x))])
```

Maple [A]
time = 0.07, size = 446, normalized size = 1.30

method	result
default	$-\frac{\left(c\,x^{2}+bx+a\right)\left(3616 a^{\frac{7}{2}} b\,c^{3} x^{5}-3360 a^{\frac{5}{2}} b^{3} c^{2} x^{5}+630 a^{\frac{3}{2}} b^{5} c\,x^{5}-960 a^{\frac{9}{2}} c^{3} x^{4}+5408 a^{\frac{7}{2}} b^{2} c^{2} x^{4}-3780 a^{\frac{5}{2}} b^{4} c\,x^{4}+630 a^{\frac{3}{2}} b^{6} x^{4}+960 \ln \left(\ldots\right.\right.}{\ldots}$
risch	$-\frac{\left(c\,x^{2}+bx+a\right)\left(292 a b c\,x^{3}-187 b^{3} x^{3}-56 a^{2} c\,x^{2}+82 a\,b^{2} x^{2}-40 a^{2} b x+16 a^{3}\right)}{64 a^{5} x^{3} \sqrt{x^{2} \left(c\,x^{2}+bx+a\right)}}+\frac{\left(\frac{c^{2}}{a^{3} \sqrt{c\,x^{2}+bx+a}}-\frac{3 c\,b^{2}}{a^{4} \sqrt{c\,x^{2}+bx+a}} \ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/128/x*(c*x^2+b*x+a)*(3616*a^(7/2)*b*c^3*x^5-3360*a^(5/2)*b^3*c^2*x^5+630
*a^(3/2)*b^5*c*x^5-960*a^(9/2)*c^3*x^4+5408*a^(7/2)*b^2*c^2*x^4-3780*a^(5/2
)*b^4*c*x^4+630*a^(3/2)*b^6*x^4+960*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/
2))/x)*(c*x^2+b*x+a)^(1/2)*a^4*c^3*x^4-3600*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*
x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^3*b^2*c^2*x^4+2100*ln((2*a+b*x+2*a^(1/
2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a^2*b^4*c*x^4-315*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)*(c*x^2+b*x+a)^(1/2)*a*b^6*x^4+1568*a^(9
/2)*b*c^2*x^3-1232*a^(7/2)*b^3*c*x^3+210*a^(5/2)*b^5*x^3-320*a^(11/2)*c^2*x
^2+416*a^(9/2)*b^2*c*x^2-84*a^(7/2)*b^4*x^2-192*a^(11/2)*b*c*x+48*a^(9/2)*b
^3*x+128*a^(13/2)*c-32*a^(11/2)*b^2)/(c*x^4+b*x^3+a*x^2)^(3/2)/a^(13/2)/(4*
a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Fricas [A]
time = 0.60, size = 866, normalized size = 2.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(15*((21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 +
 (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*x^6 + (21*a*b^6 -
140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*x^5)*sqrt(a)*log(-(8*a*b*x^2
+ (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*s
qrt(a))/x^3) - 4*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c - 1680*a^2*b^3*c^2 +
 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 2704*a^3*b^2*c^2 - 480
*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*b*c^2)*x^3 + 2*(21*a
^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3 - 4*a^5*b*c)*x)*sqrt
(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 + (a^6*b^3 - 4*a^7*b*
c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5), 1/128*(15*((21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*x^7 + (21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2
- 64*a^3*b*c^3)*x^6 + (21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*
c^3)*x^5)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(
-a)/(a*c*x^3 + a*b*x^2 + a^2*x)) - 2*(16*a^5*b^2 - 64*a^6*c - (315*a*b^5*c
- 1680*a^2*b^3*c^2 + 1808*a^3*b*c^3)*x^5 - (315*a*b^6 - 1890*a^2*b^4*c + 27
04*a^3*b^2*c^2 - 480*a^4*c^3)*x^4 - 7*(15*a^2*b^5 - 88*a^3*b^3*c + 112*a^4*
b*c^2)*x^3 + 2*(21*a^3*b^4 - 104*a^4*b^2*c + 80*a^5*c^2)*x^2 - 24*(a^4*b^3
- 4*a^5*b*c)*x)*sqrt(c*x^4 + b*x^3 + a*x^2))/((a^6*b^2*c - 4*a^7*c^2)*x^7 +
 (a^6*b^3 - 4*a^7*b*c)*x^6 + (a^7*b^2 - 4*a^8*c)*x^5)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(x^2 \left(a + bx + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x + c*x**2))**(3/2)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(c\,x^4+b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)),x)
[Out] int(1/(x^2*(a*x^2 + b*x^3 + c*x^4)^(3/2)), x)
```

3.65 $\int x^m(ax+bx^3+cx^5)\,dx$

Optimal. Leaf size=37

$$\frac{ax^{2+m}}{2+m}+\frac{bx^{4+m}}{4+m}+\frac{cx^{6+m}}{6+m}$$

```
[Out] a*x^(2+m)/(2+m)+b*x^(4+m)/(4+m)+c*x^(6+m)/(6+m)
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.056$, Rules used = {14}

$$\frac{ax^{m+2}}{m+2}+\frac{bx^{m+4}}{m+4}+\frac{cx^{m+6}}{m+6}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5),x]
[Out] (a*x^(2 + m))/(2 + m) + (b*x^(4 + m))/(4 + m) + (c*x^(6 + m))/(6 + m)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^m\left(ax+bx^3+cx^5\right)\,dx&=\int\left(ax^{1+m}+bx^{3+m}+cx^{5+m}\right)\,dx\\&=\frac{ax^{2+m}}{2+m}+\frac{bx^{4+m}}{4+m}+\frac{cx^{6+m}}{6+m}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 34, normalized size = 0.92

$$x^{2+m}\left(\frac{a}{2+m}+\frac{bx^2}{4+m}+\frac{cx^4}{6+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5),x]
[Out] x^(2 + m)*(a/(2 + m) + (b*x^2)/(4 + m) + (c*x^4)/(6 + m))
```

Maple [A]
time = 0.01, size = 47, normalized size = 1.27

method	result	size
norman	$\frac{a\,x^{2} {\mathrm e}^{m \ln \left(x \right)}}{2+m}+\frac{b\,x^{4} {\mathrm e}^{m \ln \left(x \right)}}{4+m}+\frac{c\,x^{6} {\mathrm e}^{m \ln \left(x \right)}}{6+m}$	47
gosper	$\frac{x^{2+m} \left(c\,m^{2} x^{4}+6 c m\,x^{4}+b\,m^{2} x^{2}+8 c\,x^{4}+8 b m\,x^{2}+a\,m^{2}+12 b\,x^{2}+10 a m+24 a \right)}{\left(6+m \right) \left(4+m \right) \left(2+m \right)}$	77
risch	$\frac{x^{m} \left(c\,m^{2} x^{4}+6 c m\,x^{4}+b\,m^{2} x^{2}+8 c\,x^{4}+8 b m\,x^{2}+a\,m^{2}+12 b\,x^{2}+10 a m+24 a \right) x^{2}}{\left(6+m \right) \left(4+m \right) \left(2+m \right)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] a/(2+m)*x^2*exp(m*ln(x))+b/(4+m)*x^4*exp(m*ln(x))+c/(6+m)*x^6*exp(m*ln(x))
```

Maxima [A]
time = 0.26, size = 37, normalized size = 1.00

$$\frac{cx^{m+6}}{m+6}+\frac{bx^{m+4}}{m+4}+\frac{ax^{m+2}}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] c*x^(m + 6)/(m + 6) + b*x^(m + 4)/(m + 4) + a*x^(m + 2)/(m + 2)
```

Fricas [A]
time = 0.34, size = 71, normalized size = 1.92

$$\frac{\left({\left(cm^{2}+6\,cm+8\,c\right)}x^{6}+{\left(bm^{2}+8\,bm+12\,b\right)}x^{4}+{\left(am^{2}+10\,am+24\,a\right)}x^{2}\right)x^{m}}{m^{3}+12\,m^{2}+44\,m+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 6*c*m + 8*c)*x^6 + (b*m^2 + 8*b*m + 12*b)*x^4 + (a*m^2 + 10*a*m +
 24*a)*x^2)*x^m/(m^3 + 12*m^2 + 44*m + 48)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(29) = 58$.
time = 0.29, size = 280, normalized size = 7.57

$$\begin{cases} - \frac{a}{4 x^{4}} - \frac{b}{2 x^{2}} + c \log{\left(x \right)} & \text{for}\: m = -6 \\- \frac{a}{2 x^{2}} + b \log{\left(x \right)} + \frac{c x^{2}}{2} & \text{for}\: m = -4 \\a \log{\left(x \right)} + \frac{b x^{2}}{2} + \frac{c x^{4}}{4} & \text{for}\: m = -2 \\\frac{a m^{2} x^{2} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{10 a m x^{2} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{24 a x^{2} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{b m^{2} x^{4} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{8 b m x^{4} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{12 b x^{4} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{c m^{2} x^{6} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{6 c m x^{6} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} + \frac{8 c x^{6} x^{m}}{m^{3} + 12 m^{2} + 44 m + 48} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x),x)
```

```
[Out] Piecewise((-a/(4*x**4) - b/(2*x**2) + c*log(x), Eq(m, -6)), (-a/(2*x**2) +
b*log(x) + c*x**2/2, Eq(m, -4)), (a*log(x) + b*x**2/2 + c*x**4/4, Eq(m, -2)
), (a*m**2*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + 10*a*m*x**2*x**m/(m**3
+ 12*m**2 + 44*m + 48) + 24*a*x**2*x**m/(m**3 + 12*m**2 + 44*m + 48) + b*m*
*2*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + 8*b*m*x**4*x**m/(m**3 + 12*m**2
 + 44*m + 48) + 12*b*x**4*x**m/(m**3 + 12*m**2 + 44*m + 48) + c*m**2*x**6*x
**m/(m**3 + 12*m**2 + 44*m + 48) + 6*c*m*x**6*x**m/(m**3 + 12*m**2 + 44*m +
 48) + 8*c*x**6*x**m/(m**3 + 12*m**2 + 44*m + 48), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(37) = 74$.
time = 5.29, size = 107, normalized size = 2.89

$$\frac{cm^2x^6x^m + 6\,cmx^6x^m + bm^2x^4x^m + 8\,cx^6x^m + 8\,bmx^4x^m + am^2x^2x^m + 12\,bx^4x^m + 10\,amx^2x^m + 24\,ax^2x^m}{m^3 + 12\,m^2 + 44\,m + 48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] (c*m^2*x^6*x^m + 6*c*m*x^6*x^m + b*m^2*x^4*x^m + 8*c*x^6*x^m + 8*b*m*x^4*x^
m + a*m^2*x^2*x^m + 12*b*x^4*x^m + 10*a*m*x^2*x^m + 24*a*x^2*x^m)/(m^3 + 12
*m^2 + 44*m + 48)
```

Mupad [**B**]
time = 2.08, size = 89, normalized size = 2.41

$$x^m\left(\frac{a\,x^2\,\left(m^2+10\,m+24\right)}{m^3+12\,m^2+44\,m+48}+\frac{b\,x^4\,\left(m^2+8\,m+12\right)}{m^3+12\,m^2+44\,m+48}+\frac{c\,x^6\,\left(m^2+6\,m+8\right)}{m^3+12\,m^2+44\,m+48}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^m*((a*x^2*(10*m + m^2 + 24))/(44*m + 12*m^2 + m^3 + 48) + (b*x^4*(8*m + m
^2 + 12))/(44*m + 12*m^2 + m^3 + 48) + (c*x^6*(6*m + m^2 + 8))/(44*m + 12*m
^2 + m^3 + 48))
```

3.66 $\int x^2(ax+bx^3+cx^5)\,dx$

Optimal. Leaf size=25

$$\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3+cx^5\right)\,dx&=\int\left(ax^3+bx^5+cx^7\right)\,dx\\&=\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^6}{6}+\frac{cx^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

Maple [A]
time = 0.04, size = 20, normalized size = 0.80

method	result	size
default	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
norman	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
risch	$\frac{1}{4}a\,x^{4}+\frac{1}{6}b\,x^{6}+\frac{1}{8}c\,x^{8}$	20
gosper	$\frac{x^{4}\left(3c\,x^{4}+4b\,x^{2}+6a\right)}{24}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+1/6*b*x^6+1/8*c*x^8
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.76

$$\frac{1}{8}\,cx^{8}+\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Fricas [A]
time = 0.34, size = 19, normalized size = 0.76

$$\frac{1}{8}\,cx^{8}+\frac{1}{6}\,bx^{6}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^{4}}{4}+\frac{bx^{6}}{6}+\frac{cx^{8}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6 + c*x**8/8
```

Giac [**A**]
time = 3.28, size = 19, normalized size = 0.76

$$\frac{1}{8}cx^8 + \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/8*c*x^8 + 1/6*b*x^6 + 1/4*a*x^4
```

Mupad [**B**]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^8}{8} + \frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6 + (c*x^8)/8
```

3.67 $\int x(ax+bx^3+cx^5)\,dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.062$, Rules used = $\{14\}$

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x(ax+bx^3+cx^5)\,dx &= \int (ax^2+bx^4+cx^6)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.80

method	result	size
default	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
norman	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
risch	$\frac{1}{3}a\,x^{3}+\frac{1}{5}b\,x^{5}+\frac{1}{7}c\,x^{7}$	20
gosper	$\frac{x^{3}\left(15c\,x^{4}+21b\,x^{2}+35a\right)}{105}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A]
time = 0.27, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A]
time = 0.35, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^{3}}{3}+\frac{bx^{5}}{5}+\frac{cx^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Giac [A]
time = 3.06, size = 19, normalized size = 0.76

$$\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^7}{7}+\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

3.68 $\int (ax + bx^3 + cx^5)\, dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3 + c*x^5,x]
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi steps

$$\int (ax + bx^3 + cx^5)\, dx = \frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3 + c*x^5,x]
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A]
time = 0.02, size = 20, normalized size = 0.80

method	result	size
default	$\frac{1}{2}a\,x^2 + \frac{1}{4}b\,x^4 + \frac{1}{6}c\,x^6$	20
norman	$\frac{1}{2}a\,x^2 + \frac{1}{4}b\,x^4 + \frac{1}{6}c\,x^6$	20

risch	$\frac{1}{2}a\,x^2+\frac{1}{4}b\,x^4+\frac{1}{6}c\,x^6$	20
gosper	$\frac{x^2\,(2c\,x^4+3b\,x^2+6a)}{12}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*x^5+b*x^3+a*x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A]
time = 0.33, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="fricas")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A]
time = 0.01, size = 19, normalized size = 0.76

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x**5+b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

Giac [A]
time = 3.66, size = 19, normalized size = 0.76

$$\frac{1}{6}\,cx^6+\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*x^5+b*x^3+a*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B]
time = 0.03, size = 19, normalized size = 0.76

$$\frac{c\,x^6}{6}+\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3 + c*x^5,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

3.69 $\int \frac{ax+bx^3+cx^5}{x} dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{ax + bx^3 + cx^5}{x} dx = \int \left(a + bx^2 + cx^4\right) dx$$
$$= ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Mathematica [A]
time = 0.00, size = 20, normalized size = 1.00

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.85

method	result	size
default	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
norman	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
risch	$ax+\frac{1}{3}b\,x^{3}+\frac{1}{5}c\,x^{5}$	17
gosper	$\frac{x\left(3c\,x^{4}+5b\,x^{2}+15a\right)}{15}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x,x,method=_RETURNVERBOSE)
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="maxima")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A]
time = 0.34, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^{5}+\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="fricas")
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Sympy [A]
time = 0.01, size = 15, normalized size = 0.75

$$ax+\frac{bx^{3}}{3}+\frac{cx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

Giac [A]
time = 3.39, size = 16, normalized size = 0.80

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.80

$$\frac{c\,x^5}{5} + \frac{b\,x^3}{3} + a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x,x)
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

3.70 $\int \frac{ax+bx^3+cx^5}{x^2}\,dx$

Optimal. Leaf size=21

$$\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)

Rubi [A]
time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3+cx^5}{x^2}\,dx &= \int \left(\frac{a}{x}+bx+cx^3\right)\,dx\\ &= \frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 21, normalized size = 1.00

$$\frac{bx^2}{2}+\frac{cx^4}{4}+a\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3 + c*x^5)/x^2,x]

[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]

Maple [A]
time = 0.03, size = 18, normalized size = 0.86

method	result	size
default	$\frac{b\,x^2}{2}+\frac{c\,x^4}{4}+a\ln\left(x\right)$	18
norman	$\frac{\frac{1}{2}b\,x^3+\frac{1}{4}c\,x^5}{x}+a\ln\left(x\right)$	23
risch	$\frac{c\,x^4}{4}+\frac{b\,x^2}{2}+\frac{b^2}{4c}+a\ln\left(x\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^2,x,method=_RETURNVERBOSE)
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A]
time = 0.26, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="maxima")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Fricas [A]
time = 0.32, size = 17, normalized size = 0.81

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="fricas")
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A]
time = 0.02, size = 17, normalized size = 0.81

$$a\log\left(x\right)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**2,x)
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

Giac [A]
time = 4.19, size = 20, normalized size = 0.95

$$\frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.81

$$\frac{b\,x^2}{2} + \frac{c\,x^4}{4} + a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x^2,x)
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*log(x)
```

3.71 $\int \frac{ax+bx^3+cx^5}{x^3} \, dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -a/x+b*x+1/3*c*x^3
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {14}

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)/x^3,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3+cx^5}{x^3} \, dx &= \int \left(b+\frac{a}{x^2}+cx^2\right) \, dx \\ &= -\frac{a}{x}+bx+\frac{cx^3}{3} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 18, normalized size = 1.00

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)/x^3,x]
```

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.94

method	result	size
default	$-\frac{a}{x}+bx+\frac{c x^{3}}{3}$	17
risch	$-\frac{a}{x}+bx+\frac{c x^{3}}{3}$	17
norman	$\frac{b x^{3}-ax+\frac{1}{3}c x^{5}}{x^{2}}$	21
gosper	$-\frac{-c x^{4}-3b x^{2}+3a}{3x}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A]
time = 0.27, size = 16, normalized size = 0.89

$$\frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="maxima")
[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A]
time = 0.35, size = 20, normalized size = 1.11

$$\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="fricas")
[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A]
time = 0.02, size = 12, normalized size = 0.67

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)/x**3,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

Giac [A]
time = 3.57, size = 16, normalized size = 0.89

$$\frac{1}{3}cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

Mupad [B]
time = 0.03, size = 16, normalized size = 0.89

$$b\,x - \frac{a}{x} + \frac{c\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)/x^3,x)
```

```
[Out] b*x - a/x + (c*x^3)/3
```

3.72 $\int x^m(ax+bx^3+cx^5)^2\,dx$

Optimal. Leaf size=76

$$\frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{(b^2+2ac)\,x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}$$

```
[Out] a^2*x^(3+m)/(3+m)+2*a*b*x^(5+m)/(5+m)+(2*a*c+b^2)*x^(7+m)/(7+m)+2*b*c*x^(9+
m)/(9+m)+c^2*x^(11+m)/(11+m)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {1599, 1122}

$$\frac{a^2x^{m+3}}{m+3}+\frac{x^{m+7}(2ac+b^2)}{m+7}+\frac{2abx^{m+5}}{m+5}+\frac{2bcx^{m+9}}{m+9}+\frac{c^2x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^(3 + m))/(3 + m) + (2*a*b*x^(5 + m))/(5 + m) + ((b^2 + 2*a*c)*x^(7 +
 m))/(7 + m) + (2*b*c*x^(9 + m))/(9 + m) + (c^2*x^(11 + m))/(11 + m)
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^m\left(ax+bx^3+cx^5\right)^2\,dx&=\int x^{2+m}\left(a+bx^2+cx^4\right)^2\,dx\\&=\int\left(a^2x^{2+m}+2abx^{4+m}+\left(b^2+2ac\right)x^{6+m}+2bcx^{8+m}+c^2x^{10+m}\right)dx\\&=\frac{a^2x^{3+m}}{3+m}+\frac{2abx^{5+m}}{5+m}+\frac{(b^2+2ac)\,x^{7+m}}{7+m}+\frac{2bcx^{9+m}}{9+m}+\frac{c^2x^{11+m}}{11+m}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.92

$$x^m\left(\frac{a^2x^3}{3+m}+\frac{2abx^5}{5+m}+\frac{(b^2+2ac)\,x^7}{7+m}+\frac{2bcx^9}{9+m}+\frac{c^2x^{11}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] x^m*((a^2*x^3)/(3 + m) + (2*a*b*x^5)/(5 + m) + ((b^2 + 2*a*c)*x^7)/(7 + m)
+ (2*b*c*x^9)/(9 + m) + (c^2*x^11)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(76) = 152.
time = 0.01, size = 300, normalized size = 3.95

method	result
gosper	$\frac{x^{3+m}\left(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56ac\,m^3x^4+\right.}{}$
risch	$\frac{x^m\left(c^2m^4x^8+24c^2m^3x^8+2bc\,m^4x^6+206c^2m^2x^8+52bc\,m^3x^6+744m\,x^8c^2+2ac\,m^4x^4+b^2m^4x^4+472bc\,m^2x^6+945c^2x^8+56ac\,m^3x^4+2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x^(3+m)*(c^2*m^4*x^8+24*c^2*m^3*x^8+2*b*c*m^4*x^6+206*c^2*m^2*x^8+52*b*c*m^
3*x^6+744*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+472*b*c*m^2*x^6+945*c^2*x^8+5
6*a*c*m^3*x^4+28*b^2*m^3*x^4+1772*b*c*m*x^6+2*a*b*m^4*x^2+548*a*c*m^2*x^4+2
74*b^2*m^2*x^4+2310*b*c*x^6+60*a*b*m^3*x^2+2184*a*c*m*x^4+1092*b^2*m*x^4+a^
2*m^4+640*a*b*m^2*x^2+2970*a*c*x^4+1485*b^2*x^4+32*a^2*m^3+2820*a*b*m*x^2+3
74*a^2*m^2+4158*a*b*x^2+1888*a^2*m+3465*a^2)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)
```

Maxima [A]
time = 0.26, size = 85, normalized size = 1.12

$$\frac{c^2x^{m+11}}{m+11}+\frac{2\,bcx^{m+9}}{m+9}+\frac{b^2x^{m+7}}{m+7}+\frac{2\,acx^{m+7}}{m+7}+\frac{2\,abx^{m+5}}{m+5}+\frac{a^2x^{m+3}}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] c^2*x^(m + 11)/(m + 11) + 2*b*c*x^(m + 9)/(m + 9) + b^2*x^(m + 7)/(m + 7) +
 2*a*c*x^(m + 7)/(m + 7) + 2*a*b*x^(m + 5)/(m + 5) + a^2*x^(m + 3)/(m + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(76) = 152.
time = 0.35, size = 241, normalized size = 3.17

$$\frac{((c^2m^4+24\,c^2m^3+206\,c^2m^2+744\,c^2m+945\,c^2)x^{11}+2\,(bcm^4+26\,bcm^3+236\,bcm^2+886\,bcm+1155\,bc)x^9+((b^2+2\,ac)m^4+28\,(b^2+2\,ac)m^3+274\,(b^2+2\,ac)m^2+1485\,b^2+2970\,ac+1092\,(b^2+2\,ac)m)x^7+2\,(abm^4+30\,abm^3+320\,abm^2+1410\,abm+2079\,ab)x^5+(a^2m^4+32\,a^2m^3+374\,a^2m^2+1888\,a^2m+3465\,a^2)x^3)x^m}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 24*c^2*m^3 + 206*c^2*m^2 + 744*c^2*m + 945*c^2)*x^11 + 2*(b*c*m
^4 + 26*b*c*m^3 + 236*b*c*m^2 + 886*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*
m^4 + 28*(b^2 + 2*a*c)*m^3 + 274*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c +
1092*(b^2 + 2*a*c)*m)*x^7 + 2*(a*b*m^4 + 30*a*b*m^3 + 320*a*b*m^2 + 1410*a*
b*m + 2079*a*b)*x^5 + (a^2*m^4 + 32*a^2*m^3 + 374*a^2*m^2 + 1888*a^2*m + 34
65*a^2)*x^3)*x^m/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. $2(66) = 132$.
time = 0.70, size = 1377, normalized size = 18.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Piecewise((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(4*x**4) - b
*c/x**2 + c**2*log(x), Eq(m, -11)), (-a**2/(6*x**6) - a*b/(2*x**4) - a*c/x*
*2 - b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2, Eq(m, -9)), (-a**2/(4*x**4
) - a*b/x**2 + 2*a*c*log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4, Eq(m, -
7)), (-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x**4/2 +
 c**2*x**6/6, Eq(m, -5)), (a**2*log(x) + a*b*x**2 + a*c*x**4/2 + b**2*x**4/
4 + b*c*x**6/3 + c**2*x**8/8, Eq(m, -3)), (a**2*m**4*x**3*x**m/(m**5 + 35*m
**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 32*a**2*m**3*x**3*x**m/(m**5
 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 374*a**2*m**2*x**3*x*
*m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1888*a**2*m*x
**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 3465*a*
*2*x**3*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2*a
*b*m**4*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
+ 60*a*b*m**3*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 640*a*b*m**2*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 912
9*m + 10395) + 2820*a*b*m*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
+ 9129*m + 10395) + 4158*a*b*x**5*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m*
*2 + 9129*m + 10395) + 2*a*c*m**4*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 30
10*m**2 + 9129*m + 10395) + 56*a*c*m**3*x**7*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 548*a*c*m**2*x**7*x**m/(m**5 + 35*m**4 +
470*m**3 + 3010*m**2 + 9129*m + 10395) + 2184*a*c*m*x**7*x**m/(m**5 + 35*m*
*4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 2970*a*c*x**7*x**m/(m**5 + 35
*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + b**2*m**4*x**7*x**m/(m**5
+ 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 28*b**2*m**3*x**7*x**m
/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 274*b**2*m**2*x
**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1092*b*
```

```
*2*m*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 1
485*b**2*x**7*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395)
 + 2*b*c*m**4*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 1
0395) + 52*b*c*m**3*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129
*m + 10395) + 472*b*c*m**2*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2
 + 9129*m + 10395) + 1772*b*c*m*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3010
*m**2 + 9129*m + 10395) + 2310*b*c*x**9*x**m/(m**5 + 35*m**4 + 470*m**3 + 3
010*m**2 + 9129*m + 10395) + c**2*m**4*x**11*x**m/(m**5 + 35*m**4 + 470*m**
3 + 3010*m**2 + 9129*m + 10395) + 24*c**2*m**3*x**11*x**m/(m**5 + 35*m**4 +
 470*m**3 + 3010*m**2 + 9129*m + 10395) + 206*c**2*m**2*x**11*x**m/(m**5 +
35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 744*c**2*m*x**11*x**m/(m
**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395) + 945*c**2*x**11*x**
m/(m**5 + 35*m**4 + 470*m**3 + 3010*m**2 + 9129*m + 10395), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(76) = 152$.
time = 3.85, size = 399, normalized size = 5.25

$$\frac{c^{2}m^{4}x^{11}x^{m} + 24\,c^{2}m^{3}x^{11}x^{m} + 2\,bcm^{4}x^{9}x^{m} + 206\,c^{2}m^{2}x^{11}x^{m} + 52\,bcm^{3}x^{9}x^{m} + 744\,c^{2}mx^{11}x^{m} + b^{2}m^{4}x^{7}x^{m} + 2\,acm^{4}x^{7}x^{m} + 472\,bcm^{2}x^{9}x^{m} + 945\,c^{2}x^{11}x^{m} + 28\,b^{2}m^{3}x^{7}x^{m} + 56\,acm^{3}x^{7}x^{m} + 1772\,bcmx^{9}x^{m} + 2\,abm^{4}x^{5}x^{m} + 274\,b^{2}m^{2}x^{7}x^{m} + 548\,acm^{2}x^{7}x^{m} + 2310\,bcx^{9}x^{m} + 60\,abm^{3}x^{5}x^{m} + 1092\,b^{2}mx^{7}x^{m} + 2184\,acmx^{7}x^{m} + a^{2}m^{4}x^{3}x^{m} + 640\,abm^{2}x^{5}x^{m} + 1485\,b^{2}x^{7}x^{m} + 2970\,acx^{7}x^{m} + 32\,a^{2}m^{3}x^{3}x^{m} + 2820\,abmx^{5}x^{m} + 374\,a^{2}m^{2}x^{3}x^{m} + 4158\,abx^{5}x^{m} + 1888\,a^{2}mx^{3}x^{m} + 3465\,a^{2}x^{3}x^{m}}{m^{5} + 35\,m^{4} + 470\,m^{3} + 3010\,m^{2} + 9129\,m + 10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^11*x^m + 24*c^2*m^3*x^11*x^m + 2*b*c*m^4*x^9*x^m + 206*c^2*m^2*x
^11*x^m + 52*b*c*m^3*x^9*x^m + 744*c^2*m*x^11*x^m + b^2*m^4*x^7*x^m + 2*a*c
*m^4*x^7*x^m + 472*b*c*m^2*x^9*x^m + 945*c^2*x^11*x^m + 28*b^2*m^3*x^7*x^m
+ 56*a*c*m^3*x^7*x^m + 1772*b*c*m*x^9*x^m + 2*a*b*m^4*x^5*x^m + 274*b^2*m^2
*x^7*x^m + 548*a*c*m^2*x^7*x^m + 2310*b*c*x^9*x^m + 60*a*b*m^3*x^5*x^m + 10
92*b^2*m*x^7*x^m + 2184*a*c*m*x^7*x^m + a^2*m^4*x^3*x^m + 640*a*b*m^2*x^5*x
^m + 1485*b^2*x^7*x^m + 2970*a*c*x^7*x^m + 32*a^2*m^3*x^3*x^m + 2820*a*b*m*
x^5*x^m + 374*a^2*m^2*x^3*x^m + 4158*a*b*x^5*x^m + 1888*a^2*m*x^3*x^m + 346
5*a^2*x^3*x^m)/(m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9129*m + 10395)
```

Mupad [**B**]
time = 2.19, size = 271, normalized size = 3.57

$$\frac{a^2\,x^m\,x^3\,\left(m^4+32\,m^3+374\,m^2+1888\,m+3465\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{c^2\,x^m\,x^{11}\,\left(m^4+24\,m^3+206\,m^2+744\,m+945\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{x^m\,x^7\,\left(b^2+2\,a\,c\right)\,\left(m^4+28\,m^3+274\,m^2+1092\,m+1485\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{2\,a\,b\,x^m\,x^5\,\left(m^4+30\,m^3+320\,m^2+1410\,m+2079\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}+\frac{2\,b\,c\,x^m\,x^9\,\left(m^4+26\,m^3+236\,m^2+886\,m+1155\right)}{m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (a^2*x^m*x^3*(1888*m + 374*m^2 + 32*m^3 + m^4 + 3465))/(9129*m + 3010*m^2 +
 470*m^3 + 35*m^4 + m^5 + 10395) + (c^2*x^m*x^11*(744*m + 206*m^2 + 24*m^3
+ m^4 + 945))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (x^m*x
^7*(2*a*c + b^2)*(1092*m + 274*m^2 + 28*m^3 + m^4 + 1485))/(9129*m + 3010*m
^2 + 470*m^3 + 35*m^4 + m^5 + 10395) + (2*a*b*x^m*x^5*(1410*m + 320*m^2 + 3
```

```
0*m^3 + m^4 + 2079))/(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395) +
 (2*b*c*x^m*x^9*(886*m + 236*m^2 + 26*m^3 + m^4 + 1155))/(9129*m + 3010*m^2
 + 470*m^3 + 35*m^4 + m^5 + 10395)
```

3.73 $\int x^2(ax+bx^3+cx^5)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1599, 1122}

$$\frac{a^2x^5}{5}+\frac{1}{9}x^9\left(2ac+b^2\right)+\frac{2}{7}abx^7+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax+bx^3+cx^5\right)^2\,dx &= \int x^4\left(a+bx^2+cx^4\right)^2\,dx\\ &= \int \left(a^2x^4+2abx^6+\left(b^2+2ac\right)x^8+2bcx^{10}+c^2x^{12}\right)dx\\ &= \frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{1}{9}\left(b^2+2ac\right)x^9+\frac{2}{11}bcx^{11}+\frac{c^2x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + ((b^2 + 2*a*c)*x^9)/9 + (2*b*c*x^11)/11 + (c^
2*x^13)/13
```

Maple [A]
time = 0.07, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^5}{5}+\frac{2ab\,x^7}{7}+\frac{(2ac+b^2)x^9}{9}+\frac{2bc\,x^{11}}{11}+\frac{c^2x^{13}}{13}$	45
norman	$\frac{c^2x^{13}}{13}+\frac{2bc\,x^{11}}{11}+\left(\frac{2ac}{9}+\frac{b^2}{9}\right)x^9+\frac{2ab\,x^7}{7}+\frac{a^2x^5}{5}$	46
risch	$\frac{1}{5}a^2x^5+\frac{2}{7}ab\,x^7+\frac{2}{9}x^9ac+\frac{1}{9}b^2x^9+\frac{2}{11}bc\,x^{11}+\frac{1}{13}c^2x^{13}$	47
gosper	$\frac{x^5\left(3465c^2x^8+8190bc\,x^6+10010ac\,x^4+5005b^2x^4+12870ab\,x^2+9009a^2\right)}{45045}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*(2*a*c+b^2)*x^9+2/11*b*c*x^11+1/13*c^2*x^13
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.81

$$\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,\left(b^2+2\,ac\right)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

Fricas [A]
time = 0.32, size = 44, normalized size = 0.81

$$\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,\left(b^2+2\,ac\right)x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*(b^2 + 2*a*c)*x^9 + 2/7*a*b*x^7 + 1/5*a
^2*x^5
```

Sympy [A]
time = 0.01, size = 51, normalized size = 0.94

$$\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{2bcx^{11}}{11}+\frac{c^2x^{13}}{13}+x^9\cdot\left(\frac{2ac}{9}+\frac{b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + 2*b*c*x**11/11 + c**2*x**13/13 + x**9*(2*a*c/9
 + b**2/9)
```

Giac [A]
time = 3.11, size = 46, normalized size = 0.85

$$\frac{1}{13}\,c^2x^{13}+\frac{2}{11}\,bcx^{11}+\frac{1}{9}\,b^2x^9+\frac{2}{9}\,acx^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/13*c^2*x^13 + 2/11*b*c*x^11 + 1/9*b^2*x^9 + 2/9*a*c*x^9 + 2/7*a*b*x^7 + 1
/5*a^2*x^5
```

Mupad [B]
time = 0.03, size = 45, normalized size = 0.83

$$x^9\left(\frac{b^2}{9}+\frac{2\,a\,c}{9}\right)+\frac{a^2\,x^5}{5}+\frac{c^2\,x^{13}}{13}+\frac{2\,a\,b\,x^7}{7}+\frac{2\,b\,c\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^9*((2*a*c)/9 + b^2/9) + (a^2*x^5)/5 + (c^2*x^13)/13 + (2*a*b*x^7)/7 + (2*
b*c*x^11)/11
```

3.74 $\int x(ax+bx^3+cx^5)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}\left(b^2+2ac\right)x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1599, 1128, 645}

$$\frac{a^2x^4}{4}+\frac{1}{8}x^8\left(2ac+b^2\right)+\frac{1}{3}abx^6+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + ((b^2 + 2*a*c)*x^8)/8 + (b*c*x^10)/5 + (c^2*x^1
2)/12
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int x(ax+bx^3+cx^5)^2\,dx&=\int x^3(a+bx^2+cx^4)^2\,dx\\&=\frac{1}{2}\text{Subst}\left(\int x(a+bx+cx^2)^2\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx^2+(b^2+2ac)\,x^3+2bcx^4+c^2x^5\right)\,dx,x,x^2\right)\\&=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{1}{8}(b^2+2ac)\,x^8+\frac{1}{5}bcx^{10}+\frac{c^2x^{12}}{12}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{120}x^4\left(30a^2+40abx^2+15(b^2+2ac)\,x^4+24bcx^6+10c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^4*(30*a^2 + 40*a*b*x^2 + 15*(b^2 + 2*a*c)*x^4 + 24*b*c*x^6 + 10*c^2*x^8)
)/120
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^4}{4}+\frac{ab\,x^6}{3}+\frac{(2ac+b^2)x^8}{8}+\frac{bc\,x^{10}}{5}+\frac{c^2x^{12}}{12}$	45
norman	$\frac{c^2x^{12}}{12}+\frac{bc\,x^{10}}{5}+\left(\frac{ac}{4}+\frac{b^2}{8}\right)x^8+\frac{ab\,x^6}{3}+\frac{a^2x^4}{4}$	46
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{4}x^8ac+\frac{1}{8}b^2x^8+\frac{1}{5}bc\,x^{10}+\frac{1}{12}c^2x^{12}$	47
gosper	$\frac{x^4(10c^2x^8+24bc\,x^6+30ac\,x^4+15b^2x^4+40ab\,x^2+30a^2)}{120}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*(2*a*c+b^2)*x^8+1/5*b*c*x^10+1/12*c^2*x^12
```

Maxima [A]
time = 0.26, size = 44, normalized size = 0.81

$$\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,(b^2+2\,ac)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

Fricas [A]
time = 0.35, size = 44, normalized size = 0.81

$$\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,\left(b^2+2\,ac\right)x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*(b^2 + 2*a*c)*x^8 + 1/3*a*b*x^6 + 1/4*a^
2*x^4
```

Sympy [A]
time = 0.01, size = 46, normalized size = 0.85

$$\frac{a^2x^4}{4}+\frac{abx^6}{3}+\frac{bcx^{10}}{5}+\frac{c^2x^{12}}{12}+x^8\left(\frac{ac}{4}+\frac{b^2}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b*c*x**10/5 + c**2*x**12/12 + x**8*(a*c/4 + b**2
/8)
```

Giac [A]
time = 3.54, size = 46, normalized size = 0.85

$$\frac{1}{12}\,c^2x^{12}+\frac{1}{5}\,bcx^{10}+\frac{1}{8}\,b^2x^8+\frac{1}{4}\,acx^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/12*c^2*x^12 + 1/5*b*c*x^10 + 1/8*b^2*x^8 + 1/4*a*c*x^8 + 1/3*a*b*x^6 + 1/
4*a^2*x^4
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^8\left(\frac{b^2}{8}+\frac{a\,c}{4}\right)+\frac{a^2\,x^4}{4}+\frac{c^2\,x^{12}}{12}+\frac{a\,b\,x^6}{3}+\frac{b\,c\,x^{10}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^8*((a*c)/4 + b^2/8) + (a^2*x^4)/4 + (c^2*x^12)/12 + (a*b*x^6)/3 + (b*c*x^
10)/5
```

3.75 $\int (ax+bx^3+cx^5)^2\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1608, 1122}

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\begin{aligned}
\int \left(ax+bx^3+cx^5\right)^2\,dx &= \int x^2\left(a+bx^2+cx^4\right)^2\,dx\\
&= \int \left(a^2x^2+2abx^4+\left(b^2+2ac\right)x^6+2bcx^8+c^2x^{10}\right)\,dx\\
&= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 54, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)/9 + (c^2*
x^11)/11
```

Maple [A]
time = 0.03, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{(2ac+b^2)x^7}{7}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}$	45
norman	$\frac{c^2x^{11}}{11}+\frac{2bcx^9}{9}+\left(\frac{2ac}{7}+\frac{b^2}{7}\right)x^7+\frac{2abx^5}{5}+\frac{a^2x^3}{3}$	46
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}abx^5+\frac{2}{7}x^7ac+\frac{1}{7}b^2x^7+\frac{2}{9}bcx^9+\frac{1}{11}c^2x^{11}$	47
gosper	$\frac{x^3(315c^2x^8+770bcx^6+990acx^4+495b^2x^4+1386abx^2+1155a^2)}{3465}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*x^11
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.89

$$\frac{1}{11}c^2x^{11}+\frac{2}{9}bcx^9+\frac{1}{7}b^2x^7+\frac{1}{3}a^2x^3+\frac{2}{35}\left(5cx^7+7bx^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 1/3*a^2*x^3 + 2/35*(5*c*x^7 + 7
*b*x^5)*a
```

Fricas [A]
time = 0.33, size = 44, normalized size = 0.81

$$\frac{1}{11}c^2x^{11}+\frac{2}{9}bcx^9+\frac{1}{7}\left(b^2+2ac\right)x^7+\frac{2}{5}abx^5+\frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5 + 1/3*a^2
*x^3
```

Sympy [A]
time = 0.01, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7 \cdot \left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*(2*a*c/7 +
 b**2/7)
```

Giac [A]
time = 3.44, size = 46, normalized size = 0.85

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b*x^5 + 1/3
*a^2*x^3
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^7\,\left(\frac{b^2}{7}+\frac{2\,a\,c}{7}\right)+\frac{a^2\,x^3}{3}+\frac{c^2\,x^{11}}{11}+\frac{2\,a\,b\,x^5}{5}+\frac{2\,b\,c\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] x^7*((2*a*c)/7 + b^2/7) + (a^2*x^3)/3 + (c^2*x^11)/11 + (2*a*b*x^5)/5 + (2*
b*c*x^9)/9
```

3.76 $\int \frac{(ax+bx^3+cx^5)^2}{x}\,dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}\left(b^2+2ac\right)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1599, 1121, 625}

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 + (c^2*x^10
)/10
```

Rule 625

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{(ax+bx^3+cx^5)^2}{x} \, dx = \int x(a+bx^2+cx^4)^2 \, dx$$
$$= \frac{1}{2}\text{Subst}\left(\int (a+bx+cx^2)^2 \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(a^2+2abx+b^2\left(1+\frac{2ac}{b^2}\right)x^2+2bcx^3+c^2x^4\right) dx, x, x^2\right)$$
$$= \frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{1}{6}(b^2+2ac)x^6+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2(30a^2+30abx^2+10(b^2+2ac)x^4+15bcx^6+6c^2x^8)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6*c^2*x^8))
/60
```

Maple [A]
time = 0.05, size = 45, normalized size = 0.83

method	result	size
default	$\frac{a^2x^2}{2}+\frac{ab\,x^4}{2}+\frac{(2ac+b^2)x^6}{6}+\frac{bc\,x^8}{4}+\frac{c^2x^{10}}{10}$	45
norman	$\frac{c^2x^{10}}{10}+\frac{bc\,x^8}{4}+\left(\frac{ac}{3}+\frac{b^2}{6}\right)x^6+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}$	46
risch	$\frac{1}{2}a^2x^2+\frac{1}{2}ab\,x^4+\frac{1}{3}x^6ac+\frac{1}{6}b^2x^6+\frac{1}{4}bc\,x^8+\frac{1}{10}c^2x^{10}$	47
gosper	$\frac{x^2(6c^2x^8+15bc\,x^6+20ac\,x^4+10b^2x^4+30ab\,x^2+30a^2)}{60}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*x^10
```

Maxima [A]
time = 0.28, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Fricas [A]
time = 0.35, size = 44, normalized size = 0.81

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,(b^2+2\,ac)x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4 + 1/2*a^2
*x^2
```

Sympy [A]
time = 0.01, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c/3 + b**2/
6)
```

Giac [A]
time = 3.56, size = 46, normalized size = 0.85

$$\frac{1}{10}\,c^2x^{10}+\frac{1}{4}\,bcx^8+\frac{1}{6}\,b^2x^6+\frac{1}{3}\,acx^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b*x^4 + 1/2
*a^2*x^2
```

Mupad [B]
time = 0.02, size = 45, normalized size = 0.83

$$x^6\left(\frac{b^2}{6}+\frac{a\,c}{3}\right)+\frac{a^2\,x^2}{2}+\frac{c^2\,x^{10}}{10}+\frac{a\,b\,x^4}{2}+\frac{b\,c\,x^8}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2/x,x)
```

```
[Out] x^6*((a*c)/3 + b^2/6) + (a^2*x^2)/2 + (c^2*x^10)/10 + (a*b*x^4)/2 + (b*c*x^
8)/4
```

3.77 $\int \frac{(ax+bx^3+cx^5)^2}{x^2} \, dx$

Optimal. Leaf size=49

$$a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1599, 1104}

$$a^2x + \frac{1}{5}x^5\left(2ac + b^2\right) + \frac{2}{3}abx^3 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rule 1104

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax + bx^3 + cx^5\right)^2}{x^2} \, dx &= \int \left(a + bx^2 + cx^4\right)^2 \, dx \\
&= \int \left(a^2 + 2abx^2 + b^2\left(1 + \frac{2ac}{b^2}\right)x^4 + 2bcx^6 + c^2x^8\right) dx \\
&= a^2x + \frac{2}{3}abx^3 + \frac{1}{5}\left(b^2 + 2ac\right)x^5 + \frac{2}{7}bcx^7 + \frac{c^2x^9}{9}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 49, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{1}{5}\left(b^2+2ac\right)x^5+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^2/x^2,x]
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A]
time = 0.01, size = 42, normalized size = 0.86

method	result	size
default	$a^2x+\frac{2ab\,x^3}{3}+\frac{(2ac+b^2)x^5}{5}+\frac{2bc\,x^7}{7}+\frac{c^2x^9}{9}$	42
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{2}{5}ac\,x^5+\frac{1}{5}b^2x^5+\frac{2}{7}bc\,x^7+\frac{1}{9}c^2x^9$	44
gosper	$\frac{x(35c^2x^8+90bc\,x^6+126ac\,x^4+63b^2x^4+210ab\,x^2+315a^2)}{315}$	47
norman	$\frac{a^2x^2+\left(\frac{2ac}{5}+\frac{b^2}{5}\right)x^6+\frac{c^2x^{10}}{9}+\frac{2ab\,x^4}{3}+\frac{2bc\,x^8}{7}}{x}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A]
time = 0.28, size = 41, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="maxima")
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A]
time = 0.32, size = 41, normalized size = 0.84

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,\left(b^2+2\,ac\right)x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*(b^2 + 2*a*c)*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A]
time = 0.02, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5 \cdot \left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/5
)
```

Giac [A]
time = 3.96, size = 43, normalized size = 0.88

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B]
time = 0.02, size = 42, normalized size = 0.86

$$a^2\,x + x^5\,\left(\frac{b^2}{5} + \frac{2\,a\,c}{5}\right) + \frac{c^2\,x^9}{9} + \frac{2\,a\,b\,x^3}{3} + \frac{2\,b\,c\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^2/x^2,x)
```

```
[Out] a^2*x + x^5*((2*a*c)/5 + b^2/5) + (c^2*x^9)/9 + (2*a*b*x^3)/3 + (2*b*c*x^7)
/7
```

3.78 $\int \frac{x^8}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=100

$$-\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/4*(-a*c+b^2)*ln(c*x^4+b*x^2+a)/c^3+1/2*b*(-3*a*c
+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 1128, 715, 648, 632, 212, 642}

$$\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -1/2*(b*x^2)/c^2 + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[
b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 715

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a,
 b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
 NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{ax+bx^3+cx^5}\,dx &= \int \frac{x^7}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{b}{c^2}+\frac{x}{c}+\frac{ab+(b^2-ac)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{\text{Subst}\left(\int \frac{ab+(b^2-ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}-\frac{(b(b^2-3ac))\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{(b^2-ac)\,\text{Subst}(\int \frac{b+2cx}{a+bx+cx^2}}{4c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{(b(b^2-3ac))\,\text{Subst}(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b}{2c^3}\\
&= -\frac{bx^2}{2c^2}+\frac{x^4}{4c}+\frac{b(b^2-3ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 0.93

$$\frac{cx^2(-2b+cx^2)-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2-ac)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A]
time = 0.05, size = 105, normalized size = 1.05

method	result
default	$-\frac{-\frac{1}{2}c\,x^4+b\,x^2}{2c^2}+\frac{\frac{\left(-ac+b^2\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ab-\frac{\left(-ac+b^2\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$
risch	$\frac{x^4}{4c}-\frac{b\,x^2}{2c^2}+\frac{b^2}{4c^3}-\frac{\ln\left(\left(\left(12a^2b\,c^2-7a\,b^3c+b^5+\sqrt{-b^2\left(4ac-b^2\right)\left(3ac-b^2\right)^2}\;b\right)x^2+2\sqrt{-b^2\left(4ac-b^2\right)\left(3ac\right.}\right.\right.}{c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/c^2*(-1/2*c*x^4+b*x^2)+1/2/c^2*(1/2*(-a*c+b^2)/c*ln(c*x^4+b*x^2+a)+2*(
a*b-1/2*(-a*c+b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 - integrate(-((b^2 - a*c)*x^3 + a*b*x)/(c*x^4 + b
*x^2 + a), x)/c^2
```

Fricas [A]
time = 0.35, size = 313, normalized size = 3.13

$$\left[\frac{(b^2c^2-4\,ac^3)x^4-2\,(b^3c-4\,abc^2)x^2-(b^3-3\,abc)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)}, \frac{(b^2c^2-4\,ac^3)x^4-2\,(b^3c-4\,abc^2)x^2+2\,(b^3-3\,abc)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^4-5\,ab^2c+4\,a^2c^2)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2*(b^3*c - 4*a*b*c^2)*x^2 - (b^3 - 3*a*b*c)
*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)
*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*x^4 - 2
*(b^3*c - 4*a*b*c^2)*x^2 + 2*(b^3 - 3*a*b*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^4 - 5*a*b^2*c + 4*a^2*c^2
)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(92) = 184$.
time = 1.98, size = 391, normalized size = 3.91

$$- \frac{b x^{2}}{2 c^{2}} + \left(- \frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right) \log{\left(x^{2} + \frac{2 a^{2} c - a b^{2} + 8 a c^{3} \left(- \frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right) - 2 b^{2} c^{2} \left(- \frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right)}{3 a b c - b^{3}} \right)} + \left(\frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right) \log{\left(x^{2} + \frac{2 a^{2} c - a b^{2} + 8 a c^{3} \left(\frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right) - 2 b^{2} c^{2} \left(\frac{b \sqrt{- 4 a c + b^{2}} \cdot \left(3 a c - b^{2}\right)}{4 c^{3} \cdot \left(4 a c - b^{2}\right)} - \frac{a c - b^{2}}{4 c^{3}}\right)}{3 a b c - b^{3}} \right)} + \frac{x^{4}}{4 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c -
b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(-
b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)
/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*
a*c - b**2)) - (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c +
 b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(
x**2 + (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/
(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(b*sqrt(-4*a
*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)))
/(3*a*b*c - b**3)) + x**4/(4*c)
```

Giac [A]
time = 3.69, size = 92, normalized size = 0.92

$$\frac{c x^{4} - 2 \, b x^{2}}{4 \, c^{2}} + \frac{{\left(b^{2} - a c\right)} \log\left(c x^{4} + b x^{2} + a\right)}{4 \, c^{3}} - \frac{{\left(b^{3} - 3 \, a b c\right)} \arctan\left(\frac{2 \, c x^{2} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{2 \, \sqrt{-b^{2} + 4 \, a c} c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*log(c*x^4 + b*x^2 + a)/c^3 - 1/
2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*c^3)
```

Mupad [B]
time = 2.20, size = 842, normalized size = 8.42

$$\frac{x^4}{4c}-\frac{\ln\left(cx^4+bx^2+a\right)\left(8a^2c^2-10ab^2c+2b^4\right)}{2\left(16ac^4-4b^2c^3\right)}-\frac{bx^2}{2c^2}+\frac{b\,\mathrm{atan}\left(\frac{2c^4\left(4ac-b^2\right)\left(\frac{\frac{b\left(3ac-b^2\right)\left(\frac{8a^2c^4-8ab^2c^3}{c^4}-\frac{8ac^2\left(8a^2c^2-10ab^2c+2b^4\right)}{16ac^4-4b^2c^3}\right)}{8c^3\sqrt{4ac-b^2}}-\frac{ab\left(3ac-b^2\right)\left(8a^2c^2-10ab^2c+2b^4\right)}{c\sqrt{4ac-b^2}\left(16ac^4-4b^2c^3\right)}}{a}-x^2\left(\frac{\frac{b\left(\frac{6b^3c^3-10abc^4}{c^4}+\frac{4bc^2\left(8a^2c^2-10ab^2c+2b^4\right)}{16ac^4-4b^2c^3}\right)\left(3ac-b^2\right)}{8c^3\sqrt{4ac-b^2}}}{a}+\frac{b^2\left(3ac-b^2\right)\left(8a^2c^2-10ab^2c+2b^4\right)}{2c\sqrt{4ac-b^2}\left(16ac^4-4b^2c^3\right)}+\frac{b\left(\frac{2a^2bc^2-3ab^3c+b^5}{c^4}+\frac{\left(\frac{6b^3c^3-10abc^4}{c^4}+\frac{4bc^2\left(8a^2c^2-10ab^2c+2b^4\right)}{16ac^4-4b^2c^3}\right)\left(8a^2c^2-10ab^2c+2b^4\right)}{2\left(16ac^4-4b^2c^3\right)}-\frac{b^3\left(3ac-b^2\right)^2}{2c^4\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)+\frac{b\left(\frac{\left(\frac{8a^2c^4-8ab^2c^3}{c^4}-\frac{8ac^2\left(8a^2c^2-10ab^2c+2b^4\right)}{16ac^4-4b^2c^3}\right)\left(8a^2c^2-10ab^2c+2b^4\right)}{2\left(16ac^4-4b^2c^3\right)}-\frac{a^3c^2-2a^2b^2c+ab^4}{c^4}+\frac{ab^2\left(3ac-b^2\right)^2}{c^4\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)}{9a^2b^2c^2-6ab^4c+b^6}\right)\left(3ac-b^2\right)}{2c^3\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^4/(4*c) - (log(a + b*x^2 + c*x^4)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(1
6*a*c^4 - 4*b^2*c^3)) - (b*x^2)/(2*c^2) + (b*atan((2*c^4*(4*a*c - b^2)*(((b
*(3*a*c - b^2)*((8*a^2*c^4 - 8*a*b^2*c^3)/c^4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2
 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3)))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*b
*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(c*(4*a*c - b^2)^(1/2)*(16
*a*c^4 - 4*b^2*c^3)))/a - x^2*(((b*((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2
*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(3*a*c - b^2))/(
8*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(3*a*c - b^2)*(2*b^4 + 8*a^2*c^2 - 10*a*b
^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((b^5 + 2*a
^2*b*c^2 - 3*a*b^3*c)/c^4 + (((6*b^3*c^3 - 10*a*b*c^4)/c^4 + (4*b*c^2*(2*b^
4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4 + 8*a^2*c^2 - 1
0*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b^3*(3*a*c - b^2)^2)/(2*c^4*(4*a*
c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*a^2*c^4 - 8*a*b^2*c^3)/c^
4 - (8*a*c^2*(2*b^4 + 8*a^2*c^2 - 10*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*b
^4 + 8*a^2*c^2 - 10*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*b^4 + a^3*c^2
 - 2*a^2*b^2*c)/c^4 + (a*b^2*(3*a*c - b^2)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4
*a*c - b^2)^(1/2))))/(b^6 + 9*a^2*b^2*c^2 - 6*a*b^4*c))*(3*a*c - b^2))/(2*c
^3*(4*a*c - b^2)^(1/2))
```

3.79 $\int \frac{x^7}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=203

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b+}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -b*x/c^2+1/3*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2
))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))
*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1136, 1293, 1180, 211}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*
c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
```

```
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{ax+bx^3+cx^5}\,dx &= \int \frac{x^6}{a+bx^2+cx^4}\,dx \\
&= \frac{x^3}{3c}-\frac{\int \frac{x^2\left(3a+3bx^2\right)}{a+bx^2+cx^4}\,dx}{3c} \\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\int \frac{3ab+3\left(b^2-ac\right)x^2}{a+bx^2+cx^4}\,dx}{3c^2} \\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c^2}+\frac{\left(b^2-ac+\frac{b\left(b\right.}{\sqrt{b^2}}\right.}{} \\
&= -\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2-ac+\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 250, normalized size = 1.23

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\left(-b^3+3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^3-3abc+b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/
(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 3
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 217, normalized size = 1.07

method	result
risch	$\frac{x^3}{3c}-\frac{bx}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\left(-ac+b^2\right)\_R^2+ab\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c^2}$
default	$-\frac{-\frac{1}{3}cx^3+bx}{c^2}+\frac{-\frac{\left(-ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+3abc-b^3\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\ c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{(}{}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(-1/3*c*x^3+b*x)+4/c*(-1/8*(-a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)
^(1/2)+3*a*b*c-b^3)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a*c*(-
4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)-3*a*b*c+b^3)/(-4*a*c+b^2)^(1/2)/c*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

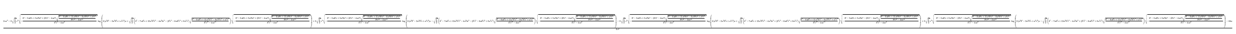
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x^4 + b*x^2
 + a), x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(167) = 334$.
time = 0.41, size = 1564, normalized size = 7.70



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b
^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c
^3 - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4
*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^
3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*
c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*
c^2)*x - sqrt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c
^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3
*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(
1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sq
rt(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^
2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b
^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sq
rt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(b^
7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6*a*b^2*c^6 + 8*a
^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*
```

```
a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b
^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A]
time = 12.60, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \mathrm{RootSum}\left(t^4\cdot\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \frac{-64t^3a^2c^7 + 48t^3ab^2c^6 - 8t^3b^4c^5 + 14ta^3bc^3 - 28ta^2b^3c^2 + 14tab^5c - 2tb^7}{a^4c^2 - 3a^3b^2c + a^2b^4}\right)\right)\right) + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b**4*c**5)
+ _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**7) + a**
5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7 + 48*_t**3*a*b**2*c**6 - 8*_t
**3*b**4*c**5 + 14*_t*a**3*b*c**3 - 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c -
 2*_t*b**7)/(a**4*c**2 - 3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2457 vs. $2(167) = 334$.
time = 5.13, size = 2457, normalized size = 12.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 -
(2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 -
4*a*c)*a^2*c^4)*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2
```

```
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*
c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 -
2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(2*sqr
t(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b
^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^
2) - 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c
^5 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b
^2 - 4*a*c)*a^2*c^4)*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2
*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c
^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(c))*arctan(
2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*
a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^
7)*c^2) + 1/3*(c^2*x^3 - 3*b*c*x)/c^3
```

Mupad [B]
time = 2.72, size = 2500, normalized size = 12.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x^3/(3*c) - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3
*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7
 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(
8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*
a^2*b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c -
3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2)*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b
*c^6)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2
 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b
^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*
b^2*c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*1i)/((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6
)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a
^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^
(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*
c^2 - 6*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
 + (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^
7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7 + b^4*(-(4*a*c
 - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 - 6
*a*b^4*c))/c^3)*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3 + 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(
a^4*c - a^3*b^2))/c^3))*(-(b^7 + b^4*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^
3 + 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c - 3*a*b^2
*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2
)*2i - atan(((((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*
c^6)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 +
```

```
 a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a
*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c
 - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2
*c^2 - 6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3
 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)
*1i - (((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((
b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^
2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2)
)/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*((b^4*(-(4*a*c - b^
2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)
^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7
+ b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6 - 2*a^3*c^3 + 9*a^2*b^2*c^2 -
6*a*b^4*c))/c^3)*((b^4*(-(4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a
^2*b^3*c^2 + a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((
((4*a*b^3*c^3 - 16*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*((b^4*(-(
4*a*c - b^2)^3)^(1/2) - b^7 + 20*a^3*b*c^3 - 25*a^2*b^3*c^2 + a^2*c^2*(-(4*
a*c - b^2)^3)^(1/2) + 9*a*b^5*c - 3*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3...
```

3.80 $\int \frac{x^6}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=81

$$\frac{x^2}{2c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1599, 1128, 717, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2
*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax+bx^3+cx^5}\,dx &= \int \frac{x^5}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{x^2}{2c}+\frac{\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{x^2}{2c}-\frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}\\
&= \frac{x^2}{2c}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c^2}\\
&= \frac{x^2}{2c}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 78, normalized size = 0.96

$$\frac{2cx^2+\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-b\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[
-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.03, size = 83, normalized size = 1.02

method	result
default	$\frac{x^2}{2c}+\frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-a+\frac{b^2}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c}$
risch	$\frac{x^2}{2c}-\frac{\ln\left(\left(\left(-8a^2c^2+6a\,b^2c-b^4+\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,b\right)x^2+2\sqrt{-\left(4ac-b^2\right)\left(2ac-b^2\right)^2}\,a\right)ab\right.}{c(4ac-b^2)}+\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c+1/2/c*(-1/2*b/c*ln(c*x^4+b*x^2+a)+2*(-a+1/2/c*b^2)/(4*a*c-b^2)^(1
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - integrate((b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A]
time = 0.36, size = 254, normalized size = 3.14

$$\left[\frac{2\,(b^2c-4\,ac^2)x^2-(b^2-2\,ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)},\ \frac{2\,(b^2c-4\,ac^2)x^2-2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^2-4\,ac^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*x^2 - (b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b
*x^2 + a)) - (b^3 - 4*a*b*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1
/4*(2*(b^2*c - 4*a*c^2)*x^2 - 2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(71) = 142$.
time = 1.31, size = 316, normalized size = 3.90

$$\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2+\frac{-ab-8ac^2\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)+2b^2c\left(-\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}\cdot(2ac-b^2)}{4c^2\cdot(4ac-b^2)}\right)}{2ac-b^2}\right)+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*
log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4*c**2) - sqrt(-4*a*c + b**2)*(
2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + (-b/(4*c**2) + sq
rt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b
- 8*a*c**2*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
 - b**2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*
c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

Giac [A]
time = 4.30, size = 75, normalized size = 0.93

$$\frac{x^2}{2\,c}-\frac{b\log\left(cx^4+bx^2+a\right)}{4\,c^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*arctan((2*
c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

Mupad [B]
time = 2.44, size = 655, normalized size = 8.09

$$\frac{x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,b^3-8\,a\,b\,c\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(\frac{\frac{\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)\,\left(2\,a\,c-b^2\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}+\frac{a\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}-x^2\,\left(\frac{\frac{\left(2\,a\,c-b^2\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{8\,c^2\,\sqrt{4\,a\,c-b^2}}-\frac{b\,\left(2\,b^3-8\,a\,b\,c\right)\,\left(2\,a\,c-b^2\right)}{2\,\sqrt{4\,a\,c-b^2}\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}}{a}+\frac{b\,\left(\frac{\left(2\,b^3-8\,a\,b\,c\right)\,\left(\frac{4\,a\,c^3-6\,b^2\,c^2}{c^2}-\frac{4\,b\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{b^3-a\,b\,c}{c^2}+\frac{b\,{\left(2\,a\,c-b^2\right)}^2}{2\,c^2\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)+\frac{b\,\left(\frac{a\,b^2}{c^2}+\frac{\left(2\,b^3-8\,a\,b\,c\right)\,\left(8\,a\,b+\frac{8\,a\,c^2\,\left(2\,b^3-8\,a\,b\,c\right)}{16\,a\,c^3-4\,b^2\,c^2}\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}-\frac{a\,{\left(2\,a\,c-b^2\right)}^2}{c^2\,\left(4\,a\,c-b^2\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)}{4\,a^2\,c^2-4\,a\,b^2\,c+b^4}\right)\,\left(2\,a\,c-b^2\right)}{2\,c^2\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(a*x + b*x^3 + c*x^5),x)

[Out] x^2/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*((((8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(1/2)) + (a*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - x^2*((((2*a*c - b^2)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + (b*(((2*b^3 - 8*a*b*c)*((4*a*c^3 - 6*b^2*c^2)/c^2 - (4*b*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b^3 - a*b*c)/c^2 + (b*(2*a*c - b^2)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((a*b^2)/c^2 + ((2*b^3 - 8*a*b*c)*(8*a*b + (8*a*c^2*(2*b^3 - 8*a*b*c))/(16*a*c^3 - 4*b^2*c^2)))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*(2*a*c - b^2)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^4 + 4*a^2*c^2 - 4*a*b^2*c))*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(1/2))

3.81 $\int \frac{x^5}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=179

$$\frac{x}{c}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^
2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arc
tan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c
+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1599, 1136, 1180, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1136

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d^3*(d*x)^(m - 3)*((a + b*x^2 + c*x^4)^(p + 1)/(c*(m + 4*p + 1))),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax+bx^3+cx^5}\,dx &= \int \frac{x^4}{a+bx^2+cx^4}\,dx \\
&= \frac{x}{c} - \frac{\int \frac{a+bx^2}{a+bx^2+cx^4}\,dx}{c} \\
&= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-}}}{2c} \\
&= \frac{x}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{}}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 202, normalized size = 1.13

$$\frac{x}{c} - \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[
```

```
b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.04, size = 169, normalized size = 0.94

method	result
risch	$\frac{x}{c}+\frac{\sum_{\_R=\operatorname{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-\_R^2 b-a\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}}{2c}$
default	$\frac{x}{c}-\frac{\left(b^2-2ac-b\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-b\sqrt{-4ac+b^2}+2ac-b^2\right)}{2\sqrt{-4ac+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] x/c-1/2*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))+1/2*(-b*(-4*a*c+b^2)^(1/2)+2*a*c-b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(143) = 286.
time = 0.37, size = 1059, normalized size = 5.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*
a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2
 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4
)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b
*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 -
4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2
 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^
7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3
- 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2
*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) -
sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*
c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2
*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (
b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(
b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [**A**]
time = 1.45, size = 129, normalized size = 0.72

$$\mathrm{RootSum}\left(t^4\cdot\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta^2c^2+8tab^2c-2tb^4}{a^2c-ab^2}\right)\right)\right)+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t, _t*log(x + (32*_t**3
*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**2 + 8*_t*a*b**2*c - 2*_t*b**4)
/(a**2*c - a*b**2)))) + x/c
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2109 vs. 2(143) = 286.
time = 5.10, size = 2109, normalized size = 11.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] x/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - (2*
b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2 - 2*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a
*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*
c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)
)/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*
b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^
2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b
```

```
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3
- 8*(b^2 - 4*a*c)*a^2*c^4)*abs(c))*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^
2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*
c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

Mupad [B]
time = 2.58, size = 3026, normalized size = 16.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] x/c - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*
(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4
*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x
*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4
*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4
*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^3 - 4
*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 + b^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3
)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c
))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*
c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)
 + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 +
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2
*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3 - 4*a*b^2*c^2)/c -
```

```
(2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c
^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 -
b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*a^2
*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4
*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^
3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)
))^(1/2)*1i)/((((16*a^2*c^3 - 4*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4
)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)
*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2
*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*a^2*c^3 - 4*a*b^2*c^2)/c + (2*x*(4*
b^3*c^3 - 16*a*b*c^4)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
- 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2))/c)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))^(1/2) + (2*x*(b^4 + 2*a^2*c^2 - 4*a*b^2*c))/c)*(-(b^5 - b^2*(-(4*
a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*a^2*b)/c))*(-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i
```

3.82 $\int \frac{x^4}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c/(-4
*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1599, 1128, 648, 632, 212, 642}

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + Log[
a + b*x^2 + c*x^4]/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax+bx^3+cx^5}\,dx &= \int \frac{x^3}{a+bx^2+cx^4}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}-\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{4c}\\
&= \frac{\log\left(a+bx^2+cx^4\right)}{4c}+\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{2c}\\
&= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 62, normalized size = 0.98

$$\frac{-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + Log[a
 + b*x^2 + c*x^4])/(4*c)
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.95

method	result
default	$\frac{\ln(c\,x^4+b\,x^2+a)}{4c}-\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2c\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2\left(4ac-b^2\right)}\,b\right)x^2+2\sqrt{-b^2\left(4ac-b^2\right)}\,a\right)a}{4ac-b^2}-\frac{\ln\left(\left(-4abc+b^3+\sqrt{-b^2\left(4ac-b^2\right)}\,b\right)x^2+\right.}{4c(4ac-b^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A]
time = 0.35, size = 197, normalized size = 3.13

$$\left[\frac{\sqrt{b^2-4\,ac}\,b\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\ \frac{2\,\sqrt{-b^2+4\,ac}\,b\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(b^2-4\,ac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*sqrt(-b^2 + 4*a*c)*b*arctan(-(2*c*x^
2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^2 - 4*a*c)*log(c*x^4 + b*x^2
+ a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(54) = 108$.
time = 0.71, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*
(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(-b*
sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)))/b) + (b*sqrt(-4*a*c +
b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b
**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/
(4*c*(4*a*c - b**2)) + 1/(4*c)))/b)
```

Giac [A]
time = 4.49, size = 59, normalized size = 0.94

$$-\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c) + 1/
4*log(c*x^4 + b*x^2 + a)/c
```

Mupad [B]
time = 0.17, size = 118, normalized size = 1.87

$$\frac{4\,a\,c\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b^2\,\ln\left(c\,x^4+b\,x^2+a\right)}{16\,a\,c^2-4\,b^2\,c}-\frac{b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^2}{\sqrt{4\,a\,c-b^2}}\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] (4*a*c*log(a + b*x^2 + c*x^4))/(16*a*c^2 - 4*b^2*c) - (b^2*log(a + b*x^2 +
c*x^4))/(16*a*c^2 - 4*b^2*c) - (b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^2)/(4
*a*c - b^2)^(1/2)))/(2*c*(4*a*c - b^2)^(1/2))
```

3.83 $\int \frac{x^3}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/
2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1599, 1144, 211}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqr
t[c]*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^3}{ax+bx^3+cx^5}\,dx = \int \frac{x^2}{a+bx^2+cx^4}\,dx$$
$$= -\left(\frac{1}{2}\left(-1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx\right)+\frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)$$
$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(-\right.}{\sqrt{2}\sqrt{c}\sqrt{b^2-}}$$

Mathematica [A]
time = 0.06, size = 165, normalized size = 1.10

$$\frac{\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.03, size = 149, normalized size = 0.99

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\_R^2\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$

default	$4c\left(-\frac{\left(-b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(b+\sqrt{-4ac+b^{2}}\right)\sqrt{2}\,\arctan\left(\right.}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(b+\right.}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/8/c*(-b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)+1/8*(b+(-4*a*c+b^2)^(1/2))/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(115) = 230$.
time = 0.34, size = 559, normalized size = 3.73

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^{2}c-4ac^{2}\right)\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^{2}c-4ac^{2}\right)\sqrt{-\frac{b+\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}+x\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^{2}c-4ac^{2}\right)\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}+x\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^{2}c-4ac^{2}\right)\sqrt{-\frac{b-\frac{b^{2}c-4ac^{2}}{\sqrt{b^{2}c^{2}-4ac^{3}}}}{b^{2}c-4ac^{2}}}}{\sqrt{b^{2}c^{2}-4ac^{3}}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/
2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqr
t(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2
*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4
*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(
b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*s
qrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a
```

```
*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b
^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A]
time = 0.77, size = 75, normalized size = 0.50

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(-16abc+4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c-2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t**2*(-16*a*
b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 16*_t**3*b**2*c - 2*
_t*b + x)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(115) = 230.
time = 4.19, size = 503, normalized size = 3.35

$$-\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,ac+2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,bc-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc+\sqrt{b^2-4ac}\,c}\,c^2-2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4-8ab^2c-2b^3c+16a^2c^2+8abc^2+b^2c^2-4ac^3\right)|c|}+\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,ac+2\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,bc-\sqrt{2}\sqrt{b^2-4ac}\sqrt{bc-\sqrt{b^2-4ac}\,c}\,c^2-2\left(b^2-4ac\right)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,x}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{c}}}\right)}{2\left(b^4-8ab^2c-2b^3c+16a^2c^2+8abc^2+b^2c^2-4ac^3\right)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a
*c)*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*
b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c)) + 1/2
*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*
c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*abs(c))
```

Mupad [B]
time = 2.21, size = 416, normalized size = 2.77

$$-2\,\mathrm{atanh}\left(\frac{\left(x\left(4ac^2-2b^2c\right)+\frac{x\left(8b^3c^2-32abc^3\right)\left(b^3+\sqrt{-{\left(4ac-b^2\right)}^3}-4abc\right)}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}\right)\sqrt{-\frac{b^3+\sqrt{-{\left(4ac-b^2\right)}^3}-4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}}{ac}\right)\sqrt{-\frac{b^3+\sqrt{-{\left(4ac-b^2\right)}^3}-4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}-2\,\mathrm{atanh}\left(\frac{\left(x\left(4ac^2-2b^2c\right)-\frac{x\left(8b^3c^2-32abc^3\right)\left(\sqrt{-{\left(4ac-b^2\right)}^3}-b^3+4abc\right)}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}\right)\sqrt{\frac{\sqrt{-{\left(4ac-b^2\right)}^3}-b^3+4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}}{ac}\right)\sqrt{\frac{\sqrt{-{\left(4ac-b^2\right)}^3}-b^3+4abc}{8\left(16a^2c^3-8ab^2c^2+b^4c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) + (x*(8*b^3*c^2 - 32*a*b*c^3)*(b^3 + (-(4
*a*c - b^2)^3)^(1/2) - 4*a*b*c))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-
(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2
*c^2)))^(1/2))/(a*c))*(-(b^3 + (-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c)/(8*(b^4*
c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh(((x*(4*a*c^2 - 2*b^2*c) - (
x*(8*b^3*c^2 - 32*a*b*c^3)*((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*b*c))/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(((-(4*a*c - b^2)^3)^(1/2) - b^3 + 4*a*
b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(a*c))*(((-(4*a*c - b^2
)^3)^(1/2) - b^3 + 4*a*b*c)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.84 $\int \frac{x^2}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1599, 1121, 632, 212}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{ax+bx^3+cx^5}\,dx &= \int \frac{x}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= -\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right) \\
&= -\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A]
time = 0.02, size = 36, normalized size = 1.00

method	result	size
default	$\frac{\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$	36
risch	$-\frac{\ln\left(\left(-b+\sqrt{-4ac+b^2}\right)x^2-2a\right)}{2\sqrt{-4ac+b^2}}+\frac{\ln\left(\left(b+\sqrt{-4ac+b^2}\right)x^2+2a\right)}{2\sqrt{-4ac+b^2}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A]
time = 0.36, size = 129, normalized size = 3.58

$$\left[\frac{\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}}, -\frac{\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a))/sqrt(b^2 - 4*a*c), -sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(34) = 68$.
time = 0.27, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*
sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c - b**2))*log(x**2 +
(4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2
```

Giac [A]
time = 3.87, size = 35, normalized size = 0.97

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

Mupad [B]
time = 2.04, size = 41, normalized size = 1.14

$$\frac{\operatorname{atan}\left(\frac{2\,a\,c\,x^2+a\,b}{a\,\sqrt{4\,a\,c-b^2}}\right)}{\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] atan((a*b + 2*a*c*x^2)/(a*(4*a*c - b^2)^(1/2)))/(4*a*c - b^2)^(1/2)
```

3.85 $\int \frac{x}{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*
a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*2^(1/2)*c^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1599, 1107, 211}

$$\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1107

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax+bx^3+cx^5}\,dx &= \int \frac{1}{a+bx^2+cx^4}\,dx\\ &= \frac{c\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{c\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]/S
qrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

Maple [A]
time = 0.03, size = 117, normalized size = 0.78

method	result	size

risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$	38
default	$4c\left(-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/(c*x^5 + b*x^3 + a*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(114) = 228$.
time = 0.35, size = 613, normalized size = 4.09

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqrt(
a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a
```

```
^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*lo
g(2*c*x - sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A]
time = 0.68, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\ \left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t**2*(-16*a*
b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2*b*c - 8*_t**3*a*b**
3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. 2(114) = 228.
time = 3.76, size = 1026, normalized size = 6.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
 - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)*
b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^
2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs
(c)) + 1/4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*
```

```
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 -
4*a*c)*b*c^2)*arctan(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4
 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c
^3)*abs(c))
```

Mupad [B]
time = 2.48, size = 763, normalized size = 5.09

$$-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}+b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{-\frac{b^3+\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}-\mathrm{atan}\left(\frac{b^4\,x\,1\mathrm{i}-b\,x\,\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}\,1\mathrm{i}+a^2\,c^2\,x\,16\mathrm{i}-a\,b^2\,c\,x\,8\mathrm{i}}{4\,a\,b^4\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}+64\,a^3\,c^2\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}-32\,a^2\,b^2\,c\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}}\right)\,\sqrt{\frac{\sqrt{-64\,a^3\,c^3+48\,a^2\,b^2\,c^2-12\,a\,b^4\,c+b^6}-b^3+4\,a\,b\,c}{128\,a^3\,c^2-64\,a^2\,b^2\,c+8\,a\,b^4}}\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] - atan((b^4*x*1i + b*x*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/
2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i)/(4*a*b^4*(-(b^3 + (b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*
a^2*b^2*c))^(1/2) + 64*a^3*c^2*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2
- 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2
) - 32*a^2*b^2*c*(-(b^3 + (b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)))*(-(b^3 + (
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c)/(8*a*b^4 +
 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i - atan((b^4*x*1i - b*x*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*c^2*x*16i - a*b^2*c*x*8i
)/(4*a*b^4*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 +
 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) + 64*a^3*c^2*(((b^6
 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^
4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2) - 32*a^2*b^2*c*(((b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2
 - 64*a^2*b^2*c))^(1/2)))*(((b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)^(1/2) - b^3 + 4*a*b*c)/(8*a*b^4 + 128*a^3*c^2 - 64*a^2*b^2*c))^(1/2)*2i
```

3.86 $\int \frac{1}{ax+bx^3+cx^5} dx$

Optimal. Leaf size=69

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{\log(x)}{a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] ln(x)/a-1/4*ln(c*x^4+b*x^2+a)/a+1/2*b*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2
))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1608, 1128, 719, 29, 648, 632, 212, 642}

$$\frac{b \tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + Log[
x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{ax+bx^3+cx^5}\,dx = \int \frac{1}{x\left(a+bx^2+cx^4\right)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right)}{2a} + \frac{\text{Subst}\left(\int \frac{-b-cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}$$
$$= \frac{\log(x)}{a} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a} - \frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4a}$$
$$= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a} + \frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2a}$$
$$= \frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}} + \frac{\log(x)}{a} - \frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

Mathematica [A]
time = 0.04, size = 113, normalized size = 1.64

$$\frac{4\sqrt{b^2-4ac}\,\log(x) - \left(b+\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right) + \left(b-\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-1),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.03, size = 65, normalized size = 0.94

method	result	size
default	$-\frac{\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{b\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a}+\frac{\ln(x)}{a}$	65
risch	$\frac{\ln(x)}{a}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4a^2c-a\,b^2\right)\_Z^2+\left(4ac-b^2\right)\_Z+c\right)}\_R\ln\left(\left(\left(10ac-3b^2\right)\_R+5c\right)x^2-ab\_R+2b\right)\right)}{2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(1/2*ln(c*x^4+b*x^2+a)+b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2)))+ln(x)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^3 + b*x)/(c*x^4 + b*x^2 + a), x)/a + log(x)/a
```

Fricas [A]
time = 0.37, size = 223, normalized size = 3.23

$$\left[\frac{\sqrt{b^2-4ac}\,b\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-(b^2-4ac)\log\left(cx^4+bx^2+a\right)+4(b^2-4ac)\log\left(x\right)}{4(ab^2-4a^2c)}, \frac{2\sqrt{-b^2+4ac}\,b\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)-(b^2-4ac)\log\left(cx^4+bx^2+a\right)+4(b^2-4ac)\log\left(x\right)}{4(ab^2-4a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b^2 - 4*a*c)*b*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x
^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^2 - 4*a*c)*log(c*x^4 +
 b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c), 1/4*(2*sqrt(-b^2 +
 4*a*c)*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^2 -
4*a*c)*log(c*x^4 + b*x^2 + a) + 4*(b^2 - 4*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(60) = 120$.
time = 9.07, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(-b*s
qrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) +
 (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2
*c*(b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqr
t(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + l
og(x)/a
```

Giac [A]
time = 2.97, size = 68, normalized size = 0.99

$$-\frac{b \arctan\left(\frac{2 \, c x^{2} + b}{\sqrt{-b^{2} + 4 \, a c}}\right)}{2 \, \sqrt{-b^{2} + 4 \, a c} a} - \frac{\log\left(c x^{4} + b x^{2} + a\right)}{4 \, a} + \frac{\log\left(x^{2}\right)}{2 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a) - 1/
4*log(c*x^4 + b*x^2 + a)/a + 1/2*log(x^2)/a
```

Mupad [B]
time = 2.70, size = 1014, normalized size = 14.70

$$\frac{\ln(x)}{a}+\frac{\ln(c x^4+b x^2+a)\,(8 a c-2 b^2)}{2\,(4 a b^2-16 a^2 c)}+\frac{b\,\mathrm{atan}\left(\frac{16 a^3 x^2\left(\frac{(3b^3-8abc)\left(\frac{(8ac-2b^2)^2\left(10bc^3-\frac{(12b^3c^2-40abc^3)(8ac-2b^2)}{2(4ab^2-16a^2c)}\right)}{4(4ab^2-16a^2c)^2}-\frac{b^2\left(10bc^3-\frac{(12b^3c^2-40abc^3)(8ac-2b^2)}{2(4ab^2-16a^2c)}\right)}{16a^2(4ac-b^2)}+\frac{b^2(12b^3c^2-40abc^3)(8ac-2b^2)}{16a^2(4ab^2-16a^2c)(4ac-b^2)}\right)}{8a^3c^2(25ac-6b^2)}-\frac{(10a^2c^2-14ab^2c+3b^4)\left(\frac{b^3(12b^3c^2-40abc^3)}{64a^3(4ac-b^2)^{3/2}}-\frac{b(12b^3c^2-40abc^3)(8ac-2b^2)^2}{16a(4ab^2-16a^2c)^2\sqrt{4ac-b^2}}+\frac{b(8ac-2b^2)\left(10bc^3-\frac{(12b^3c^2-40abc^3)(8ac-2b^2)}{2(4ab^2-16a^2c)}\right)}{4a(4ab^2-16a^2c)\sqrt{4ac-b^2}}\right)}{8a^3c^2\sqrt{4ac-b^2}\,(25ac-6b^2)}\right)(4ac-b^2)^{3/2}}{b^2c^2}+\frac{2(3b^3-8abc)(4ac-b^2)^{3/2}\left(\frac{(8ac-2b^2)^2\left(4b^2c^2-\frac{2ab^2c^2(8ac-2b^2)}{4ab^2-16a^2c}\right)}{4(4ab^2-16a^2c)^2}-\frac{b^2\left(4b^2c^2-\frac{2ab^2c^2(8ac-2b^2)}{4ab^2-16a^2c}\right)}{16a^2(4ac-b^2)}+\frac{b^4c^2(8ac-2b^2)}{4a(4ab^2-16a^2c)(4ac-b^2)}\right)}{b^2c^4(25ac-6b^2)}-\frac{2(4ac-b^2)(10a^2c^2-14ab^2c+3b^4)\left(\frac{b^5c^2}{16a^2(4ac-b^2)^{3/2}}-\frac{b^3c^2(8ac-2b^2)^2}{4(4ab^2-16a^2c)^2\sqrt{4ac-b^2}}+\frac{b(8ac-2b^2)\left(4b^2c^2-\frac{2ab^2c^2(8ac-2b^2)}{4ab^2-16a^2c}\right)}{4a(4ab^2-16a^2c)\sqrt{4ac-b^2}}\right)}{b^2c^4(25ac-6b^2)}\right)}{2a\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3 + c*x^5),x)
```

```
[Out] log(x)/a + (log(a + b*x^2 + c*x^4)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c)
) + (b*atan((16*a^3*x^2*(((3*b^3 - 8*a*b*c)*(((8*a*c - 2*b^2)^2*(10*b*c^3 -
 ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*
(4*a*b^2 - 16*a^2*c)^2) - (b^2*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a*
c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(16*a^2*(4*a*c - b^2)) + (b^2*(12*b^
3*c^2 - 40*a*b*c^3)*(8*a*c - 2*b^2))/(16*a^2*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2))))/(8*a^3*c^2*(25*a*c - 6*b^2)) - ((3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*(
(b^3*(12*b^3*c^2 - 40*a*b*c^3))/(64*a^3*(4*a*c - b^2)^(3/2)) - (b*(12*b^3*c
^2 - 40*a*b*c^3)*(8*a*c - 2*b^2)^2)/(16*a*(4*a*b^2 - 16*a^2*c)^2*(4*a*c - b
^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(10*b*c^3 - ((12*b^3*c^2 - 40*a*b*c^3)*(8*a
*c - 2*b^2))/(2*(4*a*b^2 - 16*a^2*c))))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c -
b^2)^(1/2))))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a*c - 6*b^2)))*(4*a*c - b^
2)^(3/2))/(b^2*c^2) + (2*(3*b^3 - 8*a*b*c)*(4*a*c - b^2)^(3/2)*(((8*a*c - 2
*b^2)^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))/(4*a*b^2 - 16*a^2*c)))/(
4*(4*a*b^2 - 16*a^2*c)^2) - (b^2*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^2))
/(4*a*b^2 - 16*a^2*c)))/(16*a^2*(4*a*c - b^2)) + (b^4*c^2*(8*a*c - 2*b^2))/
(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(b^2*c^4*(25*a*c - 6*b^2)) - (2*
(4*a*c - b^2)*(3*b^4 + 10*a^2*c^2 - 14*a*b^2*c)*((b^5*c^2)/(16*a^2*(4*a*c -
 b^2)^(3/2)) - (b^3*c^2*(8*a*c - 2*b^2)^2)/(4*(4*a*b^2 - 16*a^2*c)^2*(4*a*c
 - b^2)^(1/2)) + (b*(8*a*c - 2*b^2)*(4*b^2*c^2 - (2*a*b^2*c^2*(8*a*c - 2*b^
2))/(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))
/(b^2*c^4*(25*a*c - 6*b^2))))/(2*a*(4*a*c - b^2)^(1/2))
```

3.87 $\int \frac{1}{x(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=174

$$-\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x
*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4*a*c+b^2)^(1
/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1599, 1137, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sq
rt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1137

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)/(a*d*(m + 1))), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^3+cx^5\right)}\,dx &= \int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx\\ &= -\frac{1}{ax}+\frac{\int \frac{-b-cx^2}{a+bx^2+cx^4}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right.\right.}{}\\ &= -\frac{1}{ax}-\frac{\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-}}\right.}{\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 191, normalized size = 1.10

$$-\frac{\frac{2}{x}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)),x]
[Out] -1/2*(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4
```

```
*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]
*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]))/a
```

Maple [A]
time = 0.04, size = 159, normalized size = 0.91

method	result
default	$\frac{4c\left(-\frac{\left(-b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(b-\sqrt{-4ac+b^2}\right)\sqrt{2}\,\arctan\left(\frac{}{\sqrt{\left(b\right.}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-\right.}}}\right.}{a}$
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(12a^2b\,c^2-7a\,b^3c+b^5\right)\_Z^2+c^3\right)}\_R\ln\left(\left(\left(40a^5c^2-22a^4b^2c+3a^3b^4\right)\_R^4+(25a\right.\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(-1/8*(-b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)+1/8*(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1116 vs. $2(137) = 274$.
time = 0.36, size = 1116, normalized size = 6.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2
*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a
^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c))
)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 -
 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5
 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4
- 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^
2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^
3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*s
qrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*
c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c
)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A]
time = 1.99, size = 148, normalized size = 0.85

$$\operatorname{RootSum}\left(t^4\cdot\left(256a^5c^2-128a^4b^2c+16a^3b^4\right)+t^2\cdot\left(48a^2bc^2-28ab^3c+4b^5\right)+c^3,\left(t\mapsto t\log\left(x+\frac{-64t^3a^5c^2+48t^3a^4b^2c-8t^3a^3b^4-10ta^2bc^2+10tab^3c-2tb^5}{ac^3-b^2c^2}\right)\right)\right)-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) + _t**2*(48*
a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t, _t*log(x + (-64*_t**
3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*a**3*b**4 - 10*_t*a**2*b*c**2
+ 10*_t*a*b**3*c - 2*_t*b**5)/(a*c**3 - b**2*c**2)))) - 1/(a*x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1839 vs. 2(137) = 274.
time = 4.90, size = 1839, normalized size = 10.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*b^2*c
^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*
(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^
2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(1/2)*
x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*
a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(
c)) - 1/8*(2*a^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + (2*b^4*c^2 - 16*a*
b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2
 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 -
2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2)*abs(a))*arctan(2*sqrt(
1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(a)
*abs(c)) - 1/(a*x)
```

Mupad [B]

time = 2.86, size = 2997, normalized size = 17.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*
c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^
3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4
+ 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b
^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b
*c^3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 + b^2*(-(4*a
*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2
*c^3) + (-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c -
a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5
+ b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 + b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(
-(b^5 + b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c - a*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - a
tan(((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2
 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a
^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 -
7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*
```

```
b^2*c)))^(1/2)*1i + (x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^3 - 4*a^4*b^3*c
^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2)
 + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) +
12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a
^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*a^4*c^4 - 2*a^3*b^2*c^3) + (-(b^5
- b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*a^5*b*c^
3 - 4*a^4*b^3*c^2 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b^2*(-(4*a*c
- b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/
(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*(-(4*a*c - b
^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*a^4*c^4 - 2*a^3*b^2*c^
3) + (-(b^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c
*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*
(4*a^4*b^3*c^2 - 16*a^5*b*c^3 + x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5 - b
^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)))*(-(b^5 - b^2*
(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c - b^2)^3
)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*a^3*c^4))*(-(b
^5 - b^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2 - 7*a*b^3*c + a*c*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - 1/(a
*x)
```

3.88 $\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2ax^2}-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/2*(-2*a*c+b^2)*arctanh
((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1599, 1128, 723, 814, 648, 632, 212, 642}

$$-\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
```

```
[Out] -1/2*1/(a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*x^2 + c*x^4])/(4*a
^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x^2(ax+bx^3+cx^5)}\,dx = \int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \frac{-b-cx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} + \frac{\text{Subst}\left(\int \left(-\frac{b}{ax} + \frac{b^2-ac+bcx}{a(a+bx+cx^2)}\right)\,dx, x, x^2\right)}{2a}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{b^2-ac+bcx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} + \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\right.}{4a^2}$$
$$= -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2+cx^4)}{4a^2} - \frac{(b^2-2ac)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x,\right.}{2a^2}$$
$$= -\frac{1}{2ax^2} - \frac{(b^2-2ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2+cx^4)}{4a^2}$$

Mathematica [A]
time = 0.08, size = 135, normalized size = 1.52

$$\frac{-\frac{2a}{x^2} - 4b\log(x) + \frac{\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)),x]
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 + 2*a*c + b*Sqrt[b^2
- 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A]
time = 0.03, size = 85, normalized size = 0.96

method	result
default	$-\frac{-\frac{b\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(ac-\frac{b^2}{2}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2} - \frac{1}{2a\,x^2} - \frac{b\ln(x)}{a^2}$

risch	$-\frac{1}{2a\,x^2}-\frac{b\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4abc+b^3\right)\_Z+c^2\right)}\_R\ln\left(\left(\left(10a^3c-3a^2b^2\right)\_R^2-4\_Rabc+2c^2\right)x^2-a\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(c*x^5+b*x^3+a*x),x,method=_RETURNVERBOSE)

[Out] -1/2/a^2*(-1/2*b*ln(c*x^4+b*x^2+a)+2*(a*c-1/2*b^2)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2/a/x^2-b*ln(x)/a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="maxima")

[Out] -b*log(x)/a^2 + integrate((b*c*x^3 + (b^2 - a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2/(a*x^2)

Fricas [A]
time = 0.36, size = 293, normalized size = 3.29

$$\left[-\frac{(b^2-2\,ac)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2},-\frac{2\,(b^2-2\,ac)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-(b^3-4\,abc)x^2\log\left(cx^4+bx^2+a\right)+4\,(b^3-4\,abc)x^2\log\left(x\right)+2\,ab^2-8\,a^2c}{4\,(a^2b^2-4\,a^3c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="fricas")

[Out] [-1/4*((b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (b^3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2), -1/4*(2*(b^2 - 2*a*c)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (b^3 - 4*a*b*c)*x^2*log(c*x^4 + b*x^2 + a) + 4*(b^3 - 4*a*b*c)*x^2*log(x) + 2*a*b^2 - 8*a^2*c)/((a^2*b^2 - 4*a^3*c)*x^2)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(c*x**5+b*x**3+a*x),x)

[Out] Timed out

Giac [A]
time = 3.52, size = 94, normalized size = 1.06

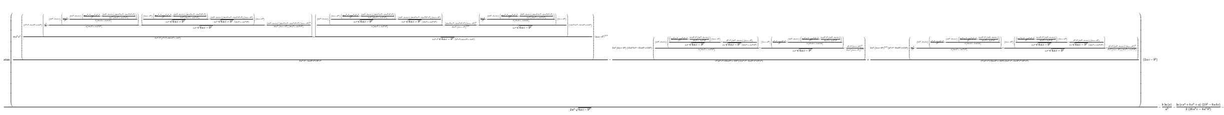
$$\frac{b\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/4*b*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*x
^2 - a)/(a^2*x^2)
```

Mupad [B]
time = 3.91, size = 2033, normalized size = 22.84



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)),x)
```

```
[Out] (atan((16*a^6*x^2*(((3*b^4 + a^2*c^2 - 9*a*b^2*c)*(c^5/a^3 + ((2*b^3 - 8*a*
b*c)*((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((((2
*a*c - b^2)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*
b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b
^2)^(1/2)) + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^
2))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a
^2*(4*a*c - b^2)^(1/2)) - ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2
)*(2*a*c - b^2)^2)/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2))))/(8*a^3*c
^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)) + ((((2*b^3 - 8*a*b*c)*(((2*a*c - b^2)*(
(20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^
3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) +
((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2))/(8*a^5*(4
*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) - ((
40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*a*c - b^2)^3)/(64*a^9*(4*a*c - b^2)^(3/2)
) + (((6*b*c^4)/a^2 + ((2*b^3 - 8*a*b*c)*((20*a^3*c^4 + 2*a^2*b^2*c^3)/a^3
+ ((2*b^3 - 8*a*b*c)*(40*a^4*b*c^3 - 12*a^3*b^3*c^2))/(2*a^3*(16*a^3*c - 4*
a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2)
^(1/2)))*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2
```

```
)*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)))*(4*a*c - b^2)^(3/2))/(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3) - (2*a^3*(4*a*c - b^2)*(3*b^5 + 13*a^2*b*c^2 - 15*a*b^3*c)
*(((2*b^3 - 8*a*b*c)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(2
*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))/(4*a^2*(4*a*c - b^2
)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2*a*(4*a*c - b^2)^(1/
2)*(16*a^3*c - 4*a^2*b^2))))/(2*(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((
a^2*c^4 - 4*a*b^2*c^3)/a^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c
^2)/a^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a
^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*a*c - b^2)^3
)/(16*a^5*(4*a*c - b^2)^(3/2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2
*c^4 + b^4*c^2 - 4*a*b^2*c^3)) + (2*a^3*(4*a*c - b^2)^(3/2)*(3*b^4 + a^2*c^
2 - 9*a*b^2*c)*((b*c^4)/a^3 - ((2*b^3 - 8*a*b*c)*((a^2*c^4 - 4*a*b^2*c^3)/a
^3 + ((2*b^3 - 8*a*b*c)*((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a^3 + (2*a*b^2*c^2*(
2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2)))/(2*(16*a^3*c - 4*a^2*b^2))))/(2*
(16*a^3*c - 4*a^2*b^2)) + ((2*a*c - b^2)*((((4*a^3*b*c^3 - 4*a^2*b^3*c^2)/a
^3 + (2*a*b^2*c^2*(2*b^3 - 8*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*a*c - b^2))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2))/(2
*a*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))))/(4*a^2*(4*a*c - b^2)^(1/2)
) + (b^2*c^2*(2*b^3 - 8*a*b*c)*(2*a*c - b^2)^2)/(8*a^3*(4*a*c - b^2)*(16*a^
3*c - 4*a^2*b^2))))/(c^2*(a^2*c^2 - 6*b^4 + 24*a*b^2*c)*(4*a^2*c^4 + b^4*c^
2 - 4*a*b^2*c^3)))*(2*a*c - b^2))/(2*a^2*(4*a*c - b^2)^(1/2)) - (b*log(x))/
a^2 - (log(a + b*x^2 + c*x^4)*(2*b^3 - 8*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
 - 1/(2*a*x^2)
```

3.89 $\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=166

$$\frac{(b^2-3ac)\,x^2}{c^2\,(b^2-4ac)}-\frac{bx^4}{2c\,(b^2-4ac)}+\frac{x^6(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}-\frac{b\,\mathrm{lo}}{}$$

```
[Out] (-3*a*c+b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*b*x^4/c/(-4*a*c+b^2)+1/2*x^6*(b*x^2+2
*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/2*b*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A]
time = 0.16, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1599, 1128, 752, 814, 648, 632, 212, 642}

$$-\frac{(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{c^3\,(b^2-4ac)^{3/2}}+\frac{x^2(b^2-3ac)}{c^2\,(b^2-4ac)}-\frac{bx^4}{2c\,(b^2-4ac)}+\frac{x^6(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{b\log\left(a+bx^2+cx^4\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((b^2 - 3*a*c)*x^2)/(c^2*(b^2 - 4*a*c)) - (b*x^4)/(2*c*(b^2 - 4*a*c)) + (x^
6*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
+ 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(c^3*(b^2 - 4*a*c)^(
3/2)) - (b*Log[a + b*x^2 + c*x^4])/(2*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^{11}}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^9}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$
$$= \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x^2(6a+2bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$
$$= \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2(b^2-3ac)}{c^2} + \frac{2bx}{c} + \frac{2(a(b^2-3ac)+b(b^2-4ac)x)}{c^2(a+bx+cx^2)}\right.\right.}{2(b^2-4ac)}$$
$$= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{a(b^2-3ac)-}{a+bx}\right.}{c^2(b^2}$$
$$= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\right.}{2c^3}$$
$$= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{b\log(a+bx^2+cx}{2c^3}$$
$$= \frac{(b^2-3ac)x^2}{c^2(b^2-4ac)} - \frac{bx^4}{2c(b^2-4ac)} + \frac{x^6(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(b^4-6ab^2c+6a^2c}{c^3(b}$$

Mathematica [A]
time = 0.13, size = 151, normalized size = 0.91

$$\frac{cx^2 + \frac{-b^4x^2-ab^2(b-4cx^2)+a^2c(3b-2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(b^4-6ab^2c+6a^2c^2)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} - b\log(a+bx^2+cx^4)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (c*x^2 + (-(b^4*x^2) - a*b^2*(b - 4*c*x^2) + a^2*c*(3*b - 2*c*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) - b*Log[a + b*x^2 + c*x
^4])/(2*c^3)
```

Maple [A]
time = 0.06, size = 209, normalized size = 1.26

method	result

default	$\frac{x^2}{2c^2} - \frac{\frac{-\frac{(2a^2c^2-4a\,b^2c+b^4)x^2}{c(4ac-b^2)}+\frac{ba(3ac-b^2)}{c(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\frac{(4abc-b^3)\ln(c\,x^4+b\,x^2+a)}{c}+\frac{4\left(3a^2c-a\,b^2-\frac{(4abc-b^3)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2c^2}$
risch	$\frac{x^2}{2c^2} + \frac{\frac{(2a^2c^2-4a\,b^2c+b^4)x^2}{2c(4ac-b^2)}-\frac{ba(3ac-b^2)}{2c(4ac-b^2)}}{c^2(c\,x^4+b\,x^2+a)} - \frac{8\ln\left(\left(-24a^3c^3+30a^2b^2c^2-10a\,b^4c+b^6+\sqrt{-(4ac-b^2)(6a^2c^2-6a\,b^2c}\right.\right.}{c(4ac-b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c^2-1/2/c^2*((-(2*a^2*c^2-4*a*b^2*c+b^4)/c/(4*a*c-b^2)*x^2+b*a/c*(3
*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(4*a*b*c-b^3)/c*l
n(c*x^4+b*x^2+a)+2*(3*a^2*c-a*b^2-1/2*(4*a*b*c-b^3)*b/c)/(4*a*c-b^2)^(1/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*b^3 - 3*a^2*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*x^2)/(a*b^2*c^3 - 4
*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*x^2/c
^2 + 2*integrate(-((b^3 - 4*a*b*c)*x^3 + (a*b^2 - 3*a^2*c)*x)/(c*x^4 + b*x^
2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(156) = 312$.
time = 0.36, size = 868, normalized size = 5.23

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b^3*c - 12*a
^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a*b^4*c + 26
*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2 + (b^4*c
- 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^2)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 +
```

```
(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^
2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b
*c^5)*x^2), 1/2*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^6 - a*b^5 + 7*a^2*b
^3*c - 12*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^4 - (b^6 - 9*a
*b^4*c + 26*a^2*b^2*c^2 - 24*a^3*c^3)*x^2 - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*
c^2 + (b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^4 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
4*a*c)) - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2 + (b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^4 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*x^2)*log(c*x^4 + b*x^2 +
 a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*
a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.88, size = 161, normalized size = 0.97

$$\frac{(b^4 - 6\,ab^2c + 6\,a^2c^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2c^3 - 4\,ac^4)\sqrt{-b^2+4\,ac}} + \frac{x^2}{2\,c^2} + \frac{b^3x^4 - 4\,abcx^4 - 2\,a^2cx^2 - a^2b}{2\,(cx^4+bx^2+a)(b^2c^2 - 4\,ac^3)} - \frac{b\log\left(cx^4+bx^2+a\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^
2*c^3 - 4*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/2*x^2/c^2 + 1/2*(b^3*x^4 - 4*a*b*c
*x^4 - 2*a^2*c*x^2 - a^2*b)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/2
*b*log(c*x^4 + b*x^2 + a)/c^3
```

Mupad [B]
time = 0.53, size = 1473, normalized size = 8.87

$$\frac{\frac{a(b^3-3abc)}{2c(4ac-b^2)} + \frac{x^2(2a^2c^2-4ab^2c+b^4)}{2c(4ac-b^2)}}{c^3x^4+bc^2x^2+ac^2} + \frac{x^2}{2c^2} + \frac{\ln(cx^4+bx^2+a)(-64a^3bc^3+48a^2b^3c^2-12ab^5c+b^7)}{2(64a^3c^6-48a^2b^2c^5+12ab^4c^4-b^6c^3)} + \frac{\mathrm{atan}\left(\text{[illegible]}\right)(6a^2c^2-6ab^2c+b^4)}{c^3(4ac-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(b^3 - 3*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(b^4 + 2*a^2*c^2 - 4*a*b^2*
c))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + x^2/(2*c^2) + (log
(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*
(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (atan(((4*a*c^5*(
4*a*c - b^2)^3 - b^2*c^4*(4*a*c - b^2)^3)*(((((16*a*b)/c + (8*a*c^2*(b^7 -
64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(64*a^3*c^6 - b^6*c^3 + 12*a*b
^4*c^4 - 48*a^2*b^2*c^5))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*c^3*(4*a*c - b^
2)^(3/2)) + (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2
*b^3*c^2 - 12*a*b^5*c))/(c*(4*a*c - b^2)^(3/2)*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) - x^2*(((((4*(6*a^2*c^5 +
3*b^4*c^3 - 14*a*b^2*c^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)
*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^4 + 6*a^2*c^2 -
6*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2)) + ((2*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6
*a^2*c^2 - 6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 + 12*a*b^
4*c^4 - 48*a^2*b^2*c^5)))/(2*a*(4*a*c - b^2)) + (b*((4*(b^5 + 3*a^2*b*c^2 -
 5*a*b^3*c))/(4*a*c^5 - b^2*c^4) + (((4*(6*a^2*c^5 + 3*b^4*c^3 - 14*a*b^2*c
^4))/(4*a*c^5 - b^2*c^4) + (2*(2*b^3*c^6 - 8*a*b*c^7)*(b^7 - 64*a^3*b*c^3 +
 48*a^2*b^3*c^2 - 12*a*b^5*c))/((4*a*c^5 - b^2*c^4)*(64*a^3*c^6 - b^6*c^3 +
 12*a*b^4*c^4 - 48*a^2*b^2*c^5)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12
*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - ((2
*b^3*c^6 - 8*a*b*c^7)*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3
*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*((4*a*b^2)/c^4 + ((
(16*a*b)/c + (8*a*c^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(
64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5))*(b^7 - 64*a^3*b*c^3
+ 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5)) - (4*a*(b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2)/(c^4*(4*a*c - b^2)^
3)))/(2*a*(4*a*c - b^2)^(3/2))))/(2*b^8 + 72*a^4*c^4 + 96*a^2*b^4*c^2 - 144
*a^3*b^2*c^3 - 24*a*b^6*c))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(c^3*(4*a*c - b^
2)^(3/2))
```

3.90 $\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=331

$$\frac{(3b^2-10ac)\,x}{2c^2\,(b^2-4ac)}-\frac{bx^3}{2c\,(b^2-4ac)}+\frac{x^5(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\left(3b^3-13abc-\frac{3b^4-19ab^2c+20a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{b\ldots}}\right.}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4\ldots}}}$$

```
[Out] 1/2*(-10*a*c+3*b^2)*x/c^2/(-4*a*c+b^2)-1/2*b*x^3/c/(-4*a*c+b^2)+1/2*x^5*(b*
x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(-20*a^2*c^2+19*a*b^2*c-3*b^4)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3-13*a*b*c+(20
*a^2*c^2-19*a*b^2*c+3*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1134, 1293, 1180, 211}

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(3b^2-10ac)}{2c^2\,(b^2-4ac)}-\frac{bx^3}{2c\,(b^2-4ac)}+\frac{x^5(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 - 4*a*c)) +
(x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3 - 13*a*
b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20
*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
```

```
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^{10}}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^4(10a+3bx^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^2(9ab+3(3b^2-10ac)x^2)}{a+bx^2+cx^4}\,dx}{6c(b^2-4ac)}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{3a(3b^2-10ac)+3b(}{a+bx^2+c}}{6c^2(b^2-}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\right.}{}$$

$$= \frac{(3b^2-10ac)x}{2c^2(b^2-4ac)} - \frac{bx^3}{2c(b^2-4ac)} + \frac{x^5(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(3b^3-13abc-\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.43, size = 327, normalized size = 0.99

$$\frac{4\sqrt{c}\,x - \frac{2\sqrt{c}\,x(2a^2c-b^3x^2-ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{2}\left(-3b^4+19ab^2c-20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}\left(3b^4-19ab^2c+20a^2c^2+3b^3\sqrt{b^2-4ac}-13abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2)))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a*b^2*c - 20*a^2*c^2
+ 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[2]*(3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 -
 4*a*c] - 13*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c
^(5/2))
```

Maple [A]
time = 0.07, size = 319, normalized size = 0.96

method	result
risch	$\frac{x}{c^2}+\frac{\frac{b(3ac-b^2)x^3}{8ac-2b^2}+\frac{a(2ac-b^2)x}{8ac-2b^2}}{c^2(cx^4+bx^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{b(13ac-3b^2)\_R^2}{4ac-b^2}-\frac{(10ac-3b^2)a}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4c^2}$
default	$\frac{x}{c^2}-\frac{\frac{-\frac{b(3ac-b^2)x^3}{2(4ac-b^2)}-\frac{a(2ac-b^2)x}{2(4ac-b^2)}}{cx^4+bx^2+a}+2c\left(-\frac{\left(13\sqrt{-4ac+b^2}\,abc-3\sqrt{-4ac+b^2}\,b^3+20a^2c^2-19ab^2c+3b^4\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{\cdots}{\sqrt{\cdots}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c\cdots}}\cdots\right)}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x/c^2-1/c^2*((-1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*(2*a*c-b^2)/(4*a*c-b
^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c-3
*(-4*a*c+b^2)^(1/2)*b^3+20*a^2*c^2-19*a*b^2*c+3*b^4)/c/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))+1/8*(13*(-4*a*c+b^2)^(1/2)*a*b*c-3*(-4*a*c+b^2)^(1/2)*
b^3-20*a^2*c^2+19*a*b^2*c-3*b^4)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b
^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/2*integrate(-(3*a*b^
2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4
*a*c^3) + x/c^2
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 2856 vs. $2(285) = 570$.
time = 0.56, size = 2856, normalized size = 8.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/2)*(a*b^2*
c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt
(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*
b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^
2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5
*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^
3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 +
336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^
6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385
*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13))
)/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2)*(a*b
^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*s
qrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12
*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051
*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*
a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818
*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6
 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c +
385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
- 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^1
3)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(
a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2
)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 -
 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 25
00*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8
818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c
```

```
^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b
^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*
c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqrt(1/2
)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*
x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2
*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2
- 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b
^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 -
 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^
5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^
2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^
3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a
^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(3*
a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^
3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3339 vs. 2(285) = 570.
time = 4.61, size = 3339, normalized size = 10.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b*c*x^3 + a*b^2*x - 2*a^2*c*x)/((c*x^4 + b*x^2 + a)*(b^2
*c^2 - 4*a*c^3)) + x/c^2 + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8
 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
```

```
 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 19
2*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50
*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 +
13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b
^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7
- 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 -
80*(b^2 - 4*a*c)*a^3*c^6)*abs(-b^2*c^2 + 4*a*c^3))*arctan(2*sqrt(1/2)*x/sqr
t((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^
2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4
*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c
^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs
(c)) + 1/16*(6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 +
 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*
c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5
*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 +
464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 +
 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7
+ 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 -
320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 1
60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 -
```

```
96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*
(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*
b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^9 - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2
*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2
+ 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2 - 2*(3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b
^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 ...
```

Mupad [B]
time = 3.76, size = 2500, normalized size = 7.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((b*x^3*(3*a*c - b^2))/(2*(4*a*c - b^2)) + (a*x*(2*a*c - b^2))/(2*(4*a*c -
b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7 + 48*a*b^8*c^
3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6
- b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(-(9*b^13 + 9*b^4*(-(4*a*c
 - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 +
 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
+ b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3
*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b
^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8 + 200*
a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 268
80*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4
4800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10
```

```
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2
*c^10)))^(1/2)*1i - (((10240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 422
4*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4
- 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*
a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 4480
0*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b
^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^
6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^
10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^
2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 3024
0*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 2
13*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2
 - 718*a^3*b^2*c^3 - 114*a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b
^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2
*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((((10
240*a^5*c^7 + 48*a*b^8*c^3 - 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4
*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*(
-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*
c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^
2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128
0*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b
*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5
*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c - 51*a*b^2*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 2
40*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))
^(1/2) - (x*(9*b^8 + 200*a^4*c^4 + 481*a^2*b^4*c^2 - 718*a^3*b^2*c^3 - 114*
a*b^6*c))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(-(9*b^13 + 9*b^4*(-(4*
a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a
^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7 + 48*a*b^8*c^3 -
 736*a^2*b^6*c^4 + 4224*a^3*b^4*c^5 - 10752*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*(-(9*b^13 + 9*b^4*(-(4*a*c -
b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30
240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 + 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) -
```

```
213*a*b^11*c - 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b
^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 128...
```

3.91 $\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=132

$$-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] -1/2*b*x^2/c/(-4*a*c+b^2)+1/2*x^4*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+
1/2*b*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)
^(3/2)+1/4*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1599, 1128, 752, 787, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b*x^2)/(c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 + c*x^4]/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^9}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^7}{(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$
$$= \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{x(4a+bx)}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\text{Subst}\left(\int \frac{-ab+(-b^2+4ac)x}{a+bx+cx^2}\,dx, x, x^2\right.}{2c(b^2-4ac)}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} - \frac{(b}{}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\log(a+bx^2+cx^4)}{4c^2} + \frac{(b(b^2-6a}{}$$
$$= -\frac{bx^2}{2c(b^2-4ac)} + \frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right.}{2c^2(b^2-4ac)^{3/2}}$$

Mathematica [A]
time = 0.12, size = 121, normalized size = 0.92

$$\frac{\frac{2(-2a^2c+b^3x^2+ab(b-3cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2b(b^2-6ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + \log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2
 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.07, size = 179, normalized size = 1.36

method	result
default	$\frac{\frac{b(3ac-b^2)x^2}{c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{(4ac-b^2)c^2}}{2c\,x^4+2b\,x^2+2a} + \frac{\frac{(4ac-b^2)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(-ab-\frac{(4ac-b^2)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c(4ac-b^2)}$

risch	$\frac{\frac{b(3ac-b^2)x^2}{2c^2(4ac-b^2)}+\frac{a(2ac-b^2)}{2(4ac-b^2)c^2}}{cx^4+bx^2+a}+\frac{4\ln\left(\left(\left(-24a^2bc^2+10ab^3c-b^5+\sqrt{-b^2(4ac-b^2)(6ac-b^2)^2}\,b\right)x^2+2\sqrt{-b^2(4ac-\right.\right.}{(4ac-b^2)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/c^2)/(c*x^
4+b*x^2+a)+1/2/c/(4*a*c-b^2)*(1/2*(4*a*c-b^2)/c*ln(c*x^4+b*x^2+a)+2*(-a*b-1
/2*(4*a*c-b^2)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^2 - 2*a^2*c + (b^3 - 3*a*b*c)*x^2)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c
^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) - integrate(-((b^2 - 4*a*c)*
x^3 + a*b*x)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(120) = 240$.
time = 0.36, size = 663, normalized size = 5.02

$$\left[\frac{2ab^4-12a^2b^2c+16a^3c^2+2(b^5-7ab^3c+12a^2bc^2)x^2+((b^3c-6abc^2)x^4+ab^3-6a^2bc+(b^4-6ab^2c)x^2)\sqrt{b^2-4ac}\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+(2cx^2+b)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+(ab^4-8a^2b^2c+16a^3c^2+(b^4c-8ab^2c^2+16a^2c^3)x^4+(b^5-8ab^3c+16a^2bc^2)x^2)\log(cx^4+bx^2+a)}{4(ab^4c^2-8a^2b^2c^3+16a^3c^4+(b^4c^3-8ab^2c^4+16a^2c^5)x^4+(b^5c^2-8ab^3c^3+16a^2bc^4)x^2)}, \frac{2ab^4-12a^2b^2c+16a^3c^2+2(b^5-7ab^3c+12a^2bc^2)x^2+2((b^3c-6abc^2)x^4+ab^3-6a^2bc+(b^4-6ab^2c)x^2)\sqrt{-b^2+4ac}\arctan\left(-\frac{(2cx^2+b)\sqrt{-b^2+4ac}}{b^2-4ac}\right)+(ab^4-8a^2b^2c+16a^3c^2+(b^4c-8ab^2c^2+16a^2c^3)x^4+(b^5-8ab^3c+16a^2bc^2)x^2)\log(cx^4+bx^2+a)}{4(ab^4c^2-8a^2b^2c^3+16a^3c^4+(b^4c^3-8ab^2c^4+16a^2c^5)x^4+(b^5c^2-8ab^3c^3+16a^2bc^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3*c + 12*a^2*b*c
^2)*x^2 + ((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*
x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2
+ b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*
b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4
 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a
^2*b*c^4)*x^2), 1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(b^5 - 7*a*b^3
*c + 12*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (
b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 +
4*a*c)/(b^2 - 4*a*c)) + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(c*x^4 +
```

```
 b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c
^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.45, size = 152, normalized size = 1.15

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}-\frac{b^2cx^4-4\,ac^2x^4-b^3x^2+2\,abcx^2-ab^2}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}+\frac{\log\left(cx^4+bx^2+a\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^2 - 4
*a*c^3)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 - b^3*x^2 + 2*a*
b*c*x^2 - a*b^2)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/4*log(c*x^4
+ b*x^2 + a)/c^2
```

Mupad [**B**]
time = 2.94, size = 1336, normalized size = 10.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((a*(2*a*c - b^2))/(2*c^2*(4*a*c - b^2)) + (b*x^2*(3*a*c - b^2))/(2*c^2*(4*
a*c - b^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3
*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4
*c^3 - 192*a^2*b^2*c^4)) + (b*atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4
*a*c - b^2)^3)*(x^2*(((b*((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((
8*b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)
)/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*
b^2*c^4)))*(6*a*c - b^2))/(8*c^2*(4*a*c - b^2)^(3/2)) + (b*(8*b^3*c^4 - 32*
a*b*c^5)*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))
```

```
/(16*c^2*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 +
 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - (b*((b^3 - 5*a*b*c)/
(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2 - 28*a*b*c^3)/(4*a*c^3 - b^2*c^2) + ((8*
b^3*c^4 - 32*a*b*c^5)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/
(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^
2*c^4)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c
^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (b^2*((b^3*c^4)/2 - 2*a
*b*c^5)*(6*a*c - b^2)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(
4*a*c - b^2)^(3/2))) - ((b*(6*a*c - b^2)*(8*a + (8*a*c^2*(2*b^6 - 128*a^3*c
^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
- 192*a^2*b^2*c^4)))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*b*(6*a*c - b^2)*(2*b^
6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/((4*a*c - b^2)^(3/2)*(256*a
^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) +
(b*(a/c^2 + ((8*a + (8*a*c^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b
^4*c))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6 -
 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(256*a^3*c^5 - 4*b^6*c^2 +
48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (a*b^2*(6*a*c - b^2)^2)/(c^2*(4*a*c - b^
2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6 + 36*a^2*b^2*c^2 - 12*a*b^4*c))*(6
*a*c - b^2))/(2*c^2*(4*a*c - b^2)^(3/2))
```

3.92 $\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=271

$$-\frac{bx}{2c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b^2-6ac-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+$$

```
[Out] -1/2*b*x/c/(-4*a*c+b^2)+1/2*x^3*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/
4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-6*a*c-b*(-8*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-
6*a*c+b*(-8*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-
4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1134, 1293, 1180, 211}

$$\frac{\left(-\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b\left(b^2-8ac\right)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{bx}{2c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b*x)/(c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
```

```
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^8}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$$

$$= \frac{x^3(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{\int \frac{x^2(6a+bx^2)}{a+bx^2+cx^4}\,dx}{2\,(b^2-4ac)}$$

$$= -\frac{bx}{2c\,(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\int \frac{ab+(b^2-6ac)x^2}{a+bx^2+cx^4}\,dx}{2c\,(b^2-4ac)}$$

$$= -\frac{bx}{2c\,(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\int}{4c\,(b^2-4ac)}$$

$$= -\frac{bx}{2c\,(b^2-4ac)} + \frac{x^3(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\left(b^2-6ac-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{2\sqrt{2}\,c^{3/2}\,(b^2-4ac)\,\sqrt{b}}$$

Mathematica [A]
time = 0.33, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}\,x(b^2x^2+a(b-2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{2}\left(-b^3+8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(b^3-8abc+b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/
(4*c^(3/2))
```

Maple [A]
time = 0.05, size = 279, normalized size = 1.03

method	result

risch	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(6ac-b^2)\_R^2}{4ac-b^2}-\frac{ab}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{(2ac-b^2)x^3}{2c(4ac-b^2)}+\frac{abx}{2c(4ac-b^2)}}{cx^4+bx^2+a}+ -\frac{\left(6ac\sqrt{-4ac+b^2}-b^2\sqrt{-4ac+b^2}-8abc+b^3\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{(-b+\sqrt{-4ac+b^2})c}}\right)}{4c\sqrt{-4ac+b^2}\sqrt{(-b+\sqrt{-4ac+b^2})c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a
)+2/(4*a*c-b^2)*(-1/8*(6*a*c*(-4*a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)-8*a*
b*c+b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*a*c*(-4*a*c+b^2)
^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+8*a*b*c-b^3)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2
*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2 - 6*a*c)*x^2 + a*b)/
(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(227) = 454$.
time = 0.43, size = 2257, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60
```

```
*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*
a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 8
8*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
- 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) +
sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x
- 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c
^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((
b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c
^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a
*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*
a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a
*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88
*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 -
640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 -
 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + s
qrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 +
48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6
- 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x -
 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8*c^
3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b
^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64
*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^
4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a
```

```
^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2736 vs. $2(227) = 454$.
time = 6.75, size = 2736, normalized size = 10.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*x^3 - 2*a*c*x^3 + a*b*x)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
- 1/16*(2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24
*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a
*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
- 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2 - 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
```

```
*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a
^2*b*c^4)*abs(b^2*c - 4*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^
2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3
)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a
^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2
*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*b^8*c^4 - 32*
a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^
5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6 - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*
a*c^3)*(b^2*c - 4*a*c^2)^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*abs(b^2*c - 4
*a*c^2))*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b
*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)
))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3
*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*
abs(b^2*c - 4*a*c^2)*abs(c))
```

Mupad [B]
time = 3.86, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x + b*x^3 + c*x^5)^2,x)

[Out] - ((x^3*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b*x)/(2*c*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) - atan(((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2)*1i - (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*
c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c
^3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2
*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b
^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(1
6*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a
^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9
*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^
10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b
^2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
- 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 61
44*a^5*b^2*c^8)))^(1/2)*1i)/((((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5
*c^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*
c^3)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^
2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a
*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*
a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^
9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b
```

```
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8)))^(1/2) - (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*
(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2)
 - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 -
 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8)))^(1/2) + (((16*a*b^7*c^2 - 1024*a^4*b*c^5 - 192*a^2*b^5*c
^3 + 768*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16
*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 1
6*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8)))^(1/2) + (x*(b^6 - 72*a^3*c^3 + 74*a^2*b^2*c^2 - 16*a*b^4*c))/(2*(b
^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^9 + b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 614
4*a^5*b^2*c^8)))^(1/2) + (5*a^2*b^4 + 216*a^4*c^2 - 66*a^3*b^2*c)/(4*(b^6*c
 - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))...
```

3.93 $\int \frac{x^7}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{x^2(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*a*arctanh((2*c*x^2+b)/(-
4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1128, 736, 632, 212}

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*ArcTanh[(b
 + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
```

```
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(ax+bx^3+cx^5)^2}\,dx &= \int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right) \\
&= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{a\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{b^2-4ac} \\
&= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(2a)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac} \\
&= \frac{x^2(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 93, normalized size = 1.19

$$\frac{b^2x^2+a(b-2cx^2)}{2c(-b^2+4ac)(a+bx^2+cx^4)} + \frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.03, size = 104, normalized size = 1.33

method	result
default	$\frac{-\frac{\left(2ac-b^{2}\right)x^{2}}{c\left(4ac-b^{2}\right)}+\frac{ab}{c\left(4ac-b^{2}\right)}}{2c\,x^{4}+2b\,x^{2}+2a}+\frac{2a\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(2ac-b^{2}\right)x^{2}}{2c\left(4ac-b^{2}\right)}+\frac{ab}{2c\left(4ac-b^{2}\right)}}{c\,x^{4}+b\,x^{2}+a}+\frac{a\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8a^{2}c-2a\,b^{2}\right)}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{a\ln\left(\left(\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}-8a^{2}c+2a\right.}{\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2*a/
(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*a*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*((b^2 - 2*a*c)
*x^2 + a*b)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b
*c^2)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(72) = 144$.
time = 0.34, size = 407, normalized size = 5.22

$$\left[-\frac{ab^{3}-4\,a^{2}bc+(b^{4}-6\,ab^{2}c+8\,a^{2}c^{2})x^{2}+2\,(ac^{2}x^{4}+abcx^{2}+a^{2}c)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\,(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{4}+(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{2})},-\frac{ab^{3}-4\,a^{2}bc+(b^{4}-6\,ab^{2}c+8\,a^{2}c^{2})x^{2}-4\,(ac^{2}x^{4}+abcx^{2}+a^{2}c)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{4}+(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2 + 2*(a*c^2*x^4
 + a*b*c*x^2 + a^2*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4*c - 8
*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5
```

```
*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(a*b^3 - 4*a^2*b*c + (b^4 - 6*a
*b^2*c + 8*a^2*c^2)*x^2 - 4*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(a*b^4*c - 8*a
^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(70) = 140$.
time = 0.87, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)+a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)+\frac{ab+x^2(-2ac+b^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*b**4*sqrt(-1/(4*a*c
- b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a
**3*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)*
*3) + a*b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + (a*b + x**2*(-2*a
*c + b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**
2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A]
time = 7.80, size = 96, normalized size = 1.23

$$-\frac{2\,a\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{b^2x^2-2\,acx^2+ab}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*a*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(b^2*x^2 - 2*a*c*x^2 + a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c
^2))
```

Mupad [B]
time = 2.20, size = 187, normalized size = 2.40

$$-\frac{\frac{x^2\,(2\,a\,c-b^2)}{2\,c\,(4\,a\,c-b^2)}-\frac{a\,b}{2\,c\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}-\frac{2\,a\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{{(4\,a\,c-b^2)}^{3/2}}-\frac{x^2\,\left(\frac{4\,a\,c^2}{{(4\,a\,c-b^2)}^{7/2}}+\frac{4\,a\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{{(4\,a\,c-b^2)}^{13/2}}\right)\,{(4\,a\,c-b^2)}^4}{8\,a^2\,c^2}\right)}{{(4\,a\,c-b^2)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - ((x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)) - (a*b)/(2*c*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) - (2*a*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*((4
*a*c^2)/(4*a*c - b^2)^(7/2) + (4*a*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(13/2))*(4*a*c - b^2)^4)/(8*a^2*c^2)))/(4*a*c - b^2)^(3/2)
```

3.94 $\int \frac{x^6}{\left(ax+bx^3+cx^5\right)^2} \, dx$

Optimal. Leaf size=237

$$\frac{x(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{b^2+4ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\ldots}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\ldots}$$

```
[Out] 1/2*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1599, 1134, 1180, 211}

$$\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(2a+bx^2)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (b^2 + 4*a*
c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 +
 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1134

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-d^3)*(d*x)^(m - 3)*(2*a + b*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2
*(p + 1)*(b^2 - 4*a*c))), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x
)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1),
```

```
x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Gt
Q[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax+bx^3+cx^5)^2}\,dx &= \int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx \\
&= \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{2a-bx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} \\
&= \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,\cdots}{4(b^2-4ac)^{3/2}} \\
&= \frac{x(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(b^2+4ac-b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x\cdots}{\sqrt{b-\sqrt{b^2-\cdots}}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\cdots}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2(2ax+bx^3)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-b^2-4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2+4ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^6/(a*x + b*x^3 + c*x^5)^2,x]

[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4

Maple [A]
time = 0.04, size = 230, normalized size = 0.97

method	result
risch	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{ax}{4ac-b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(-\frac{b\_R^2}{4ac-b^2}+\frac{2a}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$\frac{-\frac{b x^3}{2(4ac-b^2)}-\frac{ax}{4ac-b^2}}{c x^4+b x^2+a}+\frac{2c\left(-\frac{\left(-b\sqrt{-4ac+b^2}+4ac+b^2\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,c\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\cdots\right)}{4ac-b^2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)

[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(-b*(-4*a*c+b^2)^(1/2)+4*a*c+b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-b*(-4*a*c+b^2)^(1/2)-4*a*c-b^2)/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1668 vs. $2(193) = 386$.
time = 0.38, size = 1668, normalized size = 7.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^
3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))
/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*
x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*
a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3
 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/
(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b
*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12
*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b
^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(
b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*
c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*
a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2
+ 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*
c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b
^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^
2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a
^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
 48*a^2*b^2*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a
*b^2*c + 16*a^2*c^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3
+ 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*
c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 +
48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A]
time = 8.88, size = 296, normalized size = 1.25

$$\frac{-2ax-bx^3}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}+\operatorname{RootSum}\left(t^4\cdot(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144ab^{10}c^2+256b^{12}c)+t^2(-12288a^4bc^4+8192a^3b^3c^3-1536a^2b^5c^2+16b^9)+16a^3c^2+24a^2b^2c+9ab^4,\left(t\mapsto t\log\left(x+\frac{16384t^3a^3bc^4-12288t^3a^2b^3c^3+3072t^3ab^5c^2-256t^3b^7c+64ta^2c^2-128tab^2c-4tb^4}{4ac+3b^2}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] (-2*a*x - b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*
(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7 - 1572864*a**5*b**2*
c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b**6*c**4 + 61440*a**2*b**8*c**3
 - 6144*a*b**10*c**2 + 256*b**12*c) + _t**2*(-12288*a**4*b*c**4 + 8192*a**3
*b**3*c**3 - 1536*a**2*b**5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c
 + 9*a*b**4, Lambda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a
**2*b**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2*c**2
 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2132 vs. 2(193) = 386.
time = 9.38, size = 2132, normalized size = 9.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*b^7*c^2 -
 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 - (2*b^3
*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)
*b*c^2)*(b^2 - 4*a*c)^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^
```

```
3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a
*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^
3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*b^
7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4 -
 (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 -
 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2 - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2
*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 -
 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(b^2 - 4*a*c))*arc
tan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 -
 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^
2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4
- 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B]
time = 3.64, size = 2500, normalized size = 10.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4
)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^
2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(
b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
+ 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3
- 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*((
```

(-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b
^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280
*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (x*(b^4*c + 8
*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*1i - (((2048*a^4*c^5 - 32*a
*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b
^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*
b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3
c^4))/(2(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9
 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6
*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^
5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b
*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*
b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5
*b^2*c^6)))^(1/2)*1i)/((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1
536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*
(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3
*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 -
 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 -
8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*
c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2
*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
- (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))
*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^
3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1
280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) - (4*a^2*b*c
^2 + 3*a*b^3*c)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((2
048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2)
- b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 409
6*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b
^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b
*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b
^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*
(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4
 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2) + (x*(b^4*c + 8*a^2*c^3 + 2
*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(((-(4*a*c - b^2)^9)^(1/2)
 - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 40
96*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*

b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 76
8*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7
- 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6
144*a^5*b^2*c^6)))^(1/2)*2i - atan(((((2048*a^4*c^5 - 32*a*b^6*c^2 + 384*a^
2*b^4*c^3 - 1536*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b^5
*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^
2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)))^(1/2)
*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^
4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^1
0*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6)))^(1/2) - (x*(b^4*c + 8*a^2*c^3 + 2*a*b^2*c^2))/(2*(b^4 + 16*a^2*c^2
- 8*a*b^2*c)))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*
b^5*c^2 + 512*a^3*b^3*c^3)/(32*(b^12*c + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240
*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*...

3.95 $\int \frac{x^5}{(ax+bx^3+cx^5)^2} dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*arctanh((2*c*x^2+b)/(-4*a*c+
b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1128, 652, 632, 212}

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(ax+bx^3+cx^5)^2}\,dx &= \int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+bx+cx^2)^2}\,dx,x,x^2\right)\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{2\left(b^2-4ac\right)}\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac}\\ &= \frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(b + 2*c*x^
2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.03, size = 77, normalized size = 1.03

method	result
default	$\frac{-b\,x^{2}-2a}{2(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)}-\frac{b\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$
risch	$\frac{-\frac{b\,x^{2}}{2\left(4ac-b^{2}\right)}-\frac{a}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{b\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8a^{2}c-2a\,b^{2}\right)}{2\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{b\ln\left(\left(-\left(-4ac+b^{2}\right)^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}-8a^{2}c+2a\,b^{2}\right)}{2\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*arctan((2*
c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] b*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) + 1/2*(b*x^2 + 2*a)/((b
^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(69) = 138$.
time = 0.35, size = 360, normalized size = 4.80

$$\left[\frac{2\,ab^{2}-8\,a^{2}c+(b^{3}-4\,abc)x^{2}-(bcx^{4}+b^{2}x^{2}+ab)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})},\ \frac{2\,ab^{2}-8\,a^{2}c+(b^{3}-4\,abc)x^{2}-2\,(bcx^{4}+b^{2}x^{2}+ab)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - (b*c*x^4 + b^2*x^2 + a*b)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 +
 (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*
x^2), 1/2*(2*a*b^2 - 8*a^2*c + (b^3 - 4*a*b*c)*x^2 - 2*(b*c*x^4 + b^2*x^2 +
 a*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^
3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(63) = 126$.
time = 0.83, size = 269, normalized size = 3.59

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2}-\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2}+\frac{-2a-bx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/(4*a*c - b
**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5*sqrt(-1/(4*a*c - b*
*2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*a*
*2*b*c**2*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3
) + b**5*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 + (-2*a - b*x**2)/(8
*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A]
time = 6.14, size = 82, normalized size = 1.09

$$\frac{b\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{bx^2+2\,a}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*a*c
)) + 1/2*(b*x^2 + 2*a)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 0.14, size = 178, normalized size = 2.37

$$\frac{b\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{b^2\,c^2}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{b^2\,\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)}{2\,b^2\,c^2}\right)}{(4\,a\,c-b^2)^{3/2}}-\frac{\frac{a}{4\,a\,c-b^2}+\frac{b\,x^2}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b*atan((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((b^2*c^
2)/(a*(4*a*c - b^2)^(7/2)) + (b^2*(2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c))/
(2*a*(4*a*c - b^2)^(13/2))))/(2*b^2*c^2)))/(4*a*c - b^2)^(3/2) - (a/(4*a*c
- b^2) + (b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

3.96 $\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right.}{\sqrt{2}\,(b^2-4a}$$

```
[Out] -1/2*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^
2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1599, 1133, 1180, 211}

$$\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(2*b
- Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*
b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1133

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(b + 2*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*(p +
 1)*(b^2 - 4*a*c))), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
```

```
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(ax+bx^3+cx^5)^2}\,dx &= \int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)} \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\left(c\left(1+\frac{2b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2(b^2-4ac)} \\
&= -\frac{x(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4}}}\right.}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A]
time = 0.28, size = 222, normalized size = 1.00

$$\frac{-bx-2cx^3}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(-2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(2b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(-2*b +
 Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*b
 + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.08, size = 271, normalized size = 1.23

method	result
risch	$\frac{\frac{c x^3}{4ac-b^2}+\frac{bx}{8ac-2b^2}}{c x^4+b x^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\frac{2c\_R^2}{4ac-b^2}-\frac{b}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_R b}\right)}{4}$
default	$16c^2\left(\frac{\frac{\sqrt{-4ac+b^2}\,x}{8c\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\sqrt{2}\,\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4c(4ac-b^2)\sqrt{-4ac+b^2}}+\frac{\sqrt{-}}{8c\left(x^2+\frac{b}{2c}-\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(1/8/c*(-4*a*c+b^2)^(1/2)*x/(x
^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/4*(b+1/2*(-4*a*c+b^2)^(1/2))*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))+1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(1/8/c*(-4*a*c+b^2)^(1/2)
*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))-1/4*(-b+1/2*(-4*a*c+b^2)^(1/2))*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*
b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*
c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1680 vs. $2(180) = 360$.
time = 0.39, size = 1680, normalized size = 7.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 -
 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)
)/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*
c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6
*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3
*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) - sqrt(1/2)*
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3
+ 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*
b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c +
48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256
*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(
-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqr
t(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a
^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^
3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c
^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6
- 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2
- 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(
a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2
)*x - 1/2*sqrt(1/2)*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c
+ 128*a^4*b^2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
```

```
^2 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c
^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3))) + 2*b*x)/((b^2*
c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1970 vs. 2(180) = 360.
time = 7.96, size = 1970, normalized size = 8.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c)) + 1/8*(4*b^6*c^2 -
 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*
c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)
^2 + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c - 2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 32*a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c
^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3
- 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/
((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^
```

```
4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4
*a*c)*abs(c)) - 1/8*(4*b^6*c^2 - 32*a*b^4*c^3 + 64*a^2*b^2*c^4 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*(b^2 - 4*a*c)*b^4*c^2 + 16*(b^2 - 4*a*
c)*a*b^2*c^3 - (2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*
(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2 - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*
c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*abs(b^2 - 4*a*c))*arctan(2*sqrt(1/2)*x/
sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c -
 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3
*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b
^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c))
```

Mupad [B]
time = 3.36, size = 2500, normalized size = 11.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] atan(((((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(((-(4*a*c - b^2)^9)^(1
/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 +
 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*
b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)
^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*
b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
```

b^2*c^5)))^(1/2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2
*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(((-(4*
a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^
3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*
b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(
((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*
b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 128
0*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (x*(4*a*c^4
- 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) -
b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*
a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*1i)/((4*a*c^4 + 3*b^2*c^3)/(2*(b^6 - 64*a^
3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a
^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)) + (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^
2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*(8
*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*
b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240
*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1
/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c
- b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(
32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2) + (((8*b^7*c^2 - 96*a*b^
5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*
c^2 - 12*a*b^4*c)) - (x*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 +
96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*
c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^
5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4))/(b
^4 + 16*a^2*c^2 - 8*a*b^2*c))*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*
c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*
b^2*c^5)))^(1/2) + (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c)
)*(((-(4*a*c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a
^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 -
1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)))*(((-(4*a*
c - b^2)^9)^(1/2) - b^9 + 768*a^4*b*c^4 + 96*a^2*b^5*c^2 - 512*a^3*b^3*c^3)
/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^
6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/2)*2i + atan(((((8*b^7*c^
2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*c^3 +
48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 614

```
4*a^6*b^2*c^5)))^(1/2)*(8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*
b^3*c^4))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2)
 - 768*a^4*b*c^4 - 96*a^2*b^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7
*c^6 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^
4 - 6144*a^6*b^2*c^5)))^(1/2) - (x*(4*a*c^4 - 5*b^2*c^3))/(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))*(-(b^9 + (-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^4 - 96*a^2*b
^5*c^2 + 512*a^3*b^3*c^3)/(32*(a*b^12 + 4096*a^7*c^6 - 24*a^2*b^10*c + 240*
a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5)))^(1/
2)*1i - (((8*b^7*c^2 - 96*a*b^5*c^3 - 512*a^3*b*c^5 + 384*a^2*b^3*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)...
```

3.97 $\int \frac{x^3}{(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(-2*c*x^2-b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*c*arctanh((2*c*x^2+b)/(-4*a
*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1121, 628, 632, 212}

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*(b + 2*c*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right) \\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{c\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^2\right)}{b^2-4ac} \\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{(2c)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^2\right)}{b^2-4ac} \\
&= -\frac{b+2cx^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 79, normalized size = 1.07

$$
-\frac{\frac{b+2cx^2}{a+bx^2+cx^4}+\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] -1/2*((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/Sqrt[-b
^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(b^2 - 4*a*c)
```

Maple [A]
time = 0.04, size = 75, normalized size = 1.01

method	result	size
default	$\frac{2c\,x^{2}+b}{2(4ac-b^{2})(c\,x^{4}+b\,x^{2}+a)}+\frac{2c\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$	75
risch	$\frac{\frac{c\,x^{2}}{4ac-b^{2}}+\frac{b}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{c\ln\left(\left((-4ac+b^{2})^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8a^{2}c-2a\,b^{2}\right)}{(-4ac+b^{2})^{\frac{3}{2}}}-\frac{c\ln\left(\left((-4ac+b^{2})^{\frac{3}{2}}-4abc+b^{3}\right)x^{2}-8a^{2}c+2a\,b^{2}\right)}{(-4ac+b^{2})^{\frac{3}{2}}}$	151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -2*c*integrate(x/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c) - 1/2*(2*c*x^2 + b)/
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(68) = 136$.
time = 0.37, size = 361, normalized size = 4.88

$$\left[-\frac{b^{3}-4\,abc+2\,(b^{2}c-4\,ac^{2})x^{2}+2\,(c^{2}x^{4}+bcx^{2}+ac)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac-(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})},\ -\frac{b^{3}-4\,abc+2\,(b^{2}c-4\,ac^{2})x^{2}-4\,(c^{2}x^{4}+bcx^{2}+ac)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)}{2\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + 2*(c^2*x^4 + b*c*x^2 + a*c
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^
2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*x^2), -1/2*(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - 4*(c^2*x^4 + b*c*x
^2 + a*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(66) = 132$.
time = 0.79, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)+c\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2c^3\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^4c\sqrt{-\frac{1}{(4ac-b^2)^3}}+bc}{2c^2}\right)+\frac{b+2cx^2}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(4*a*c - b*
*2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b**4*c*sqrt(-1/(4*a*c
- b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (16*
a**2*c**3*sqrt(-1/(4*a*c - b**2)**3) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)
**3) + b**4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/
(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3)
)
```

Giac [A]
time = 7.91, size = 82, normalized size = 1.11

$$-\frac{2\,c\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{2\,cx^2+b}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -2*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(-b^2 + 4*
a*c)) - 1/2*(2*c*x^2 + b)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 2.16, size = 172, normalized size = 2.32

$$\frac{\frac{b}{2\,(4\,a\,c-b^2)}+\frac{c\,x^2}{4\,a\,c-b^2}}{c\,x^4+b\,x^2+a}-\frac{2\,c\,\mathrm{atan}\left(\frac{b^3-4\,a\,b\,c}{(4\,a\,c-b^2)^{3/2}}-\frac{x^2\,(4\,a\,c-b^2)^4\left(\frac{4\,c^4}{a\,(4\,a\,c-b^2)^{7/2}}+\frac{4\,c^2\,(b^3\,c^2-4\,a\,b\,c^3)\,(b^3-4\,a\,b\,c)}{a\,(4\,a\,c-b^2)^{13/2}}\right)}{8\,c^4}\right)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] (b/(2*(4*a*c - b^2)) + (c*x^2)/(4*a*c - b^2))/(a + b*x^2 + c*x^4) - (2*c*at
an((b^3 - 4*a*b*c)/(4*a*c - b^2)^(3/2) - (x^2*(4*a*c - b^2)^4*((4*c^4)/(a*(
4*a*c - b^2)^(7/2)) + (4*c^2*(b^3*c^2 - 4*a*b*c^3)*(b^3 - 4*a*b*c))/(a*(4*a
*c - b^2)^(13/2))))/(8*c^4)))/(4*a*c - b^2)^(3/2)
```

3.98 $\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x(b^2-2ac+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(b^2-12a\right.}{2\sqrt{}}$$

```
[Out] 1/2*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1599, 1106, 1180, 211}

$$\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(-2ac+b^2+bcx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt
[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1106

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x)*(b^2
- 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))
), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b
^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x^2}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{b^2-2ac-2\left(b^2-4ac\right)-bcx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(c\left(b^2-12ac-b\sqrt{b^2-4ac}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4a}}}{4a\left(b^2-4ac\right)^{3/2}}$$

$$= \frac{x\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{b-}}\right.}{2\sqrt{2}\,a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4a}}}$$

Mathematica [A]
time = 0.28, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(b^2-2ac+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.07, size = 320, normalized size = 1.27

method	result
risch	$\frac{-\frac{b c x^3}{2a(4ac-b^2)}+\frac{(2ac-b^2)x}{2a(4ac-b^2)}}{c x^4+b x^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2 b+a)}\frac{\left(-\frac{bc\_R^2}{4ac-b^2}+\frac{6ac-b^2}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_R b}}{4a}$
default	$16c^2\left(-\frac{\frac{\left(b\sqrt{-4ac+b^2}+4ac-b^2\right)x}{16ac^2\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}+\frac{\left(b\sqrt{-4ac+b^2}+12ac-b^2\right)\sqrt{2}\arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{16ac\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}}{4\sqrt{-4ac+b^2}\,(4ac-b^2)}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*(1/16/a/c^2*(b*(-4*a*c+b^2)^(1/
2)+4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+1/16*(b*(-4*a*c+b^2)
^(1/2)+12*a*c-b^2)/a/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^
2)*(-1/16/a/c^2*(4*a*c-b^2-b*(-4*a*c+b^2)^(1/2))*x/(x^2+1/2*b/c-1/2/c*(-4*a
*c+b^2)^(1/2))-1/16*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a/c*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^
3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2 + b^2 - 6*a*c)/(c*x
^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2309 vs. $2(206) = 412$.
time = 0.43, size = 2309, normalized size = 9.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6
- 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^
3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 67
2*a^3*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 4
48*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b
^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c
+ 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqr
t((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
- 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*
x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 8
64*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 +
512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 +
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2
*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x + 1/2*sqrt(1/2)
*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3
*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sq
```

```
rt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/
(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c +
 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt
((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log(
(5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*
c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7
*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^
2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*
sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7
*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^
2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2
*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. 2(206) = 412.
time = 13.39, size = 2682, normalized size = 10.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*x^3 + b^2*x - 2*a*c*x)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1
/16*(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(
```

b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4 + (2*b^
3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c
)*b*c^2)*(a*b^2 - 4*a^2*c)^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b
^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c
)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))*arctan(2*sq
rt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2
 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*
a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 6
4*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)
abs(c)) - 1/16(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5
*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7
+ 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 -
 4*a*c)*a^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b
*c^4 + (2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*
(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4

```
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c + 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*abs(a*b^2 - 4*a^2*c))
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c))
```

Mupad [B]
time = 3.85, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] ((x*(2*a*c - b^2))/(2*a*(4*a*c - b^2)) - (b*c*x^3)/(2*a*(4*a*c - b^2)))/(a
+ b*x^2 + c*x^4) + atan(((((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 +
 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4
*c + 48*a^4*b^2*c^2)) - (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^
5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*
c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^
2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4
*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 38
40*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*
b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c
- b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 384
0*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12
+ 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*
a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*a^5*c^6 + 16*a*b^8*c^2
 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*
a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2
)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4
*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 409
6*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b
^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a
^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-
(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1
504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^
```

(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 128
0*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c
^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(
b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 15
04*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(
1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280
*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i)/((((6144*a^
5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4 - 5632*a^4*b^2*c^
5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(b^11
+ b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^
3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 - 16
*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^
(1/2) + (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*b^4 + 16*a^4*c^2
- 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 +
288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 24
0*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(
1/2) + (((6144*a^5*c^6 + 16*a*b^8*c^2 - 288*a^2*b^6*c^3 + 1920*a^3*b^4*c^4
- 5632*a^4*b^2*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^
2)) + (x*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) - 3840*a^5*b*c^5 + 288*a^2*
b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27*a*b^9*c - 9*a*c*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^
8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
 3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 2
7*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5)))^(1/2) - (x*(72*a^2*c^5 + b^4*c^3 - 14*a*b^2*c^4))/(2*(a^2*
b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(b^11 + b^2*(-(4*a*c - b^2)^9)^(1/2) -
3840*a^5*b*c^5 + 288*a^2*b^7*c^2 - 1504*a^3*b^5*c^3 + 3840*a^4*b^3*c^4 - 27
*a*b^9*c - 9*a*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^3*b^12 + 4096*a^9*c^6 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144
*a^8*b^2*c^5)))^(1/2) + (5*b^3*c^4 - 36*a*b*c^5...

3.99 $\int \frac{x}{(ax+bx^3+cx^5)^2} \, dx$

Optimal. Leaf size=122

$$\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] 1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*b*(-6*a*c+b^2)*a
rctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+ln(x)/a^2-1/4
*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A]
time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {1599, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (b*(b^2 -
 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{x}{\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-b^2+4ac-bcx}{x(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-b^2+4ac}{ax} + \frac{b(b^2-5ac)+c(b^2-4ac)x}{a(a+bx+cx^2)}\right)\,dx, x, x^2 \cdots}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b(b^2-5ac)+c(b^2-4ac)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2} - \frac{(b(b^2-6 \cdots}{\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\log(x)}{a^2} - \frac{\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{(b(b^2-6ac))\,\text{Sub}\cdots}{2\cdots}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}} + \frac{\log(x)}{a^2} - \frac{\text{lo}\cdots}{\cdots}$$

Mathematica [A]

time = 0.22, size = 207, normalized size = 1.70

$$\frac{\frac{2a(b^2-2ac+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)} + 4\log(x) - \frac{\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3 + c*x^5)^2,x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*Log[
x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((b^3 - 6*a*b*c -
b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A]

time = 0.05, size = 185, normalized size = 1.52

method	result

default	$-\frac{\frac{\frac{abc\,x^2}{4ac-b^2}-\frac{a(2ac-b^2)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{(4c^2a-b^2c)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(5abc-b^3-\frac{(4c^2a-b^2c)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}+\frac{\ln(x)}{a^2}$
risch	$\frac{-\frac{bc\,x^2}{2a(4ac-b^2)}+\frac{2ac-b^2}{2a(4ac-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left((64a^5c^3-48a^4b^2c^2+12a^3b^4c-b^6a^2)\_Z^2+(64a^3c^3-48a^2b^2c^2+12a\,b^4c-b^6)\_Z+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)

[Out] -1/2/a^2*((a*b*c/(4*a*c-b^2)*x^2-a*(2*a*c-b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)
+1/(4*a*c-b^2)*(1/2*(4*a*c^2-b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(5*a*b*c-b^3-1/2*
(4*a*c^2-b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)
)))+ln(x)/a^2

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")

[Out] 1/2*(b*c*x^2 + b^2 - 2*a*c)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
+ (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-((b^2*c - 4*a*c^2)*x^3 + (b^3 - 5*a
*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + log(x)/a^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(112) = 224.
time = 0.44, size = 813, normalized size = 6.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")

[Out] [1/4*(2*a*b^4 - 12*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 +
((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt
(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt
(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^
2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(x

```
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^
4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*a*b^4 - 12
*a^2*b^2*c + 16*a^3*c^2 + 2*(a*b^3*c - 4*a^2*b*c^2)*x^2 + 2*((b^3*c - 6*a*b
*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b^2*c)*x^2)*sqrt(-b^2 + 4*a*c)*a
rctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^
2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*
a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)
]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 8.45, size = 166, normalized size = 1.36

$$-\frac{(b^3-6\,abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^2b^2-4\,a^3c)\sqrt{-b^2+4\,ac}}+\frac{b^2cx^4-4\,ac^2x^4+b^3x^2-2\,abcx^2+3\,ab^2-8\,a^2c}{4\,(cx^4+bx^2+a)(a^2b^2-4\,a^3c)}-\frac{\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3 - 6*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^2*b^2 - 4
*a^3*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(b^2*c*x^4 - 4*a*c^2*x^4 + b^3*x^2 - 2*a*
b*c*x^2 + 3*a*b^2 - 8*a^2*c)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c)) - 1/
4*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Mupad [B]
time = 6.31, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] log(x)/a^2 + ((2*a*c - b^2)/(2*a*(4*a*c - b^2)) - (b*c*x^2)/(2*a*(4*a*c - b
^2)))/(a + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6 - 128*a^3*c^3 +
```

96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*atan((x^2*((((((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (b*((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^3*(6*a*c - b^2)^3*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(3*b^6 - 40*a^3*c^3 + 69*a^2*b^2*c^2 - 27*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)) + (3*b*(b^4 + 11*a^2*c^2 - 7*a*b^2*c)*(((((6*a*b^5*c^4 + 80*a^3*b*c^6 - 44*a^2*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (b*(6*a*c - b^2)*((b*((320*a^5*b*c^6 - 2*a^2*b^7*c^3 + 36*a^3*b^5*c^4 - 192*a^4*b^3*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (b*(6*a*c - b^2)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 10

```
56*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 6
4*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^
3*b^4*c + 192*a^4*b^2*c^2))))/(4*a^2*(4*a*c - b^2)^(3/2)) + (b^2*(6*a*c - b
^2)^2*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4*c)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))))/(8*a^3*c^2
*(4*a*c - b^2)^3*(6*b^6 - 400*a^3*c^3 + 291*a^2*b^2*c^2 - 72*a*b^4*c)))*(16
*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b
^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2)))/(b^6*c^2 -
 12*a*b^4*c^3 + 36*a^2*b^2*c^4) + (((b*((4*a*b^4*c^3 - 17*a^2*b^2*c^4)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) - (((4*a^2*b^6*c^2 - 36*a^3*b^4*c^3 + 80*a
^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5
*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a*b^4
*c))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6 - 128*a^3*c^3 + 96*a^2*b^2*c^2 - 24*a
*b^4*c))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(6
*a*c - b^2))/(4*a^2*(4*a*c - b^2)^(3/2)) - (((b*((4*a^2*b^6*c^2 - 36*a^3*b^
4*c^3 + 80*a^4*b^2*c^4)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6 -...
```

3.100 $\int \frac{1}{(ax+bx^3+cx^5)^2} \, dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2(b^2-4ac)x}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)}-\frac{\sqrt{c}\left(3b^3-16abc+(3b^2-10ac)\sqrt{b^2-4ac}\right)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4a}}}$$

```
[Out] 1/2*(10*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2
)/x/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c
+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-10*a*c+3*b^2)*(-
4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]
time = 0.93, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1608, 1135, 1295, 1180, 211}

$$-\frac{\sqrt{c}\left((3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-(3b^2-10ac)\sqrt{b^2-4ac}-16abc+3b^3\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] -1/2*(3*b^2 - 10*a*c)/(a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*
(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 - 16*a*b*c + (3*b^2
- 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
+ (Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
```

```
1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{1}{(ax+bx^3+cx^5)^2}\,dx = \int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx$$
$$= \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\int \frac{-3b^2+10ac-3bcx^2}{x^2(a+bx^2+cx^4)}\,dx}{2a(b^2-4ac)}$$
$$= -\frac{3b^2-10ac}{2a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} + \frac{\int \frac{-b(3b^2-13ac)-c(3b^2-10ac)x^{2}}{a+bx^2+cx^4}}{2a^2(b^2-4ac)}$$
$$= -\frac{3b^2-10ac}{2a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\left(c\left(3b^2-10ac+\frac{3b^3}{\sqrt{b^2-}}\right.\right.}{}$$
$$= -\frac{3b^2-10ac}{2a^2(b^2-4ac)x} + \frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x(a+bx^2+cx^4)} - \frac{\sqrt{c}\left(3b^2-10ac+\frac{3}{\sqrt{b^2-}}\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.40, size = 302, normalized size = 0.98

$$\frac{-\frac{4}{x}-\frac{2x(b^3-3abc+b^2cx^2-2ac^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(-2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4
*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2
]*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a
*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [A]
time = 0.06, size = 294, normalized size = 0.95

method	result

default	$-\frac{\frac{c(2ac-b^2)x^3}{8ac-2b^2}+\frac{b(3ac-b^2)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(10ac\sqrt{-4ac+b^2}-3b^2\sqrt{-4ac+b^2}+16abc-3b^3\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{(-b+\sqrt{-4ac}}}\right.}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{a^2}$
risch	$\frac{-\frac{c(10ac-3b^2)x^4}{2a^2(4ac-b^2)}-\frac{b(11ac-3b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{a}}{x(c\,x^4+b\,x^2+a)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4096a^{11}c^6-6144a^{10}b^2c^5+3840a^9b^4c^4-1280a^8b^6c^3+240a^7b^8c^2-24a^6b^{10}c+a^5b^{12}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*c*(2*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*b*(3*a*c-b^2)/(4*a*c-b^2)*x)
/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4
*a*c+b^2)^(1/2)+16*a*b*c-3*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/
8*(10*a*c*(-4*a*c+b^2)^(1/2)-3*b^2*(-4*a*c+b^2)^(1/2)-16*a*b*c+3*b^3)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))-1/a^2/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*a*b*c)*x^2
)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a
^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c + (3*b^2*c - 10*a*c^2)*x^2)/(c*
x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2912 vs. $2(260) = 520$.
time = 0.56, size = 2912, normalized size = 9.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3 - 11*a*b*c
)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
+ (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^
6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4
+ 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^
5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*
a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c
^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b
^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^
5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7
*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sq
rt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 144
08*a^4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*
c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*
c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/
2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*
a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b
^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^
7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*
c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10
 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1
280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^
3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c
```

```
+ 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 -
4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 -
1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
- 5200*a^5*b*c^5 + (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*
b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3
*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2
)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3087 vs. 2(260) = 520.
time = 7.11, size = 3087, normalized size = 10.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*x^4 - 10*a*c^2*x^4 + 3*b^3*x^2 - 11*a*b*c*x^2 + 2*a*b^2 - 8*a
^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*(6*a^4*b^8*c^2 - 8
0*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
```

t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 5
6*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + (6*b^4*c^2 -
44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^
2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*
a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2 + 2*(3*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c
- 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 5
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 -
304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 41
6*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 10
4*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a^3*c))*arctan(2*sqrt(1/2)*x/sqr
t((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*
c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b
^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8
*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*ab
s(c)) + 1/16*(6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^
2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8
 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c +
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 17
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 5
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 25
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 1
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 +
28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a
*c)*a^6*b^2*c^4 + (6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*

```
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*
b^2 - 4*a^3*c)^2 - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 3
7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*
b^5*c^2 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50
*(b^2 - 4*a*c)*a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*abs(a^2*b^2 - 4*a
^3*c))*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a...
```

Mupad [B]
time = 2.64, size = 2500, normalized size = 8.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3 + c*x^5)^2,x)
```

```
[Out] - atan((((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 207
7*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(85
1968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4
 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 + x*(-(
9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^
2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*
a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b
*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680
*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x*(204800*a^
12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*
a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*
b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b
^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i - ((-(9*b^13 - 9*b^4*(-(4*
```

a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^
3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 +
4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c
^2 - 4672*a^9*b^11*c^3 + 47360*a^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*
a^12*b^5*c^6 - 1261568*a^13*b^3*c^7 - x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9
)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^
4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a
*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*
a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*
c^6 - 1572864*a^15*b^3*c^7)) - x*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264
*a^7*b^10*c^4 + 30112*a^8*b^8*c^5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^
7 - 458752*a^11*b^2*c^8))*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2688
0*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44
800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a
*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^1
0*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5)))^(1/2)*1i)/(((-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*
b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^
5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))
)^(1/2)*(851968*a^14*b*c^8 + 192*a^8*b^13*c^2 - 4672*a^9*b^11*c^3 + 47360*a
^10*b^9*c^4 - 256000*a^11*b^7*c^5 + 778240*a^12*b^5*c^6 - 1261568*a^13*b^3*
c^7 + x*(-(9*b^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077
*a^2*b^9*c^2 - 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 -
25*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c -
b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*
c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(104
8576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c
^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)) + x
*(204800*a^12*c^9 + 144*a^6*b^12*c^3 - 3264*a^7*b^10*c^4 + 30112*a^8*b^8*c^
5 - 143360*a^9*b^6*c^6 + 365568*a^10*b^4*c^7 - 458752*a^11*b^2*c^8))*(-(9*b
^13 - 9*b^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 -
 10656*a^3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))
/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) + ((-(9*b^13 - 9*b
^4*(-(4*a*c - b^2)^9)^(1/2) + 26880*a^6*b*c^6 + 2077*a^2*b^9*c^2 - 10656*a^
3*b^7*c^3 + 30240*a^4*b^5*c^4 - 44800*a^5*b^3*c^5 - 25*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c + 51*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3

```
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)...
```

3.101 $\int \frac{1}{x(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=162

$$-\frac{b^2-3ac}{a^2(b^2-4ac)x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^2(a+bx^2+cx^4)}-\frac{(b^4-6ab^2c+6a^2c^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}-\frac{2b\log(x}{a^3}$$

```
[Out] (3*a*c-b^2)/a^2/(-4*a*c+b^2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^2
/(c*x^4+b*x^2+a)-(6*a^2*c^2-6*a*b^2*c+b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-2*b*ln(x)/a^3+1/2*b*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A]
time = 0.17, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1599, 1128, 754, 814, 648, 632, 212, 642}

$$\frac{b\log(a+bx^2+cx^4)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2(b^2-4ac)}-\frac{(6a^2c^2-6ab^2c+b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^
2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c + 6*a^2*c^2)*ArcTan
h[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2 - 4*a*c)^(3/2)) - (2*b*Log[x]
)/a^3 + (b*Log[a + b*x^2 + c*x^4])/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x\left(ax+bx^3+cx^5\right)^2}\,dx = \int \frac{1}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2(b^2-3ac)-2bcx}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)}{2a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\Big(\int \Big(\frac{2(-b^2+3ac)}{ax^2} - \frac{2b(-b^2+4ac)}{a^2x} + \frac{2(-b^4+5}{}}{2a\left(b^2-4ac\right.}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} - \frac{\text{Subst}\Big(\int}{}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\text{Subst}\Big(\int}{}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+}{2a}$$

$$= -\frac{b^2-3ac}{a^2\left(b^2-4ac\right)x^2} + \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\left(b^4-6ab^2c+6a^2c^2\right)\text{ta}}{a^3\left(b^2-\right.}$$

Mathematica [A]
time = 0.18, size = 248, normalized size = 1.53

$$\frac{-\frac{a}{x^2} - \frac{a\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)}{(b^2-4ac)(a+bx^2+cx^4)} - 4b\log(x) + \frac{\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 -
4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(
b^2 - 4*a*c)^(3/2))/(2*a^3)
```

Maple [A]
time = 0.06, size = 213, normalized size = 1.31

method	result

default	$-\frac{\frac{\frac{ac(2ac-b^2)x^2}{4ac-b^2}+\frac{ab(3ac-b^2)}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\frac{(-4abc^2+b^3c)\ln(cx^4+bx^2+a)}{c}+\frac{4\left(3a^2c^2-5ab^2c+b^4-\frac{(-4abc^2+b^3c)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^3}-$
risch	$\frac{-\frac{c(3ac-b^2)x^4}{a^2(4ac-b^2)}-\frac{b(7ac-2b^2)x^2}{2(4ac-b^2)a^2}-\frac{1}{2a}}{x^2(cx^4+bx^2+a)}-\frac{2b\ln(x)}{a^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((64a^6c^3-48b^2a^5c^2+12a^4b^4c-a^3b^6)\_Z^2+(-64a^3bc^3+48a^2b^3c^2-\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*((a*c*(2*a*c-b^2)/(4*a*c-b^2)*x^2+a*b*(3*a*c-b^2)/(4*a*c-b^2))/(c*
x^4+b*x^2+a)+2/(4*a*c-b^2)*(1/2*(-4*a*b*c^2+b^3*c)/c*ln(c*x^4+b*x^2+a)+2*(3
*a^2*c^2-5*a*b^2*c+b^4-1/2*(-4*a*b*c^2+b^3*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan
((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b^2*c - 3*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (2*b^3 - 7*a*b*c)*x^2)/((
a^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c
)*x^2) - 2*integrate(-((b^3*c - 4*a*b*c^2)*x^3 + (b^4 - 5*a*b^2*c + 3*a^2*c
^2)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c) - 2*b*log(x)/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(154) = 308$.
time = 0.52, size = 1007, normalized size = 6.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12
*a^3*c^3)*x^4 + (2*a*b^5 - 15*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + ((b^4*c - 6*a
*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 -
6*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b
^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)
```

```
*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*b^2*
c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*b^4*c -
8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^
4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2), -1/2*(a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + 2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*x^4 + (2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*x^2 + 2*((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 +
 (b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^
2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - ((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*a^2*
b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2
+ a) + 4*((b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + (b^6 - 8*a*b^4*c + 16*
a^2*b^2*c^2)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(x))/((a^3*
b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b
*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 8.71, size = 182, normalized size = 1.12

$$\frac{(b^4-6\,ab^2c+6\,a^2c^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3b^2-4\,a^4c)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cx^4-6\,ac^2x^4+2\,b^3x^2-7\,abcx^2+ab^2-4\,a^2c}{2\,(cx^6+bx^4+ax^2)(a^2b^2-4\,a^3c)}+\frac{b\log\left(cx^4+bx^2+a\right)}{2\,a^3}-\frac{b\log\left(x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] (b^4 - 6*a*b^2*c + 6*a^2*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^
3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*b^2*c*x^4 - 6*a*c^2*x^4 + 2*b
^3*x^2 - 7*a*b*c*x^2 + a*b^2 - 4*a^2*c)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 -
 4*a^3*c)) + 1/2*b*log(c*x^4 + b*x^2 + a)/a^3 - b*log(x^2)/a^3
```

Mupad [**B**]
time = 6.77, size = 2500, normalized size = 15.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3 + c*x^5)^2),x)

[Out] (log(a + b*x^2 + c*x^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (1/(2*a) - (x
^2*(2*b^3 - 7*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(3*a*c - b^2))/(a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) - (2*b*log(x))/a^3 + (atan(((2*a^9*b^
6*(4*a*c - b^2)^(9/2) - 128*a^12*c^3*(4*a*c - b^2)^(9/2) - 24*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 96*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(3*b^6 - 3*a^3*c^3
 + 36*a^2*b^2*c^2 - 21*a*b^4*c)*((4*(2*b^5*c^4 - 12*a*b^3*c^5 + 18*a^2*b*c^
6))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) + (((4*(9*a^5*c^6 - 4*a^2*b^6*c^3
+ 29*a^3*b^4*c^4 - 54*a^4*b^2*c^5))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) -
(((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c^4))/(a^6
*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9
*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^6*b^4 + 1
6*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*
c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^
3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2)) + (((((4*(24*a^7*b*c^5 - 2*a^4*b^7*c^2 + 18*a^5*b^5*c^3 - 46*a^6*b^3*c
^4))/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (2*(a^7*b^6*c^2 - 8*a^8*b^4*c^3
 + 16*a^9*b^2*c^4)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/((a^
6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2))
- ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(a^3*(4*a*c - b^2)
^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*
b^4*c + 48*a^5*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^
2)^(3/2)) - ((a^7*b^6*c^2 - 8*a^8*b^4*c^3 + 16*a^9*b^2*c^4)*(b^4 + 6*a^2*c^
2 - 6*a*b^2*c)^2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*a^6
*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(a^3*b^6 - 64*a^6*c^3
 - 12*a^4*b^4*c + 48*a^5*b^2*c^2))))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4
- 6*b^8 - 288*a^2*b^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)*(36*a^4*c^6 + b^8
*c^2 - 12*a*b^6*c^3 + 48*a^2*b^4*c^4 - 72*a^3*b^2*c^5)) - (x^2*((((4*(54*a^
3*c^8 - 2*b^6*c^5 + 18*a*b^4*c^6 - 54*a^2*b^2*c^7))/(a^6*b^6 - 64*a^9*c^3 -
 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (((4*(276*a^5*b*c^7 - 6*a^2*b^7*c^4 + 65*
a^3*b^5*c^5 - 233*a^4*b^3*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2) - (((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*
c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c
^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6
 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a
^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3
- 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 1
2*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(b^
7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c))/(2*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)) - (((((4*(480*a^8*c^7 - a^4*b^8*c^3 + 6*a^
```

```
5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9*c^3 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a
*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4
- 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2
- 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)*
(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*
b^9*c^2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(a^3*(4*a*c
- b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^
6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 + 6*a^2*c^2 - 6*a*b^
2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) + ((b^4 + 6*a^2*c^2 - 6*a*b^2*c)^2*(b^7 -
 64*a^3*b*c^3 + 48*a^2*b^3*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^
2 - 46*a^7*b^7*c^3 + 264*a^8*b^5*c^4 - 672*a^9*b^3*c^5))/(2*a^6*(4*a*c - b^
2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(a^6*b^6 - 64*a
^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(3*b^6 - 3*a^3*c^3 + 36*a^2*b^2*c
^2 - 21*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(9*a^4*c^4 - 6*b^8 - 288*a^2*b
^4*c^2 + 382*a^3*b^2*c^3 + 72*a*b^6*c)) - (b*((((((4*(480*a^8*c^7 - a^4*b^8
*c^3 + 6*a^5*b^6*c^4 + 30*a^6*b^4*c^5 - 272*a^7*b^2*c^6))/(a^6*b^6 - 64*a^9
*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - (2*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3
*c^2 - 12*a*b^5*c)*(640*a^10*b*c^6 + 3*a^6*b^9*c^2 - 46*a^7*b^7*c^3 + 264*a
^8*b^5*c^4 - 672*a^9*b^3*c^5))/((a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)))*(b^4 +
 6*a^2*c^2 - 6*a*b^2*c))/(2*a^3*(4*a*c - b^2)^(3/2)) - ((b^4 + 6*a^2*c^2 -
6*a*b^2*c)*(b^7 - 64*a^3*b*c^3 + 48*a^2*b^3*c^2...
```

3.102 $\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx$

Optimal. Leaf size=361

$$-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+b(5b^2\right.}{2\sqrt{2}\,a^3\left(b^2-\right.}$$

```
[Out] 1/6*(14*a*c-5*b^2)/a^2/(-4*a*c+b^2)/x^3+1/2*b*(-19*a*c+5*b^2)/a^3/(-4*a*c+b
^2)/x+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+
28*a^2*c^2+b*(-19*a*c+5*b^2)*(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4-29*a*b^2*c+28*a^2*c^2-b*(-19*a*c+5*b^2)*(-
4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A]
time = 2.08, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1599, 1135, 1295, 1180, 211}

$$\frac{b(5b^2-19ac)}{2a^3x\left(b^2-4ac\right)}-\frac{5b^2-14ac}{6a^2x^3\left(b^2-4ac\right)}+\frac{\sqrt{c}\left(28a^2c^2-29ab^2c+b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(28a^2c^2-29ab^2c-b(5b^2-19ac)\sqrt{b^2-4ac}+5b^4\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -1/6*(5*b^2 - 14*a*c)/(a^2*(b^2 - 4*a*c)*x^3) + (b*(5*b^2 - 19*a*c))/(2*a^3
*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x
^2 + c*x^4)) + (Sqrt[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + b*(5*b^2 - 19*a*
c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - b*(5*b^2 - 19*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3+cx^5\right)^2}\,dx=\int \frac{1}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\int \frac{-5b^2+14ac-5bcx^2}{x^4(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}+\frac{\int \frac{-3b(5b^2-19ac)-3c(5b^2-1}{x^2(a+bx^2+cx^4)}}{6a^2\left(b^2-4ac\right)}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\int$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\Big(c}{}$$

$$=-\frac{5b^2-14ac}{6a^2\left(b^2-4ac\right)x^3}+\frac{b(5b^2-19ac)}{2a^3\left(b^2-4ac\right)x}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}}{}$$

Mathematica [A]
time = 0.47, size = 344, normalized size = 0.95

$$\frac{-\frac{4a}{x^3}+\frac{24b}{x}+\frac{6x\left(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(-5b^4+29ab^2c-28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] ((-4*a)/x^3 + (24*b)/x + (6*x*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*
a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b
^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4 + 29
*a*b^2*c - 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A]
time = 0.08, size = 334, normalized size = 0.93

method	result

default	$-\frac{\frac{-\frac{bc\left(3ac-b^2\right)x^3}{2\left(4ac-b^2\right)}+\frac{\left(2a^2c^2-4a\,b^2c+b^4\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(5b^4-29a\,b^2c+28a^2c^2+5\sqrt{-4ac+b^2}\,b^3-19\sqrt{-4ac+b^2}\,abc\right)\sqrt{2}\,\operatorname{arctanh}\cdots}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\cdots\right.}}\cdots\right.}{\cdots}}{\cdots}$
risch	$\frac{\frac{cb\left(19ac-5b^2\right)x^6}{2\left(4ac-b^2\right)a^3}-\frac{\left(14a^2c^2-62a\,b^2c+15b^4\right)x^4}{6a^3\left(4ac-b^2\right)}+\frac{5b\,x^2}{3a^2}-\frac{1}{3a}}{x^3\left(c\,x^4+b\,x^2+a\right)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(4096a^{13}c^6-6144a^{12}b^2c^5+3840a^{11}b^4c^4-1280a^{10}b^6c^3+240a^9b\cdots\right.\right.}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((-1/2*b*c*(3*a*c-b^2)/(4*a*c-b^2)*x^3+1/2*(2*a^2*c^2-4*a*b^2*c+b^4)
/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(5*b^4-29*a*b^2*c+28*
a^2*c^2+5*(-4*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-19*(-4*a*c+b^2)^(1/2)*a*b*c+5*(-4*a*c+b^2
)^(1/2)*b^3-28*a^2*c^2+29*a*b^2*c-5*b^4)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))))-1/3/a^2/x^3+2*b/a^3/x
```

<u>Maxima</u> [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 19*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x^4
- 2*a^2*b^2 + 8*a^3*c + 10*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2
)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) - 1/2*integrat
e(-(5*b^4 - 24*a*b^2*c + 14*a^2*c^2 + (5*b^3*c - 19*a*b*c^2)*x^2)/(c*x^4 +
b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

<u>Fricas</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 3435 vs. 2(311) = 622.
time = 0.80, size = 3435, normalized size = 9.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(6*(5*b^3*c - 19*a*b*c^2)*x^6 + 2*(15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*x
^4 - 4*a^2*b^2 + 16*a^3*c + 20*(a*b^3 - 4*a^2*b*c)*x^2 + 3*sqrt(1/2)*((a^3*
b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^
3)*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260
*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((
625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^
4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
64*a^10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 504
21*a^3*b^2*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c
+ 18940*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*
b^4*c^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c +
700*a^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*
sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76
686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4
*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^
2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 4
8*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8
*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6
*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6
 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c
 - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sq
rt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*
b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b
^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4
*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^1
6*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^
10*c^3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^
3*b^2*c^7 + 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 189
40*a^2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c
^5 + 79408*a^6*b^2*c^6 - 10976*a^7*c^7 - (5*a^7*b^11 - 94*a^8*b^9*c + 700*a
^9*b^7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt(
(625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a
^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5
*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9
*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2
- 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)
/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12
*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*
a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(
25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4
 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 -
```

```
 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4
- 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2
*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^
3))*log((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2
*c^7 + 9604*a^4*c^8)*x + 1/2*sqrt(1/2)*(125*b^14 - 2425*a*b^12*c + 18940*a^
2*b^10*c^2 - 75579*a^3*b^8*c^3 + 160932*a^4*b^6*c^4 - 172990*a^5*b^4*c^5 +
79408*a^6*b^2*c^6 - 10976*a^7*c^7 + (5*a^7*b^11 - 94*a^8*b^9*c + 700*a^9*b^
7*c^2 - 2576*a^10*b^5*c^3 + 4672*a^11*b^3*c^4 - 3328*a^12*b*c^5)*sqrt((625*
b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^
4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(25*b^9 - 315*a*b^7*c + 1386*a^2*b^5*c^2
- 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*
c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250*a*b^10*c + 39525*a^2*b^8*c^2 - 836
30*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 24108*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^1
4*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*
b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))) - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c
^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(25*b^
9 - 315*a*b^7*c + 1386*a^2*b^5*c^2 - 2415*a^3*b^3*c^3 + 1260*a^4*b*c^4 - (a
^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((625*b^12 - 8250
*a*b^10*c + 39525*a^2*b^8*c^2 - 83630*a^3*b^6*c^3 + 76686*a^4*b^4*c^4 - 241
08*a^5*b^2*c^5 + 2401*a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2
- 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*l
og((1125*b^8*c^4 - 12325*a*b^6*c^5 + 43410*a^2*b^4*c^6 - 50421*a^3*b^2*c^7
+ 9604*a^4*c^8)*x - 1/2*sqrt(1/2)*(125*b^14 - 2...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3651 vs. $2(311) = 622$.
time = 7.37, size = 3651, normalized size = 10.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b*c^2*x^3 + b^4*x - 4*a*b^2*c*x + 2*a^2*c^2*x)/((a^3*b
^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*(10*a^6*b^9*c^2 - 138*a^7*b^7*c^3
```

+ 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7
c^2 + 98(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*c^4 + 224*(
b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 -
4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2 + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c -
10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 -
44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 +
224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a
^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*abs
(a^3*b^2 - 4*a^4*c))*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt(
(a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(
a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^
2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b
^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/16*(10*a^6*b^9*c^2
- 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2

```
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5 + (10*b^5*c^2 - 78*a*b^3*c^3 + 152*
a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 1
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3
*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a...
```

Mupad [B]
time = 4.91, size = 2500, normalized size = 6.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] atan((((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 636
6*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^
5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c
 - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 -
1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^1
2*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 4
68480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*
a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b
*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*
a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 61
5*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*
c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*
b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2
)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17
*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7
```

)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + ((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9 - 320*a^12*b^14*c^2 + 7936*a^13*b^12*c^3 - 82816*a^14*b^10*c^4 + 468480*a^15*b^8*c^5 - 1536000*a^16*b^6*c^6 + 2867200*a^17*b^4*c^7 - 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^13*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15*b^2*c^9))*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(320*a^12*b^14*c^2 - 917504*a^19*c^9 - 7936*a^13*b^12*c^3 + 82816*a^14*b^10*c^4 - 468480*a^15*b^8*c^5 + 1536000*a^16*b^6*c^6 - 2867200*a^17*b^4*c^7 + 2719744*a^18*b^2*c^8 + x*(-(25*b^15 - 25*b^6*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7 + 6366*a^2*b^11*c^2 - 35767*a^3*b^9*c^3 + 116928*a^4*b^7*c^4 - 219744*a^5*b^5*c^5 + 215040*a^6*b^3*c^6 + 49*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c - 246*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^

```
6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 614
4*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^
5*c^6 - 1572864*a^20*b^3*c^7)) - x*(401408*a^16*c^10 - 400*a^9*b^14*c^3 + 9
440*a^10*b^12*c^4 - 92816*a^11*b^10*c^5 + 488096*a^12*b^8*c^6 - 1458688*a^1
3*b^6*c^7 + 2401280*a^14*b^4*c^8 - 1871872*a^15...
```

3.103 $\int \frac{1}{x^3(ax+bx^3+cx^5)^2}\,dx$

Optimal. Leaf size=219

$$-\frac{3b^2-8ac}{4a^2(b^2-4ac)x^4}+\frac{b(3b^2-11ac)}{2a^3(b^2-4ac)x^2}+\frac{b^2-2ac+bcx^2}{2a(b^2-4ac)x^4(a+bx^2+cx^4)}+\frac{b(3b^4-20ab^2c+30a^2c^2)\tanh^{-1}\left(\right.}{2a^4(b^2-4ac)^{3/2}}$$

```
[Out] 1/4*(8*a*c-3*b^2)/a^2/(-4*a*c+b^2)/x^4+1/2*b*(-11*a*c+3*b^2)/a^3/(-4*a*c+b^
2)/x^2+1/2*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^4/(c*x^4+b*x^2+a)+1/2*b*(30
*a^2*c^2-20*a*b^2*c+3*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*
a*c+b^2)^(3/2)+(-2*a*c+3*b^2)*ln(x)/a^4-1/4*(-2*a*c+3*b^2)*ln(c*x^4+b*x^2+a
)/a^4
```

Rubi [A]
time = 0.23, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1599, 1128, 754, 814, 648, 632, 212, 642}

$$-\frac{(3b^2-2ac)\log(a+bx^2+cx^4)}{4a^4}+\frac{\log(x)(3b^2-2ac)}{a^4}+\frac{b(3b^2-11ac)}{2a^3x^2(b^2-4ac)}-\frac{3b^2-8ac}{4a^2x^4(b^2-4ac)}+\frac{b(30a^2c^2-20ab^2c+3b^4)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4(b^2-4ac)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^4(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] -1/4*(3*b^2 - 8*a*c)/(a^2*(b^2 - 4*a*c)*x^4) + (b*(3*b^2 - 11*a*c))/(2*a^3*
(b^2 - 4*a*c)*x^2) + (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*x^4*(a + b*
x^2 + c*x^4)) + (b*(3*b^4 - 20*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(3/2)) + ((3*b^2 - 2*a*c)*Log[x])/
a^4 - ((3*b^2 - 2*a*c)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(ax+bx^3+cx^5\right)^2}\,dx &= \int \frac{1}{x^5\left(a+bx^2+cx^4\right)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right) \\
&= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \frac{-3b^2+8ac-3bcx}{x^3(a+bx+cx^2)}\,dx,x,x^2\right)}{2a\left(b^2-4ac\right)} \\
&= \frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \left(\frac{-3b^2+8ac}{ax^3}+\frac{3b^3-11abc}{a^2x^2}+\frac{(b^2-4ac)(-3b}{a^3x}\right.\right.}{2} \\
&= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}+\frac{(3}{} \\
&= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}+\frac{(3}{} \\
&= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}+\frac{(3}{} \\
&= -\frac{3b^2-8ac}{4a^2\left(b^2-4ac\right)x^4}+\frac{b(3b^2-11ac)}{2a^3\left(b^2-4ac\right)x^2}+\frac{b^2-2ac+bcx^2}{2a\left(b^2-4ac\right)x^4\left(a+bx^2+cx^4\right)}+\frac{b(}{}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 328, normalized size = 1.50

$$
\frac{-\frac{a^2}{x^4}+\frac{4ab}{x^2}+\frac{2a(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2)}{(b^2-4ac)(a+bx^2+cx^4)}+4(3b^2-2ac)\log(x)-\frac{\left(3b^5-20ab^3c+30a^2bc^2+3b^4\sqrt{b^2-4ac}-14ab^2c\sqrt{b^2-4ac}+8a^2c^2\sqrt{b^2-4ac}\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(3b^5-20ab^3c+30a^2bc^2-3b^4\sqrt{b^2-4ac}+14ab^2c\sqrt{b^2-4ac}-8a^2c^2\sqrt{b^2-4ac}\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3 + c*x^5)^2),x]
```

```
[Out] (-(a^2/x^4) + (4*a*b)/x^2 + (2*a*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 -
 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(3*b^2 - 2*a*c)*Lo
g[x] - ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2 + 3*b^4*Sqrt[b^2 - 4*a*c] - 14*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 8*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 -
4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((3*b^5 - 20*a*b^3*c + 30*a^2*b*c^
2 - 3*b^4*Sqrt[b^2 - 4*a*c] + 14*a*b^2*c*Sqrt[b^2 - 4*a*c] - 8*a^2*c^2*Sqrt
[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(
4*a^4)
```

Maple [A]
time = 0.07, size = 263, normalized size = 1.20

method	result
default	$\frac{\frac{\frac{acb(3ac-b^2)x^2}{4ac-b^2}-\frac{a(2a^2c^2-4ab^2c+b^4)}{4ac-b^2}}{cx^4+bx^2+a}+\frac{\frac{(8a^2c^3-14ab^2c^2+3b^4c)\ln(cx^4+bx^2+a)}{2c}+\frac{2\left(19a^2bc^2-17ab^3c+3b^5-\frac{(8a^2c^3-14ab^2c^2+3b^4c)b}{2c}\right)a}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^4}$
risch	$\frac{\frac{bc(11ac-3b^2)x^6}{2a^3(4ac-b^2)}-\frac{(8a^2c^2-25ab^2c+6b^4)x^4}{4a^3(4ac-b^2)}+\frac{3bx^2}{4a^2}-\frac{1}{4a}}{x^4(cx^4+bx^2+a)}-\frac{2\ln(x)c}{a^3}+\frac{3b^2\ln(x)}{a^4}+\left(\sum_{\_R=\text{RootOf}((64a^7c^3-48a^6b^2c^2+12a^5b^4c-b^6a^4)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(c*x^5+b*x^3+a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^4*((a*c*b*(3*a*c-b^2)/(4*a*c-b^2)*x^2-a*(2*a^2*c^2-4*a*b^2*c+b^4)/(4*
a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(8*a^2*c^3-14*a*b^2*c^2+3*b^4*
c)/c*ln(c*x^4+b*x^2+a)+2*(19*a^2*b*c^2-17*a*b^3*c+3*b^5-1/2*(8*a^2*c^3-14*a
*b^2*c^2+3*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))))-1/4/a^2/x^4+(-2*a*c+3*b^2)*ln(x)/a^4+b/a^3/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*b^3*c - 11*a*b*c^2)*x^6 + (6*b^4 - 25*a*b^2*c + 8*a^2*c^2)*x^4 -
a^2*b^2 + 4*a^3*c + 3*(a*b^3 - 4*a^2*b*c)*x^2)/((a^3*b^2*c - 4*a^4*c^2)*x^8
 + (a^3*b^3 - 4*a^4*b*c)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4) - integrate(((3*b^4
*c - 14*a*b^2*c^2 + 8*a^2*c^3)*x^3 + (3*b^5 - 17*a*b^3*c + 19*a^2*b*c^2)*x)
/(c*x^4 + b*x^2 + a), x)/(a^4*b^2 - 4*a^5*c) + (3*b^2 - 2*a*c)*log(x)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(205) = 410$.
time = 0.52, size = 1242, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 23*a^2*b^3*c^2 +
 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3
```

```
)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 + ((3*b^5*c - 20*a*b^3
*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*x^6 + (3*a
*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + ((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^
7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4
*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c
 - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c +
64*a^2*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4)*log(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8
 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16
*a^7*c^2)*x^4), -1/4*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 - 2*(3*a*b^5*c - 2
3*a^2*b^3*c^2 + 44*a^3*b*c^3)*x^6 - (6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c
^2 - 32*a^4*c^3)*x^4 - 3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2 - 2*((3
*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*x^8 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^
2*c^2)*x^6 + (3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*x^4)*sqrt(-b^2 + 4*a*c
)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((3*b^6*c - 26*
a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2
*b^3*c^2 - 32*a^3*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 3
2*a^4*c^3)*x^4)*log(c*x^4 + b*x^2 + a) - 4*((3*b^6*c - 26*a*b^4*c^2 + 64*a^
2*b^2*c^3 - 32*a^3*c^4)*x^8 + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3
*b*c^3)*x^6 + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*x^4)*l
og(x))/((a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3
*c + 16*a^6*b*c^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(c*x**5+b*x**3+a*x)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 7.07, size = 274, normalized size = 1.25

$$-\frac{(3\,b^5-20\,ab^3c+30\,a^2bc^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^4b^2-4\,a^5c)\sqrt{-b^2+4\,ac}}+\frac{3\,b^4cx^4-14\,ab^2c^2x^4+8\,a^2c^3x^4+3\,b^5x^2-12\,ab^3cx^2+2\,a^2bc^2x^2+5\,ab^4-22\,a^2b^2c+12\,a^3c^2}{4\,(a^4b^2-4\,a^5c)(cx^4+bx^2+a)}-\frac{(3\,b^2-2\,ac)\log\left(cx^4+bx^2+a\right)}{4\,a^4}+\frac{(3\,b^2-2\,ac)\log\left(x^2\right)}{2\,a^4}-\frac{9\,b^2x^4-6\,acx^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(c*x^5+b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5 - 20*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*x^4 - 14*a*b
```

```
^2*c^2*x^4 + 8*a^2*c^3*x^4 + 3*b^5*x^2 - 12*a*b^3*c*x^2 + 2*a^2*b*c^2*x^2 +
 5*a*b^4 - 22*a^2*b^2*c + 12*a^3*c^2)/((a^4*b^2 - 4*a^5*c)*(c*x^4 + b*x^2 +
 a)) - 1/4*(3*b^2 - 2*a*c)*log(c*x^4 + b*x^2 + a)/a^4 + 1/2*(3*b^2 - 2*a*c)
*log(x^2)/a^4 - 1/4*(9*b^2*x^4 - 6*a*c*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B]
time = 7.47, size = 2500, normalized size = 11.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3 + c*x^5)^2),x)
```

```
[Out] (b*atan((x^2*((((((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 1
08*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c +
 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 +
 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*
c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^
4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^
2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (
b*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184
*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4
 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3
/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 -
256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*
a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 -
48*a^5*b^4*c + 192*a^6*b^2*c^2)) + (b*((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 - 6
57*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 +
 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^
3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 -
 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4
*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^
12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5
*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)
) + (b^3*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^3*(2560*a^13*b*c^6 + 12*a^9*b^9*
c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5))/(64*a^12*(
4*a*c - b^2)^(9/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2
)))*(9*b^8 + 80*a^4*c^4 + 270*a^2*b^4*c^2 - 285*a^3*b^2*c^3 - 87*a*b^6*c))/
(8*a^3*c^2*(4*a*c - b^2)^(7/2)*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 -
 7200*a^3*b^4*c^3 + 5775*a^4*b^2*c^4 - 720*a*b^8*c)) + (3*b*((27*b^9*c^5 -
297*a*b^7*c^6 + 1089*a^2*b^5*c^7 - 1331*a^3*b^3*c^8)/(a^9*b^6 - 64*a^12*c^3
 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - (((1760*a^7*b*c^8 + 54*a^3*b^9*c^4 -
```

```
657*a^4*b^7*c^5 + 2775*a^5*b^5*c^6 - 4484*a^6*b^3*c^7)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((2240*a^10*b*c^7 - 6*a^6*b^9*c^3
+ 40*a^7*b^7*c^4 + 108*a^8*b^5*c^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c
^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2
- 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^
4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a
^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^
5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a
^3*b^2*c^3 - 76*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*
a^6*b^2*c^2)))*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 7
6*a*b^6*c))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2))
+ (b*((b*((2240*a^10*b*c^7 - 6*a^6*b^9*c^3 + 40*a^7*b^7*c^4 + 108*a^8*b^5*c
^5 - 1248*a^9*b^3*c^6)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2
*c^2) - ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b
^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^
3*b^2*c^3 - 76*a*b^6*c))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^1
1*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*
b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a^4*(4*a*c - b^2)^(3/2)) - (b*(3*b^4 + 3
0*a^2*c^2 - 20*a*b^2*c)*(2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^
3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b
^4*c^2 - 576*a^3*b^2*c^3 - 76*a*b^6*c))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6
 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3
- 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c))/(4*a
^4*(4*a*c - b^2)^(3/2)) - (b^2*(3*b^4 + 30*a^2*c^2 - 20*a*b^2*c)^2*(2560*a^
13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^1
2*b^3*c^5)*(6*b^8 + 256*a^4*c^4 + 336*a^2*b^4*c^2 - 576*a^3*b^2*c^3 - 76*a*
b^6*c))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48
*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))
*(3*b^6 - 25*a^3*c^3 + 50*a^2*b^2*c^2 - 23*a*b^4*c))/(8*a^3*c^2*(4*a*c - b^
2)^3*(54*b^10 - 1600*a^5*c^5 + 3480*a^2*b^6*c^2 - 7200*a^3*b^4*c^3 + 5775*a
^4*b^2*c^4 - 720*a*b^8*c)))*(16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^
3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c
^2*(4*a*c - b^2)^(9/2)))/(9*b^10*c^2 - 120*a*b^8*c^3 + 580*a^2*b^6*c^4 - 12
00*a^3*b^4*c^5 + 900*a^4*b^2*c^6) + (((b*((36*a^3*b^8*c^3 - 309*a^4*b^6*c^4
 + 778*a^5*b^4*c^5 - 473*a^6*b^2*c^6)/(a^9*b^4 ...
```

3.104 $\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=142

$$\frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 2/3*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1944, 1155, 524}

$$\frac{2x^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^5]
)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1944

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Dist[(a*x^q + b*x^n + c*x^(2*n - q))^p/(x^(p*q)*(a + b*x^(n
 - q) + c*x^(2*(n - q)))^p), Int[x^(m + p*q)*(a + b*x^(n - q) + c*x^(2*(n -
 q)))^p, x], x] /; FreeQ[{a, b, c, m, n, p, q}, x] && EqQ[r, 2*n - q] &&  !
IntegerQ[p] && PosQ[n - q]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{x}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\cdots}}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{2x^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\cdots\right.}{3\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A]
time = 10.07, size = 170, normalized size = 1.20

$$\frac{2x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)}{3\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1
/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(3*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{c\,x^5+b\,x^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(a + bx^{2} + cx^{4}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.105 $\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx$

Optimal. Leaf size=380

$$-\frac{2(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\,(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c}+\frac{2\sqrt[4]{a}\,(b^2-3ac)\,\sqrt{x}\,\left(\sqrt{a}\right.+}{}$$

```
[Out] -2/15*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c
*x^5+b*x^3+a*x)^(1/2)+1/15*(3*c*x^2+b)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/
15*a^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-
b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a
^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/30*a^(1/4)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^
2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1933, 1967, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}\sqrt{x}\,(b^2-3ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2x^{3/2}(b^2-3ac)\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(15*c^(3/2)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b + 3*c*x^2)*Sqrt[a*x + b*x^
3 + c*x^5])/(15*c) + (2*a^(1/4)*(b^2 - 3*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^
2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(
c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a*x + b
*x^3 + c*x^5]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(7
/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int x^{3/2}\sqrt{ax+bx^3+cx^5}\,dx = \frac{\sqrt{x}\,(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\int \frac{\sqrt{x}\,(-ab-2(b^2-3ac)x^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{15c}$$

$$= \frac{\sqrt{x}\,(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{-ab-2(b^2-3ac)}{\sqrt{a+bx^2+c}}}{15c\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}\,(b+3cx^2)\sqrt{ax+bx^3+cx^5}}{15c} + \frac{\left(2\sqrt{a}\,(b^2-3ac)\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)}{15c^{3/2}\sqrt{ax+bx^3+c}}$$

$$= -\frac{2(b^2-3ac)\,x^{3/2}(a+bx^2+cx^4)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\,(b+3cx^2)\sqrt{ax+bx^3+c}}{15c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.32, size = 486, normalized size = 1.28

$$\frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b+3cx^2)(a+bx^2+cx^4)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{30c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(
-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b
^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
- 4*a*c])]))/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*
x^4)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1041 vs. 2(364) = 728.
time = 0.07, size = 1042, normalized size = 2.74

method	result

risch	$\frac{x^{\frac{3}{2}}(3cx^2+b)(cx^4+bx^2+a)}{15c\sqrt{x(cx^4+bx^2+a)}} - \frac{\left((6ac-2b^2)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x}{a}}\right.}{}$
default	$-\frac{\sqrt{x(cx^4+bx^2+a)}\left(-6\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,bc^2x^7-6\sqrt{-4ac+b^2}\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,c^2x^7-8\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/30*(x*(c*x^4+b*x^2+a))^(1/2)*(-6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b*c^2
*x^7-6*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*c^2*x^7-8*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c*x^5-8*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*b*c*x^5-6*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c*x^3-6*
(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*c*x^3-2*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2)*b^3*x^3-2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*b^2*x^3+12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*
(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1
/2))*a^2*c-3*b^2*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-
4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2
))+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(
1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*(-4*a*c+b
^2)^(1/2)-12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+
b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2
*c+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/
2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^2-2*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^2*x-2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*a*b*x)/x^(1/2)/(c*x^4+b*x^2+a)/c/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{\frac{3}{2}}\sqrt{x\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.106 $\int \sqrt{x} \sqrt{ax+bx^3+cx^5}\, dx$

Optimal. Leaf size=129

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{(b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] -1/16*(-4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x
^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/8*(2*c*x^2+b
)*(c*x^5+b*x^3+a*x)^(1/2)/c/x^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1932, 1928, 1121, 635, 212}

$$\frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}}-\frac{\sqrt{x}\,(b^2-4ac)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(8*c*Sqrt[x]) - ((b^2 - 4*a*c)*Sq
rt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x
^2 + c*x^4])])/(16*c^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \sqrt{x}\,\sqrt{ax+bx^3+cx^5}\,dx = \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{(b^2-4ac)\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{8c}$$

$$= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\int \frac{\cdots}{\sqrt{a+\cdots}}}{8c\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \cdots\right.}{16c\sqrt{ax+bx^3+c\cdots}}$$

$$= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{\left((b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \cdots\right.}{8c\sqrt{ax+bx^3+\cdots}}$$

$$= \frac{(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{8c\sqrt{x}} - \frac{(b^2-4ac)\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.}{16c^{3/2}\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A]
time = 0.18, size = 123, normalized size = 0.95

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\,(b+2cx^2)\sqrt{a+bx^2+cx^4}+(b^2-4ac)\log\left(c\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)\right)}{16c^{3/2}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
c*x^4] + (b^2 - 4*a*c)*Log[c*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])]))/(16*c^(3/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 157, normalized size = 1.22

method	result
risch	$\frac{(2c\,x^2+b)\,(c\,x^4+b\,x^2+a)\,\sqrt{x}}{8c\sqrt{x\,(c\,x^4+b\,x^2+a)}}+\frac{\left(\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)a}{4\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b^2}{16c^{\frac{3}{2}}}\right)\sqrt{c\,x^4+}}{\sqrt{x\,(c\,x^4+b\,x^2+a)}}$
default	$\frac{\sqrt{x\,(c\,x^4+b\,x^2+a)}\,\left(4c^{\frac{3}{2}}x^2\sqrt{c\,x^4+b\,x^2+a}+4\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)ac-\ln\left(\frac{2c\,x^2+2\sqrt{c\,x}}{}\right.\right.}{16c^{\frac{3}{2}}\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(x*(c*x^4+b*x^2+a))^(1/2)/c^(3/2)*(4*c^(3/2)*x^2*(c*x^4+b*x^2+a)^(1/2)
+4*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*c-ln(1/2*(
2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^2+2*b*(c*x^4+b*x^2+a)
^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x), x)
```

Fricas [A]
time = 0.35, size = 232, normalized size = 1.80

$$\left[-\frac{(b^2-4\,ac)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\,\sqrt{x}+(b^2+4\,ac)x}{x}\right)-4\,\sqrt{cx^5+bx^3+ax}\,(2\,c^2x^2+bc)\sqrt{x}}{32\,c^2x},\ \frac{(b^2-4\,ac)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\,\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)+2\,\sqrt{cx^5+bx^3+ax}\,(2\,c^2x^2+bc)\sqrt{x}}{16\,c^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5
+ b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) - 4*sqrt
(c*x^5 + b*x^3 + a*x)*(2*c^2*x^2 + b*c)*sqrt(x))/(c^2*x), 1/16*((b^2 - 4*a*
c)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*s
qrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c^2*x^
2 + b*c)*sqrt(x))/(c^2*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x}\,\sqrt{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 5.27, size = 127, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}\,b\sqrt{c}}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*log(abs(-2*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - 1/16*(b^2*lo
g(abs(-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2
*sqrt(a)*b*sqrt(c))/c^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.107 $\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$

Optimal. Leaf size=347

$$\frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}-\frac{\sqrt[4]{a}\,b\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+c}{(\sqrt{a}+\sqrt{c}}}}{3c^{3/4}\sqrt{ax}}$$

```
[Out] 1/3*b*x^(3/2)*(c*x^4+b*x^2+a)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*
x)^(1/2)+1/3*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)-1/3*a^(1/4)*b*(cos(2*arctan(c^
(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2
*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c
^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*
x^5+b*x^3+a*x)^(1/2)+1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(b+2*a^(1/2)*c^(1/
2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^5+
b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1935, 1967, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\,\sqrt{x}\,\left(2\sqrt{a}\,\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\,b\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{ax+bx^3+cx^5}}+\frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5}+\frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (b*x^(3/2)*(a + b*x^2 + c*x^4))/(3*Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5]) + (Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])/3 - (a^(1/4)*b*Sqr
t[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^
2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(3*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[c])*
Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]
*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(6*c^(3/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5} + \frac{1}{3}\int \frac{\sqrt{x}\,(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$$

$$= \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5} + \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5} + \frac{\left(\sqrt{a}\left(2\sqrt{a}+\frac{b}{\sqrt{c}}\right)\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{}{\sqrt{a+}}}{3\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{bx^{3/2}(a+bx^2+cx^4)}{3\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{1}{3}\sqrt{x}\,\sqrt{ax+bx^3+cx^5} - \frac{\sqrt[4]{a}\,b\sqrt{x}\,(}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.60, size = 452, normalized size = 1.30

$$\frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(a+bx^2+cx^4)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{12c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*(
-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2
- 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*S
qrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])]))/(12*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[
x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 508, normalized size = 1.46

method	result

risch	$\frac{x^{\frac{3}{2}}\left(c\,x^{4}+b\,x^{2}+a\right)}{3\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}}+\frac{\left(a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^{2}}\right)x^{2}}{a}}\;\mathrm{Elliptic}\ldots\right.}{6\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{c\ldots}}$
default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,c\,x^{5}+\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,bc\,x^{5}+\sqrt{\frac{-b+\sqrt{-\ldots}}{\ldots}}\ldots\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(x*(c*x^4+b*x^2+a))^(1/2)/x^(1/2)*(((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-
4*a*c+b^2)^(1/2)*c*x^5+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b*c*x^5+((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*b*x^3+((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*b^2*x^3+a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4
*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2)
)*(-4*a*c+b^2)^(1/2)+b*a*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((
x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c
)^(1/2))+((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*x+((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*a*b*x)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a+bx^2+cx^4\right)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/sqrt(x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/sqrt(x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^5+b\,x^3+a\,x}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2),x)
```

```
[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(1/2), x)
```

3.108 $\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tan}{4\sqrt{c}\sqrt{ax}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)*x^(1/2)
*(c*x^4+b*x^2+a)^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+1/4*b*arctanh(1/2*(2*c*x^2+b
)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(c*x
^5+b*x^3+a*x)^(1/2)+1/2*(c*x^5+b*x^3+a*x)^(1/2)/x^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1935, 1967, 1265, 857, 635, 212, 738}

$$\frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}}-\frac{\sqrt{a}\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] Sqrt[a*x + b*x^3 + c*x^5]/(2*Sqrt[x]) - (Sqrt[a]*Sqrt[x]*Sqrt[a + b*x^2 + c
*x^4]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a
*x + b*x^3 + c*x^5]) + (b*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*Sqrt[a*x + b*x^3 + c*
x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3+cx^5}}{x^{3/2}}\,dx = \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{1}{2}\int \frac{2a+bx^2}{\sqrt{x}\,\sqrt{ax+bx^3+cx^5}}\,dx$$

$$= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{2a+bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx}{2\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{2a+bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x\right.}{4\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} + \frac{\left(a\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x,\right.}{2\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+}}\right.}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{ax+bx^3+cx^5}}{2\sqrt{x}} - \frac{\sqrt{a}\,\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A]
time = 0.20, size = 154, normalized size = 0.79

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\,\sqrt{a+bx^2+cx^4}+4\sqrt{a}\,\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)-b\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{4\sqrt{c}\,\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3 + c*x^5]/x^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4] + 4*Sqr
t[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] - b*L
og[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(4*Sqrt[c]*Sqrt[x*(a
+ b*x^2 + c*x^4)])
```

Maple [A]
time = 0.03, size = 136, normalized size = 0.70

method	result
default	$-\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\left(2\sqrt{a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)\sqrt{c}-b\ln\left(\frac{2c\,x^2+2\sqrt{c\,x^4+b\,x^2+a}}{2\sqrt{c}}\right.\right.}{4\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(x*(c*x^4+b*x^2+a))^(1/2)*(2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)*c^(1/2)-b*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2
)+b)/c^(1/2))-2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2
)/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^5 + b*x^3 + a*x)/x^(3/2), x)
```

Fricas [A]
time = 0.39, size = 666, normalized size = 3.43

$$\left[\frac{b\sqrt{c}\,x\log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}\,(2cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4ac)x}{x}\right)+2\sqrt{a}\,cx\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)+4\sqrt{cx^5+bx^3+ax}\,c\sqrt{x}}{8cx},\ -\frac{b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2cx^2+b)\sqrt{-c}\sqrt{x}}{2(c^2x^5+bcx^3+acx)}\right)-\sqrt{a}\,cx\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)-2\sqrt{cx^5+bx^3+ax}\,c\sqrt{x}}{4cx},\ \frac{4\sqrt{-a}\,cx\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)}\right)+b\sqrt{c}\,x\log\left(-\frac{8c^2x^5+8bcx^3+4\sqrt{cx^5+bx^3+ax}\,(2cx^2+b)\sqrt{c}\sqrt{x}+(b^2+4ac)x}{x}\right)+4\sqrt{cx^5+bx^3+ax}\,c\sqrt{x}}{8cx},\ \frac{2\sqrt{-a}\,cx\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)}\right)-b\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2cx^2+b)\sqrt{-c}\sqrt{x}}{2(c^2x^5+bcx^3+acx)}\right)+2\sqrt{cx^5+bx^3+ax}\,c\sqrt{x}}{4cx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)
*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 2*sqrt(a)*c*x*log(-(
(b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^
2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*
x), -1/4*(b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*s
qrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - sqrt(a)*c*x*log(-((b^2 + 4*a
*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*s
qrt(a)*sqrt(x))/x^5) - 2*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/8*(4
*sqrt(-a)*c*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*s
qrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + b*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x
^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a
*c)*x)/x) + 4*sqrt(c*x^5 + b*x^3 + a*x)*c*sqrt(x))/(c*x), 1/4*(2*sqrt(-a)*c
*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c
*x^5 + a*b*x^3 + a^2*x)) - b*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x
)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*sqrt(c*x^
5 + b*x^3 + a*x)*c*sqrt(x))/(c*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\,(a+bx^2+cx^4)}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2 + c*x**4))/x**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(t_
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^5+b\,x^3+a\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2),x)
```

```
[Out] int((a*x + b*x^3 + c*x^5)^(1/2)/x^(3/2), x)
```

3.109 $\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx$

Optimal. Leaf size=244

$$\frac{\left(15b^4-100ab^2c+128a^2c^2\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{x^{3/2}\left(b\left(5b^2-4ac\right)+4c\left(5b^2-16ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{640c^2}+$$

```
[Out] 1/80*(8*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)*x^(1/2)/c-3/512*b*(-4*a*c+b^2)^2
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^
2+a)^(1/2)/c^(7/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/640*x^(3/2)*(b*(-4*a*c+5*b^2)+
4*c*(-16*a*c+5*b^2)*x^2)*(c*x^5+b*x^3+a*x)^(1/2)/c^2+1/1280*(128*a^2*c^2-10
0*a*b^2*c+15*b^4)*(c*x^5+b*x^3+a*x)^(1/2)/c^3/x^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {1933, 1959, 1963, 12, 1928, 1121, 635, 212}

$$\frac{\left(128a^2c^2-100ab^2c+15b^4\right)\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}}-\frac{3b\sqrt{x}\left(b^2-4ac\right)^2\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}\sqrt{ax+bx^3+cx^5}}-\frac{x^{3/2}\left(4cx^2\left(5b^2-16ac\right)+b\left(5b^2-4ac\right)\right)\sqrt{ax+bx^3+cx^5}}{640c^2}+\frac{\sqrt{x}\left(3b+8cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{80c}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*Sqrt[a*x + b*x^3 + c*x^5])/(1280*c^3*
Sqrt[x]) - (x^(3/2)*(b*(5*b^2 - 4*a*c) + 4*c*(5*b^2 - 16*a*c)*x^2)*Sqrt[a*x
 + b*x^3 + c*x^5])/(640*c^2) + (Sqrt[x]*(3*b + 8*c*x^2)*(a*x + b*x^3 + c*x^
5)^(3/2))/(80*c) - (3*b*(b^2 - 4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Arc
Tanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2)*Sqrt[
a*x + b*x^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1963

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[B*x^(m - n + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(c*(m + p*q + (n - q)*(2*p + 1) + 1))), x] -
Dist[1/(c*(m + p*q + (n - q)*(2*p + 1) + 1)), Int[x^(m - n + q)*Simp[a*B*(m
 + p*q - n + q + 1) + (b*B*(m + p*q + (n - q)*p + 1) - A*c*(m + p*q + (n -
q)*(2*p + 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x]
/; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Integ
erQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[p, -1] && LtQ[p, 0] && R
ationalQ[m, q] && GeQ[m + p*q, n - q - 1] && NeQ[m + p*q + (n - q)*(2*p + 1
) + 1, 0]
```

Rubi steps

$$\int x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}\,dx = \frac{\sqrt{x}\,(3b+8cx^2)\,(ax+bx^3+cx^5)^{3/2}}{80c} + \frac{3\int\sqrt{x}\,(-2ab-(5b^2-16ac)\,x^2)\,\sqrt{a\cdots}}{80c}$$

$$= -\frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2-16ac)\,x^2)\,\sqrt{ax+bx^3+cx^5}}{640c^2} + \frac{\sqrt{x}\,(3b+8cx\cdots}{\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

$$= \frac{(15b^4-100ab^2c+128a^2c^2)\,\sqrt{ax+bx^3+cx^5}}{1280c^3\sqrt{x}} - \frac{x^{3/2}(b(5b^2-4ac)+4c(5b^2\cdots}{6\cdots}$$

Mathematica [**A**]

time = 0.43, size = 181, normalized size = 0.74

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\left(2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\,\left(15b^4-10b^3cx^2+128c^2(a+cx^4)^2+4b^2c(-25a+2cx^4)+8bc^2x^2(7a+22cx^4)\right)+15b(b^2-4ac)^2\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{2560c^{7/2}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^4
 - 10*b^3*c*x^2 + 128*c^2*(a + c*x^4)^2 + 4*b^2*c*(-25*a + 2*c*x^4) + 8*b*c
^2*x^2*(7*a + 22*c*x^4)) + 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 - 2*Sqrt[c]
*Sqrt[a + b*x^2 + c*x^4]]))/(2560*c^(7/2)*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 369, normalized size = 1.51

method	result
risch	$\frac{(128c^4x^8+176b\,c^3x^6+256a\,x^4c^3+8b^2c^2x^4+56ab\,c^2x^2-10b^3c\,x^2+128a^2c^2-100a\,b^2c+15b^4)\,(c\,x^4+b\,x^2+a)\,\sqrt{x}}{1280c^3\sqrt{x\,(c\,x^4+b\,x^2+a)}} + \frac{\left(-\frac{3a^2b\ln\left(\frac{b}{2}+ \dots\right.}{\sqrt{\dots}}\right.}{\dots}$
default	$-\frac{\sqrt{x\,(c\,x^4+b\,x^2+a)}\,\left(-256c^{\frac{9}{2}}x^8\sqrt{c\,x^4+b\,x^2+a}-352b\,c^{\frac{7}{2}}x^6\sqrt{c\,x^4+b\,x^2+a}-512a\,c^{\frac{7}{2}}x^4\sqrt{c\,x^4+b \dots}\right.}{\dots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2560*(x*(c*x^4+b*x^2+a))^(1/2)/c^(7/2)*(-256*c^(9/2)*x^8*(c*x^4+b*x^2+a)
^(1/2)-352*b*c^(7/2)*x^6*(c*x^4+b*x^2+a)^(1/2)-512*a*c^(7/2)*x^4*(c*x^4+b*x
^2+a)^(1/2)-16*b^2*c^(5/2)*x^4*(c*x^4+b*x^2+a)^(1/2)-112*a*b*c^(5/2)*x^2*(c
*x^4+b*x^2+a)^(1/2)+20*b^3*c^(3/2)*x^2*(c*x^4+b*x^2+a)^(1/2)+240*ln(1/2*(2*
c*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a^2*b*c^2-120*ln(1/2*(2*c
*x^2+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*b^3*c+15*ln(1/2*(2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*b^5-256*a^2*c^(5/2)*(c*x^4+b*x
^2+a)^(1/2)+200*a*b^2*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-30*b^4*(c*x^4+b*x^2+a)^
(1/2)*c^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2), x)

Fricas [**A**]
time = 0.38, size = 396, normalized size = 1.62

$$\left[\frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3-4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\,\sqrt{x}+(b^2+4\,ac)x}{x}\right)+4\,(128\,c^5x^8+176\,bc^4x^6+15\,b^4c-100\,ab^2c^2+128\,a^2c^3+8\,(b^2c^3+32\,ac^4)x^4-2\,(5\,b^3c^2-28\,abc^3)x^2)\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{5120\,c^4x},\ \frac{15\,(b^5-8\,ab^3c+16\,a^2bc^2)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\,\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)+2\,(128\,c^5x^8+176\,bc^4x^6+15\,b^4c-100\,ab^2c^2+128\,a^2c^3+8\,(b^2c^3+32\,ac^4)x^4-2\,(5\,b^3c^2-28\,abc^3)x^2)\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{2560\,c^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")

[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*x^3 - 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x) + 4*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x), 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) + 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3 + 8*(b^2*c^3 + 32*a*c^4)*x^4 - 2*(5*b^3*c^2 - 28*a*b*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^4*x)]

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(3/2)*(c*x**5+b*x**3+a*x)**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4060 deep

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(210) = 420.
time = 4.06, size = 662, normalized size = 2.71

$$\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3\,b^2-8\,ac}{c^2}\right)-\frac{3\,(b^3-4\,abc)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{3\,b^3\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-12\,abc\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+6\sqrt{a}\,b^2\sqrt{c}-16\,a^{\frac{3}{2}}c^{\frac{3}{2}}}{c^{\frac{5}{2}}}\right)a+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5\,b^2c-12\,ac^2}{c^3}\right)x^2+\frac{15\,b^3-52\,abc}{c^3}\right)+\frac{3\,(5\,b^4-24\,ab^2c+16\,a^2c^2)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}-\frac{15\,b^4\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-72\,ab^2c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+48\,a^2c^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+30\sqrt{a}\,b^3\sqrt{c}-104\,a^{\frac{3}{2}}bc^{\frac{3}{2}}}{c^{\frac{7}{2}}}\right)b+\frac{1}{7680}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7\,b^2c^2-16\,ac^3}{c^4}\right)x^2+\frac{35\,b^3c-116\,abc^2}{c^4}\right)x^2-\frac{105\,b^4-460\,ab^2c+256\,a^2c^2}{c^4}\right)-\frac{15\,(7\,b^5-40\,ab^3c+48\,a^2bc^2)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{9}{2}}}+\frac{105\,b^5\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-600\,ab^3c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+720\,a^2bc^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+210\sqrt{a}\,b^4\sqrt{c}-920\,a^{\frac{3}{2}}b^2c^{\frac{3}{2}}+512\,a^{\frac{5}{2}}c^{\frac{5}{2}}}{c^{\frac{9}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3/2)*(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")

[Out] 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sqrt(c) - 16*a^(3/2)*c^(3/2)) /c^(5/2))*a + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5

```
*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c)/c^3) + 3*(5*b^4 - 24*a*b^
2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2) - (15*b^4*log(abs(-b + 2*sqrt(a)*sqrt(c))) - 72*a*b^2*c*log
(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c)))
 + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2))/c^(7/2))*b + 1/7680*(2*s
qrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^2*c^2 - 16*a*c^3)/
c^4)*x^2 + (35*b^3*c - 116*a*b*c^2)/c^4)*x^2 - (105*b^4 - 460*a*b^2*c + 256
*a^2*c^2)/c^4) - 15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*log(abs(-2*(sqrt(c)
*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2) + (105*b^5*log(abs(-b
 + 2*sqrt(a)*sqrt(c))) - 600*a*b^3*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 720
*a^2*b*c^2*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 210*sqrt(a)*b^4*sqrt(c) - 920
*a^(3/2)*b^2*c^(3/2) + 512*a^(5/2)*c^(5/2))/c^(9/2))*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3/2}\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.110 $\int \sqrt{x} \left(ax+bx^3+cx^5\right)^{3/2} dx$

Optimal. Leaf size=487

$$\frac{\left(8b^4-57ab^2c+84a^2c^2\right)x^{3/2}\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{x}\left(b\left(4b^2-9ac\right)+6c\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{(3b}{}$$

```
[Out] 1/63*(7*c*x^2+3*b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(1/2)+1/315*(84*a^2*c^2-57*a
*b^2*c+8*b^4)*x^(3/2)*(c*x^4+b*x^2+a)/c^(5/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+
b*x^3+a*x)^(1/2)-1/315*(b*(-9*a*c+4*b^2)+6*c*(-7*a*c+2*b^2)*x^2)*x^(1/2)*(c
*x^5+b*x^3+a*x)^(1/2)/c^2-1/315*a^(1/4)*(84*a^2*c^2-57*a*b^2*c+8*b^4)*(cos(
2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elli
pticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^
(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/
c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)+1/630*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b
^4-57*a*b^2*c+84*a^2*c^2+4*b*(-6*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+
b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1933, 1959, 1967, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+4\sqrt{a}\,b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{a}\sqrt{x}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{ax+bx^3+cx^5}}+\frac{x^{3/2}\left(84a^2c^2-57ab^2c+8b^4\right)\left(a+bx^2+cx^4\right)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{x}\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x^(3/2)*(a + b*x^2 + c*x^4))/(315*c^(5/2
)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (Sqrt[x]*(b*(4*b^2 -
 9*a*c) + 6*c*(2*b^2 - 7*a*c)*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(315*c^2) + (
(3*b + 7*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(63*c*Sqrt[x]) - (a^(1/4)*(8*b
^4 - 57*a*b^2*c + 84*a^2*c^2)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
 + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*Sqrt[a]*b*Sqrt[c]*(b^2 - 6
*a*c))*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sq
rt[c]))/4])/(630*c^(11/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1933

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b*(n - q)*p + c*(m + p*q + (n - q)*
(2*p - 1) + 1)*x^(n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n
 - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), x] + Dist[(n - q)*(p/(c*(m
+ p*(2*n - q) + 1)*(m + p*q + (n - q)*(2*p - 1) + 1))), Int[x^(m - (n - 2*q
))*Simp[(-a)*b*(m + p*q - n + q + 1) + (2*a*c*(m + p*q + (n - q)*(2*p - 1)
+ 1) - b^2*(m + p*q + (n - q)*(p - 1) + 1))*x^(n - q), x]*(a*x^q + b*x^n +
c*x^(2*n - q))^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] &&
 PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p
, 0] && RationalQ[m, q] && GtQ[m + p*q + 1, n - q] && NeQ[m + p*(2*n - q) +
 1, 0] && NeQ[m + p*q + (n - q)*(2*p - 1) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
```

```
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \sqrt{x}\left(ax+bx^3+cx^5\right)^{3/2}\,dx = \frac{\left(3b+7cx^2\right)\left(ax+bx^3+cx^5\right)^{3/2}}{63c\sqrt{x}}+\frac{\int \frac{\left(-ab-2\left(2b^2-7ac\right)x^2\right)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx}{21c}$$

$$= -\frac{\sqrt{x}\left(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)(a}{63}$$

$$= -\frac{\sqrt{x}\left(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)(a}{63}$$

$$= -\frac{\sqrt{x}\left(b(4b^2-9ac)+6c(2b^2-7ac)\,x^2\right)\sqrt{ax+bx^3+cx^5}}{315c^2}+\frac{\left(3b+7cx^2\right)(a}{63}$$

$$= \frac{\left(8b^4-57ab^2c+84a^2c^2\right)x^{3/2}(a+bx^2+cx^4)}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{x}\left(b(4b^2-9ac)+6c(2b^2-\right.}{31}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.45, size = 609, normalized size = 1.25

$$\frac{\sqrt{x}\left(4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(-4b^4x^2-b^3cx^4+53b^2c^2x^6+85bc^3x^8+35c^4x^{10}+a^2c(24b+77cx^2)+a(-4b^3+27b^2cx^2+151bc^2x^4+112c^3x^6)\right)+i\left(8b^4-57ab^2c+84a^2c^2\right)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-8b^5+65ab^3c-132a^2bc^2+8b^4\sqrt{b^2-4ac}-57ab^2c\sqrt{b^2-4ac}+84a^2c^2\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{1260c^3\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 + 5
3*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*x^2) + a*(-
4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) + I*(8*b^4 - 57*a*b^2*
c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 +
65*a*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqrt[b^2
- 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(1260*c^3*Sq
rt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1877 vs. $2(465) = 930$.
time = 0.05, size = 1878, normalized size = 3.86

method	result
risch	$\frac{x^{\frac{3}{2}}\left(35c^3x^6+50b\,c^2x^4+77a\,c^2x^2+3b^2c\,x^2+24abc-4b^3\right)\left(c\,x^4+b\,x^2+a\right)}{315c^2\sqrt{x\left(c\,x^4+b\,x^2+a\right)}} - \frac{\left(\left(84a^2c^2-57a\,b^2c+8b^4\right)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac}\right.}{a}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/315*(x*(c*x^4+b*x^2+a))^(1/2)*(4*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^4
*x+4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^4*x^3+((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*b^4*c*x^5-53*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3
*c^2*x^7-35*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b*c^4*x^11-85*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*b^2*c^3*x^9-35*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*c^4*x^11+12*(-4*a*c+b^2)^(1/2)*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^
2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1
/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*b*c-24*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*a^2*b*c*x-45*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2
```

```
)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)
/a/c)^(1/2))*a^2*b^2*c+57*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*(
(x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/
c)^(1/2))*a^2*b^2*c-151*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*a*b*c^2*x^5-27*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^2
*c*x^3-2*(-4*a*c+b^2)^(1/2)*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)
*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+
(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/
a/c)^(1/2))*a*b^3+84*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*
(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1
/2))*a^3*c^2+6*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*
c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a
*b^4-84*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^
(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^3*c^2-
8*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+
b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^4+4*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*b^5*x^3+4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*a*b^3*x-24*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*b^2*c*x-11
2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*c^3*x^7-53*(-4*a*c
+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c^2*x^7-112*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*a*b*c^3*x^7+(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*b^3*c*x^5-151*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^2*c^2*x^5-85
*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b*c^3*x^9-77*(-4*a*c+
b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*c^2*x^3-77*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*a^2*b*c^2*x^3-27*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^3
*c*x^3)/x^(1/2)/(c*x^4+b*x^2+a)/c^2/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-
4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Fricas [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \left(x \left(a + bx^2 + cx^4\right)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{x}\, \left(c\,x^5 + b\,x^3 + a\,x\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.111 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=177

$$-\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{(b+2cx^2)(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}}+\frac{3(b^2-4ac)^2\sqrt{x}\sqrt{a+bx^2+cx}}{256c^{5/2}\sqrt{a}}$$

[Out] 1/16*(2*c*x^2+b)*(c*x^5+b*x^3+a*x)^(3/2)/c/x^(3/2)+3/256*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(c*x^5+b*x^3+a*x)^(1/2)-3/128*(-4*a*c+b^2)*(2*c*x^2+b)*(c*x^5+b*x^3+a*x)^(1/2)/c^2/x^(1/2)

Rubi [A]
time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1932, 1928, 1121, 635, 212}

$$\frac{3\sqrt{x}\,(b^2-4ac)^2\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}\sqrt{ax+bx^3+cx^5}}-\frac{3(b^2-4ac)(b+2cx^2)\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}}+\frac{(b+2cx^2)(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a*x + b*x^3 + c*x^5])/(128*c^2*Sqrt[x]
) + ((b + 2*c*x^2)*(a*x + b*x^3 + c*x^5)^(3/2))/(16*c*x^(3/2)) + (3*(b^2 -
4*a*c)^2*Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1932

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - n + q + 1)*(b + 2*c*x^(n - q))*((a*x^q + b*x^n
+ c*x^(2*n - q))^p/(2*c*(n - q)*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*
c*(2*p + 1))), Int[x^(m + q)*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x
] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p]
 && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && Eq
Q[m + p*q + 1, n - q]
```

Rubi steps

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{\sqrt{x}}\,dx = \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} - \frac{(3(b^2-4ac))\int \sqrt{x}\,\sqrt{ax+bx^3+cx^5}\,dx}{16c}$$

$$= -\frac{3(b^2-4ac)\,(b+2cx^2)\,\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx^2)\,\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx^2)\,\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx^2)\,\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} +$$

$$= -\frac{3(b^2-4ac)\,(b+2cx^2)\,\sqrt{ax+bx^3+cx^5}}{128c^2\sqrt{x}} + \frac{(b+2cx^2)\,(ax+bx^3+cx^5)^{3/2}}{16cx^{3/2}} +$$

Mathematica [A]

time = 0.35, size = 150, normalized size = 0.85

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\left(2\sqrt{c}\,(b+2cx^2)\sqrt{a+bx^2+cx^4}\,(-3b^2+8bcx^2+4c(5a+2cx^4))-3(b^2-4ac)^2\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{256c^{5/2}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*(2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
c*x^4]*(-3*b^2 + 8*b*c*x^2 + 4*c*(5*a + 2*c*x^4)) - 3*(b^2 - 4*a*c)^2*Log[b
 + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(256*c^(5/2)*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 295, normalized size = 1.67

method	result
risch	$\frac{(16c^3x^6+24bc^2x^4+40ac^2x^2+2b^2cx^2+20abc-3b^3)(cx^4+bx^2+a)\sqrt{x}}{128c^2\sqrt{x(cx^4+bx^2+a)}}+\frac{\left(\frac{3a^2\ln\left(\frac{\frac{b}{2}+cx^2}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)}{16\sqrt{c}}-\frac{3ab^2\ln\left(\frac{b}{2}\right.}{}\right.}{}$
default	$\frac{\sqrt{x(cx^4+bx^2+a)}\left(32c^{\frac{7}{2}}x^6\sqrt{cx^4+bx^2+a}+48bc^{\frac{5}{2}}x^4\sqrt{cx^4+bx^2+a}+80ac^{\frac{5}{2}}x^2\sqrt{cx^4+bx^2+a}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/256*(x*(c*x^4+b*x^2+a))^(1/2)/c^(5/2)*(32*c^(7/2)*x^6*(c*x^4+b*x^2+a)^(1/
2)+48*b*c^(5/2)*x^4*(c*x^4+b*x^2+a)^(1/2)+80*a*c^(5/2)*x^2*(c*x^4+b*x^2+a)^
(1/2)+4*b^2*c^(3/2)*x^2*(c*x^4+b*x^2+a)^(1/2)+48*ln(1/2*(2*c*x^2+2*(c*x^4+b
*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a^2*c^2-24*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2
+a)^(1/2)*c^(1/2)+b)/c^(1/2))*a*b^2*c+3*ln(1/2*(2*c*x^2+2*(c*x^4+b*x^2+a)^(
1/2)*c^(1/2)+b)/c^(1/2))*b^4+40*a*b*c^(3/2)*(c*x^4+b*x^2+a)^(1/2)-6*b^3*c^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/sqrt(x), x)
```

Fricas [A]
time = 0.36, size = 332, normalized size = 1.88

$$\left[\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{c}\,x\log\left(-\frac{8\,c^2x^5+8\,bcx^3+4\,\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{c}\,\sqrt{x}+(b^2+4\,ac)x}{x}\right)+4\,(16\,c^4x^6+24\,bc^3x^4-3\,b^3c+20\,abc^2+2\,(b^2c^2+20\,ac^3)x^2)\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{512\,c^3x},\ -\frac{3\,(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-c}\,x\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(2\,cx^2+b)\sqrt{-c}\,\sqrt{x}}{2\,(c^2x^5+bcx^3+acx)}\right)-2\,(16\,c^4x^6+24\,bc^3x^4-3\,b^3c+20\,abc^2+2\,(b^2c^2+20\,ac^3)x^2)\sqrt{cx^5+bx^3+ax}\,\sqrt{x}}{256\,c^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(c)*x*log(-(8*c^2*x^5 + 8*b*c*
x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(c)*sqrt(x) + (b^2 + 4*
a*c)*x)/x) + 4*(16*c^4*x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c
^2 + 20*a*c^3)*x^2)*sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x), -1/256*(3*(
b^4 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-c)*x*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*
x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3 + a*c*x)) - 2*(16*c^4*
x^6 + 24*b*c^3*x^4 - 3*b^3*c + 20*a*b*c^2 + 2*(b^2*c^2 + 20*a*c^3)*x^2)*sqr
t(c*x^5 + b*x^3 + a*x)*sqrt(x))/(c^3*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x(a+bx^2+cx^4)\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
```

```
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/sqrt(x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(149) = 298.
time = 4.08, size = 518, normalized size = 2.93

$$\frac{1}{16}\left(2\sqrt{cx^4+bx^2+a}\left(2x^2+\frac{b}{c}\right)+\frac{(b^2-4ac)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}-\frac{b^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-4ac\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+2\sqrt{a}b\sqrt{c}}{c^{\frac{3}{2}}}\right)a+\frac{1}{96}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{3(b^3-4abc)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{3b^3\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-12abc\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+6\sqrt{a}b^2\sqrt{c}-16a^{\frac{3}{2}}c^{\frac{3}{2}}}{c^{\frac{5}{2}}}\right)b+\frac{1}{768}\left(2\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6x^2+\frac{b}{c}\right)x^2-\frac{5b^2c-12ac^2}{c^3}\right)x^2+\frac{15b^3-52abc}{c^3}\right)+\frac{3(5b^4-24ab^2c+16a^2c^2)\log\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{7}{2}}}-\frac{15b^4\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)-72ab^2c\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+48a^2c^2\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)+30\sqrt{a}b^3\sqrt{c}-104a^{\frac{3}{2}}bc^{\frac{3}{2}}}{c^{\frac{7}{2}}}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + (b^2 - 4*a*c)*log(abs(-2*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2) - (b^2*log(abs(
-b + 2*sqrt(a)*sqrt(c))) - 4*a*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 2*sqrt(
a)*b*sqrt(c))/c^(3/2))*a + 1/96*(2*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)
*x^2 - (3*b^2 - 8*a*c)/c^2) - 3*(b^3 - 4*a*b*c)*log(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2) + (3*b^3*log(abs(-b + 2*sqrt(
a)*sqrt(c))) - 12*a*b*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 6*sqrt(a)*b^2*sq
```

```
rt(c) - 16*a^(3/2)*c^(3/2))/c^(5/2))*b + 1/768*(2*sqrt(c*x^4 + b*x^2 + a)*(
2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c - 12*a*c^2)/c^3)*x^2 + (15*b^3 - 52*a*b*c
)/c^3) + 3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(7/2) - (15*b^4*log(abs(-b + 2*sqrt(a)
*sqrt(c))) - 72*a*b^2*c*log(abs(-b + 2*sqrt(a)*sqrt(c))) + 48*a^2*c^2*log(a
bs(-b + 2*sqrt(a)*sqrt(c))) + 30*sqrt(a)*b^3*sqrt(c) - 104*a^(3/2)*b*c^(3/2
))/c^(7/2))*c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}}{\sqrt{x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(1/2), x)
```

3.112 $\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=425

$$-\frac{2b(b^2-8ac)\,x^{3/2}(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\,(b^2+10ac+3bcx^2)\,\sqrt{ax+bx^3+cx^5}}{35c}+\frac{(ax+bx^3+cx^5)^{3}}{7\sqrt{x}}$$

```
[Out] 1/7*(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2)-2/35*b*(-8*a*c+b^2)*x^(3/2)*(c*x^4+b*x^
2+a)/c^(3/2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+1/35*(3*b*c*x^2+
10*a*c+b^2)*x^(1/2)*(c*x^5+b*x^3+a*x)^(1/2)/c+2/35*a^(1/4)*b*(-8*a*c+b^2)*(
cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*
EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))
*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1
/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)-1/70*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^
(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^
(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2
*b*(-8*a*c+b^2)+(-20*a*c+b^2)*a^(1/2)*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^
(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1935, 1959, 1967, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\,\sqrt{x}\left(\sqrt{a}\,\sqrt{c}\,(b^2-20ac)+2b(b^2-8ac)\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt[4]{a}\,b\sqrt{x}\,(b^2-8ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{ax+bx^3+cx^5}}-\frac{2bx^{3/2}(b^2-8ac)\,(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\,(10ac+b^2+3bcx^2)\,\sqrt{ax+bx^3+cx^5}}{35c}+\frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(35*c^(3/2)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (Sqrt[x]*(b^2 + 10*a*c + 3*b*c*x^2)*
Sqrt[a*x + b*x^3 + c*x^5])/(35*c) + (a*x + b*x^3 + c*x^5)^(3/2)/(7*Sqrt[x])
 + (2*a^(1/4)*b*(b^2 - 8*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a*x + b*x^3 + c*x^5]) -
 (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*Sqrt[x]*(Sqr
t[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(70*c^(
7/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1935

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^p/(m + p*(2
*n - q) + 1)), x] + Dist[(n - q)*(p/(m + p*(2*n - q) + 1)), Int[x^(m + q)*(
2*a + b*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /; FreeQ
[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^
2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q] && GtQ[m + p*q
+ 1, -(n - q)] && NeQ[m + p*(2*n - q) + 1, 0]
```

Rule 1959

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[x^(m + 1)*(b*B*(n - q)*p +
A*c*(m + p*q + (n - q)*(2*p + 1) + 1) + B*c*(m + p*q + 2*(n - q)*p + 1)*x^(
n - q))*((a*x^q + b*x^n + c*x^(2*n - q))^p/(c*(m + p*(2*n - q) + 1)*(m + p*
q + (n - q)*(2*p + 1) + 1))), x] + Dist[(n - q)*(p/(c*(m + p*(2*n - q) + 1)
*(m + p*q + (n - q)*(2*p + 1) + 1))), Int[x^(m + q)*Simp[2*a*A*c*(m + p*q +
 (n - q)*(2*p + 1) + 1) - a*b*B*(m + p*q + 1) + (2*a*B*c*(m + p*q + 2*(n -
q)*p + 1) + A*b*c*(m + p*q + (n - q)*(2*p + 1) + 1) - b^2*B*(m + p*q + (n -
 q)*p + 1))*x^(n - q), x]*(a*x^q + b*x^n + c*x^(2*n - q))^(p - 1), x], x] /
; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q] && EqQ[j, 2*n - q] &&  !Intege
rQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GtQ[p, 0] && RationalQ[m, q]
&& GtQ[m + p*q, -(n - q) - 1] && NeQ[m + p*(2*n - q) + 1, 0] && NeQ[m + p*q
 + (n - q)*(2*p + 1) + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{(ax+bx^3+cx^5)^{3/2}}{x^{3/2}}\,dx = \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{3}{7}\int \frac{(2a+bx^2)\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

$$= \frac{\sqrt{x}\,(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\int \frac{\sqrt{x}\,(-a}{\sqrt{c}}}{}$$

$$= \frac{\sqrt{x}\,(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(\sqrt{x}\,\sqrt{a}\right.}{}$$

$$= \frac{\sqrt{x}\,(b^2+10ac+3bcx^2)\sqrt{ax+bx^3+cx^5}}{35c} + \frac{(ax+bx^3+cx^5)^{3/2}}{7\sqrt{x}} + \frac{\left(2\sqrt{a}\,b(b\right.}{}$$

$$= -\frac{2b(b^2-8ac)\,x^{3/2}(a+bx^2+cx^4)}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}} + \frac{\sqrt{x}\,(b^2+10ac+3bcx^2)\sqrt{ax+b}}{35c}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 540, normalized size = 1.27

$$\frac{\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(15a^2c+a(b^2+23bcx^2+20c^2x^4)+x^2(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6))-ib(b^2-8ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^4+9ab^2c-20a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{70c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x]
```

```
[Out] (Sqrt[x]*(2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*c
*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c^3*x^6)) -
I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 +
```

```
9*a*b^2*c - 20*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
- 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSi
nh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])]))/(70*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a +
b*x^2 + c*x^4)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1393 vs. $2(403) = 806$.
time = 0.04, size = 1394, normalized size = 3.28 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/70*(x*(c*x^4+b*x^2+a))^(1/2)*(-10*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b*c^
3*x^9-10*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*c^3*x^9-26*((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c^2*x^7-26*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2)*(-4*a*c+b^2)^(1/2)*b*c^2*x^7-40*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b
*c^2*x^5-40*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*c^2*x^5-
18*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*c*x^5-18*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(-4*a*c+b^2)^(1/2)*b^2*c*x^5-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*
a*b^2*c*x^3-46*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*b*c*x
^3-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^4*x^3-2*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(-4*a*c+b^2)^(1/2)*b^3*x^3+12*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a
)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2
*a*c+b^2)/a/c)^(1/2))*a^2*b*c-20*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^
(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+
b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*c-3*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x
^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(
1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^3+(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)
^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c
+b^2)/a/c)^(1/2))*(-4*a*c+b^2)^(1/2)*a*b^2-32*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b
*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)
^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*b*c+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2
*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)
-2*a*c+b^2)/a/c)^(1/2))*a*b^3-30*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*b*c*
x-30*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a^2*c*x-2*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^3*x-2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*
a*c+b^2)^(1/2)*a*b^2*x)/x^(1/2)/(c*x^4+b*x^2+a)/c/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**5+b*x**3+a*x)**(3/2)/x**(3/2),x)
[Out] Integral((x*(a + b*x**2 + c*x**4))**(3/2)/x**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^5+b*x^3+a*x)^(3/2)/x^(3/2),x, algorithm="giac")
[Out] integrate((c*x^5 + b*x^3 + a*x)^(3/2)/x^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2),x)
[Out] int((a*x + b*x^3 + c*x^5)^(3/2)/x^(3/2), x)
```

3.113 $\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*x^(1/2)*(c*x^4+b
*x^2+a)^(1/2)/c^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1928, 1121, 635, 212}

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(2*Sqrt[c]*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{ax+bx^3+cx^5}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{ax+bx^3+cx^5}} \\
&= \frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,\sqrt{ax+bx^3+cx^5}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 0.98

$$-\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}\,\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4]])/(Sqrt[c]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.02, size = 72, normalized size = 0.88

method	result	size

default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\,\ln\left(\frac{2c\,x^{2}+2\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}+b}{2\sqrt{c}}\right)}{2\sqrt{x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}\,\sqrt{c}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*ln(1/2*(2*c*x^2
+2*(c*x^4+b*x^2+a)^(1/2)*c^(1/2)+b)/c^(1/2))/c^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [A]
time = 0.36, size = 135, normalized size = 1.65

$$\left[\frac{\log\left(-\frac{8\,c^{2}x^{5}+8\,bcx^{3}+4\,\sqrt{cx^{5}+bx^{3}+ax}\,(2\,cx^{2}+b)\sqrt{c}\,\sqrt{x}+(b^{2}+4\,ac)x}{x}\right)}{4\,\sqrt{c}},-\frac{\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{5}+bx^{3}+ax}\,(2\,cx^{2}+b)\sqrt{-c}\,\sqrt{x}}{2\,(c^{2}x^{5}+bcx^{3}+acx)}\right)}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(8*c^2*x^5 + 8*b*c*x^3 + 4*sqrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b
)*sqrt(c)*sqrt(x) + (b^2 + 4*a*c)*x)/x)/sqrt(c), -1/2*sqrt(-c)*arctan(1/2*s
qrt(c*x^5 + b*x^3 + a*x)*(2*c*x^2 + b)*sqrt(-c)*sqrt(x)/(c^2*x^5 + b*c*x^3
+ a*c*x))/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{x\,(a+bx^{2}+cx^{4})}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 3.37, size = 60, normalized size = 0.73

$$-\frac{\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}+\frac{\log\left(\left|-b+2\sqrt{a}\sqrt{c}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/sqrt(
c) + 1/2*log(abs(-b + 2*sqrt(a)*sqrt(c)))/sqrt(c)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.114 $\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

```
[Out] 1/2*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/
4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(
1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^
2)^(1/2)/a^(1/4)/c^(1/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1928, 1117}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
```

```
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right.}{2\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 193, normalized size = 1.60

$$-\frac{i\sqrt{x}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] ((-I)*Sqrt[x]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x
^4)])
```

Maple [A]
time = 0.02, size = 177, normalized size = 1.46

method	result
default	$\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\sqrt{-\frac{2\left(x^2\sqrt{-4ac+b^2}-b\,x^2-2a\right)}{a}}\sqrt{\frac{x^2\sqrt{-4ac+b^2}+b\,x^2+2a}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\sqrt{}}{}\right.}{2\sqrt{x}\,\left(c\,x^4+b\,x^2+a\right)\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)/((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2)*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+
b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [**A**]
time = 0.09, size = 122, normalized size = 1.01

$$\frac{\sqrt{\frac{1}{2}}\left(c\sqrt{\frac{b^2-4\,ac}{c^2}}+b\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\,\mathrm{ellipticF}\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x},\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac}\right)}{2\,a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*(c*sqrt((b^2 - 4*a*c)/c^2) + b)*sqrt((c*sqrt((b^2 - 4*a*c)/c^
2) - b)/c)*ellipticF(sqrt(1/2)*sqrt((c*sqrt((b^2 - 4*a*c)/c^2) - b)/c)/x, 1
/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(a*c))/(a*sqrt(c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.115 $\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/
2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx = -\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\,(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 1.57

$$\frac{\sqrt{x}\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 72, normalized size = 1.41

method	result	size
default	$-\frac{\sqrt{x\,(c\,x^4+b\,x^2+a)}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,\sqrt{a}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)/(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)), x)

Fricas [A]

time = 0.36, size = 137, normalized size = 2.69

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{a}\,\sqrt{x}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{-a}\,\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,(a+bx^2+cx^4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(c*x**5+b*x**3+a*x)**(1/2),x)

[Out] Integral(1/(sqrt(x)*sqrt(x*(a + b*x**2 + c*x**4))), x)

Giac [A]

time = 4.33, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)

[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)

3.116 $\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=330

$$\frac{\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}-\frac{\sqrt[4]{c}\,\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\Big(}{a^{3/4}\sqrt{ax+bx^3+}$$

```
[Out] x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3+a*x)^(
1/2)-(c*x^5+b*x^3+a*x)^(1/2)/a/x^(3/2)-c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1
/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1
/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1
/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*
x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1943, 12, 1928, 1153, 1117, 1209}

$$\frac{\sqrt[4]{c}\,\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\,\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x +
 b*x^3 + c*x^5]) - Sqrt[a*x + b*x^3 + c*x^5]/(a*x^(3/2)) - (c^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2
]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a
^(3/4)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2
)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*Sqrt[a*x + b*x
^3 + c*x^5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1928

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]/Sqrt[a
*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)/Sqrt[a + b*x^(n - q) + c*x^
(2*(n - q))], x], x] /; FreeQ[{a, b, c, m, n, q}, x] && EqQ[r, 2*n - q] &&
PosQ[n - q] && ((EqQ[m, 1] && EqQ[n, 3] && EqQ[q, 2]) || ((EqQ[m + 1/2] ||
EqQ[m, 3/2] || EqQ[m, 1/2] || EqQ[m, 5/2]) && EqQ[n, 3] && EqQ[q, 1]))
```

Rule 1943

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[x^(m - q + 1)*((a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
/(a*(m + p*q + 1))), x] - Dist[1/(a*(m + p*q + 1)), Int[x^(m + n - q)*(b*(m
 + p*q + (n - q)*(p + 1) + 1) + c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n -
q))*(a*x^q + b*x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c}, x] && EqQ
[r, 2*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ
[n, 0] && GeQ[p, -1] && LtQ[p, 0] && RationalQ[m, q] && LtQ[m + p*q + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx &= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\int \frac{cx^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{c\int \frac{x^{5/2}}{\sqrt{ax+bx^3+cx^5}}\,dx}{a} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(c\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\sqrt{ax+bx^3+cx^5}} \\
&= -\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}+\frac{\left(\sqrt{c}\,\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{a}\,\sqrt{ax+bx^3+cx^5}} - \\
&= \frac{\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt{ax+bx^3+cx^5}}{ax^{3/2}}-\frac{\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}\,+\right.}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.33, size = 303, normalized size = 0.92

$$
\frac{-4(a+bx^2+cx^4)+\frac{i\sqrt{2}\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{4a\sqrt{x}\,\sqrt{x(a+bx^2+cx^4)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5]),x]
```

```
[Out] (-4*(a + b*x^2 + c*x^4) + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])
/(b - Sqrt[b^2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[x]*S
qrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.04, size = 508, normalized size = 1.54

method	result

risch	$-\frac{c x^4+b x^2+a}{a\sqrt{x}\,\sqrt{x\,(c x^4+b x^2+a)}}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\Bigg(}{\Bigg(}$
default	$\frac{\left(-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{-4ac+b^2}\,c x^4-\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,b c x^4-c\sqrt{-\frac{2\left(x^2\sqrt{-4ac+b^2}-b x^2-2a\right)}{a}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*c*x^4-((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2)*b*c*x^4-c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/
2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*x*EllipticF(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c
+b^2)/a/c)^(1/2))+c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(
-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*a*x*EllipticE(1/2*x*2^(1/2)*((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)
^(1/2))-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*b*x^2-((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*b^2*x^2-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c
+b^2)^(1/2)*a-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b)/x^(3/2)*(x*(c*x^4+b*x^
2+a))^(1/2)/(c*x^4+b*x^2+a)/a/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(1/2),x)
[Out] Integral(1/(x**(3/2)*sqrt(x*(a + b*x**2 + c*x**4))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(c*x^5 + b*x^3 + a*x)*x^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3/2}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(1/2)), x)
```

3.117 $\int \frac{x^{3/2}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx$

Optimal. Leaf size=391

$$\frac{x^{3/2}(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{b\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{b\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{}}{a}$$

```
[Out] x^(3/2)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-b*x^(3/2
)*(c*x^4+b*x^2+a)*c^(1/2)/a/(-4*a*c+b^2)/(a^(1/2)+x^2*c^(1/2))/(c*x^5+b*x^3
+a*x)^(1/2)+b*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(
1/2)+x^2*c^(1/2))^2)^(1/2)/a^(3/4)/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2
*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/
a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/
2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1
/2))^2)^(1/2)/a^(3/4)/(b-2*a^(1/2)*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1938, 1967, 1211, 1117, 1209}

$$\frac{b\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\sqrt[4]{c}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{ax+bx^3+cx^5}}+\frac{x^{3/2}(-2ac+b^2+bcx^2)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{b\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - (b*Sqrt[c]*x^(3/2)*(a + b*x^2 + c*x^4))/(a*(b^2 - 4*a*c)*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) + (b*c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*Arc
Tan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2 - 4*a
*c)*Sqrt[a*x + b*x^3 + c*x^5]) - (c^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*S
qrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*(b - 2*Sqrt[a]*Sq
rt[c])*Sqrt[a*x + b*x^3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{\sqrt{x}\,(2ac+bcx^2)}{\sqrt{ax+bx^3+cx^5}}\,dx}{a\,(b^2-4ac)}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{2ac+bcx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} + \frac{\left(b\sqrt{c}\,\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx}}}{\sqrt{a}\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{x^{3/2}(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\sqrt{ax+bx^3+cx^5}} - \frac{b\sqrt{c}\,x^{3/2}(a+bx^2+cx^4)}{a\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.70, size = 463, normalized size = 1.18

$$-\frac{\sqrt{x}\left(-4\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(b^2-2ac+bcx^2)+ib\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-b^2+4ac+b\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{4a(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[x]*(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2)
 + I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x
], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*S
qrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b
^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(a*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b
^2 - 4*a*c])]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.03, size = 533, normalized size = 1.36

method	result

default	$\frac{\sqrt{x\left(c\,x^{4}+b\,x^{2}+a\right)}\left(-\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,\sqrt{-4ac+b^{2}}\,bc\,x^{3}-\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}}\,b^{2}c\,x^{3}+c\sqrt{-\frac{2}{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/x^(1/2)*(x*(c*x^4+b*x^2+a))^(1/2)*(-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(-4
*a*c+b^2)^(1/2)*b*c*x^3-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^2*c*x^3+c*(-2*(
x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2
*a)/a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a*(-4*a*c+b^2)^(1/2)+
b*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2
)+b*x^2+2*a)/a)^(1/2)*a*EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))+2*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(-4*a*c+b^2)^(1/2)*a*c*x-((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(-4*a*c+b^2)^(1/2)*b^2*x+2*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*
c*x-((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*x)/(c*x^4+b*x^2+a)/a/(4*a*c-b^2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^(3/2)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3/2}}{\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
[Out] int(x^(3/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.118 $\int \frac{\sqrt{x}}{(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{x}\,(b^2-2ac+bcx^2)}{a\,(b^2-4ac)\,\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(3/
2)+(b*c*x^2-2*a*c+b^2)*x^(1/2)/a/(-4*a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1936, 1927, 212}

$$\frac{\sqrt{x}\,(-2ac+b^2+bcx^2)}{a\,(b^2-4ac)\,\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5
]) - ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/(2*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 1936

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
```

```
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[(2*a*c - b^2*(p + 2))/(a*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(a*x^q
+ b*x^n + c*x^(2*n - q))^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2
*n - q] && PosQ[n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0
] && LtQ[p, -1] && RationalQ[m, p, q] && EqQ[m + p*q + 1, (-(n - q))*(2*p +
 3)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\left(ax+bx^3+cx^5\right)^{3/2}}\,dx &= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx^3+cx^5}}\,dx}{a}\\ &= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\operatorname{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{\sqrt{x}\left(2a+bx^2\right)}{\sqrt{ax+bx^3+cx^5}}\right)}{a}\\ &= \frac{\sqrt{x}\left(b^2-2ac+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\left(2a+bx^2\right)}{2\sqrt{a}\sqrt{ax+bx^3+cx^5}}\right)}{2a^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 123, normalized size = 1.19

$$-\frac{\sqrt{x}\left(\sqrt{a}\left(b^2-2ac+bcx^2\right)+\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)\right)}{a^{3/2}\left(-b^2+4ac\right)\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3 + c*x^5)^(3/2),x]
```

```
[Out] -((Sqrt[x]*(Sqrt[a]*(b^2 - 2*a*c + b*c*x^2) + (b^2 - 4*a*c)*Sqrt[a + b*x^2
+ c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]))/(a^(3/2
)*(-b^2 + 4*a*c)*Sqrt[x*(a + b*x^2 + c*x^4)]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(87) = 174.
time = 0.04, size = 179, normalized size = 1.74

method	result
default	$-\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\left(2bc\,x^2\sqrt{a}+4\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)ac\sqrt{c\,x^4+b\,x^2+a}-\ln\left(\frac{2a+b}{}\right.\right.}{2a^{\frac{3}{2}}\sqrt{x}\,\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/a^(3/2)*(2*b*c*x^2*a^(1/2)+4*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*c*(c*x^4+b*x^2+a)^(1/2)-ln((2*a+b*x^
2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^2*(c*x^4+b*x^2+a)^(1/2)-4*a^(3/2)
*c+2*b^2*a^(1/2))/x^(1/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^5 + b*x^3 + a*x)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(87) = 174$.
time = 0.41, size = 424, normalized size = 4.12

$$\left[\frac{((b^2c-4\,ac^2)x^5+(b^3-4\,abc)x^3+(ab^2-4\,a^2c)x)\sqrt{a}\,\log\left(-\frac{(b^2+4\,ac)x^5+8\,abx^3+8\,a^2x-4\,\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{a}\,\sqrt{x}}{x^5}\right)+4\,\sqrt{cx^5+bx^3+ax}\,(abcx^2+ab^2-2\,a^2c)\sqrt{x}}{4\,((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x)},\ \frac{((b^2c-4\,ac^2)x^5+(b^3-4\,abc)x^3+(ab^2-4\,a^2c)x)\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2\,a)\sqrt{-a}\,\sqrt{x}}{2\,(acx^5+abx^3+a^2x)}\right)+2\,\sqrt{cx^5+bx^3+ax}\,(abcx^2+ab^2-2\,a^2c)\sqrt{x}}{2\,((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^5 + (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*s
qrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3
 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) + 4*sqrt(c*x^5 + b*x^3 + a*x)*(
a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x), 1/2*(((b^2*c - 4*a*c^2)*x^5 +
 (b^3 - 4*a*b*c)*x^3 + (a*b^2 - 4*a^2*c)*x)*sqrt(-a)*arctan(1/2*sqrt(c*x^5
+ b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))
+ 2*sqrt(c*x^5 + b*x^3 + a*x)*(a*b*c*x^2 + a*b^2 - 2*a^2*c)*sqrt(x))/((a^2*
b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2 + c*x**4))**(3/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(87) = 174.
time = 3.70, size = 193, normalized size = 1.87

$$\frac{\frac{abcx^2}{a^2b^2-4\,a^3c}+\frac{ab^2-2\,a^2c}{a^2b^2-4\,a^3c}}{\sqrt{cx^4+bx^2+a}}-\frac{ab^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)-4\,a^2c\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\,b^2-2\,\sqrt{-a}\,a^{\frac{3}{2}}c}{\sqrt{-a}\,a^2b^2-4\,\sqrt{-a}\,a^3c}+\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] (a*b*c*x^2/(a^2*b^2 - 4*a^3*c) + (a*b^2 - 2*a^2*c)/(a^2*b^2 - 4*a^3*c))/sqr
t(c*x^4 + b*x^2 + a) - (a*b^2*arctan(sqrt(a)/sqrt(-a)) - 4*a^2*c*arctan(sqr
t(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*b^2 - 2*sqrt(-a)*a^(3/2)*c)/(sqrt(-a)*a^2
*b^2 - 4*sqrt(-a)*a^3*c) + arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3 + c*x^5)^(3/2), x)
```

3.119 $\int \frac{1}{\sqrt{x} \left(ax+bx^3+cx^5\right)^{3/2}} \, dx$

Optimal. Leaf size=468

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)\sqrt{x}\sqrt{ax+bx^3+cx^5}}+\frac{2\sqrt{c}\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\left(b^2-3ac\right)\sqrt{ax+b}}{a^2\left(b^2-4ac\right)x}$$

```
[Out] 2*(-3*a*c+b^2)*x^(3/2)*(c*x^4+b*x^2+a)*c^(1/2)/a^2/(-4*a*c+b^2)/(a^(1/2)+x^
2*c^(1/2))/(c*x^5+b*x^3+a*x)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(1/
2)/(c*x^5+b*x^3+a*x)^(1/2)-2*(-3*a*c+b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a
*c+b^2)/x^(3/2)-2*c^(1/4)*(-3*a*c+b^2)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)
^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^
(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*x^(1/2)*((c*
x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*a*c+b^2)/(c*x^5+b*x
^3+a*x)^(1/2)+1/2*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*
arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(
2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b^2-6*a*c+b*a^(1/2)*c^
(1/2))*x^(1/2)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(-4*
a*c+b^2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 468, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1938, 1965, 1967, 1211, 1117, 1209}

$$\frac{\sqrt[4]{c}\sqrt{x}\left(\sqrt{a}\,b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\sqrt[4]{c}\sqrt{x}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}-\frac{2\left(b^2-3ac\right)\sqrt{ax+bx^3+cx^5}}{a^2x^{3/2}\left(b^2-4ac\right)}+\frac{2\sqrt{c}\,x^{3/2}\left(b^2-3ac\right)\left(a+bx^2+cx^4\right)}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{-2ac+b^2+bcx^2}{a\sqrt{x}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[x]*Sqrt[a*x + b*x^3 + c*x^5])
 + (2*Sqrt[c]*(b^2 - 3*a*c)*x^(3/2)*(a + b*x^2 + c*x^4))/(a^2*(b^2 - 4*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[a*x + b*x^3 + c*x^5]) - (2*(b^2 - 3*a*c)*Sqrt
[a*x + b*x^3 + c*x^5])/(a^2*(b^2 - 4*a*c)*x^(3/2)) - (2*c^(1/4)*(b^2 - 3*a*
c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c
]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^3 + c*x^5]) + (c^(1/4)*(2*b^2
 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*Sqrt[x]*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x
^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a*x + b*x^
3 + c*x^5])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rule 1967

```
Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(j_.)))/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x
_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] :> Dist[x^(q/2)*(Sqrt[a + b*x^(n -
q) + c*x^(2*(n - q))]/Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]), Int[x^(m - q/2)
*((A + B*x^(n - q))/Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), x], x] /; Fre
eQ[{a, b, c, A, B, m, n, q}, x] && EqQ[j, n - q] && EqQ[r, 2*n - q] && PosQ
[n - q] && (EqQ[m, 1/2] || EqQ[m, -2^(-1)]) && EqQ[n, 3] && EqQ[q, 1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\,(ax+bx^3+cx^5)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\sqrt{x}\,\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-2b^2+6ac-bcx^2}{x^{3/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\,(b^2-4ac)}$$

$$= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\sqrt{x}\,\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2\,(b^2-4ac)\,x^{3/2}} + \ldots$$

$$= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\sqrt{x}\,\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2\,(b^2-4ac)\,x^{3/2}} + \ldots$$

$$= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\sqrt{x}\,\sqrt{ax+bx^3+cx^5}} - \frac{2(b^2-3ac)\sqrt{ax+bx^3+cx^5}}{a^2\,(b^2-4ac)\,x^{3/2}} - \ldots$$

$$= \frac{b^2-2ac+bcx^2}{a\,(b^2-4ac)\sqrt{x}\,\sqrt{ax+bx^3+cx^5}} + \frac{2\sqrt{c}\,(b^2-3ac)\,x^{3/2}(a+bx^2 \ldots}{a^2\,(b^2-4ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+\ldots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.89, size = 519, normalized size = 1.11

$$-\frac{2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(-4a^2c+2b^2x^2(b+cx^2)+a(b^2-7bcx^2-6c^2x^4)\right)-i(b^2-3ac)\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2a^2\,(b^2-4ac)\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x}\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] -1/2*(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*x^2) +
 a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c]
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
```

```
b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c
*Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c
])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sq
rt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(a^2*(b^2 - 4*a*c)*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(454) = 908$.
time = 0.05, size = 1136, normalized size = 2.43

method	result
default	$-\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\left(12\sqrt{-4ac+b^2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,a\,c^2x^4-4\sqrt{-4ac+b^2}\,\sqrt{\frac{-b+\sqrt{-4ac+\cdots}}{a}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(x*(c*x^4+b*x^2+a))^(1/2)/x^(3/2)*(12*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*a*c^2*x^4-4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*b^2*c*x^4+12*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b*c^2*x^4-4*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*b^3*c*x^4+a*b*c*(-2*(x^2*(-4*a*c+b^2)^(1/2)-
b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticF(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2
)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x*(-4*a*c+b^2)^(1/2)+12*(-2*(x^2*(-4*a*c+b^2
)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4
*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x-3*a*b^2*c*(-2*(x^2*(-4*a*c
+b^2)^(1/2)-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b
*(-4*a*c+b^2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*x-12*(-2*(x^2*(-4*a*c+b^2)^(1/2)
-b*x^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^
2)^(1/2)-2*a*c+b^2)/a/c)^(1/2))*a^2*c^2*x+4*(-2*(x^2*(-4*a*c+b^2)^(1/2)-b*x
^2-2*a)/a)^(1/2)*((x^2*(-4*a*c+b^2)^(1/2)+b*x^2+2*a)/a)^(1/2)*EllipticE(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*2^(1/2)*((b*(-4*a*c+b^2)^(
1/2)-2*a*c+b^2)/a/c)^(1/2))*a*b^2*c*x+14*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*a*b*c*x^2-4*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*b^3*x^2+14*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a*b^2*c*x^2-4*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*b^4*x^2+8*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*a^2*c-2*(-4*a*c+b^2)^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
```

```
*a*b^2+8*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*a^2*b*c-2*((-b+(-4*a*c+b^2)^(1/2
))/a)^(1/2)*a*b^3)/(c*x^4+b*x^2+a)/a^2/(4*a*c-b^2)/((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="maxima")
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*sqrt(x)/(c^2*x^11 + 2*b*c*x^9 + (b^2 + 2
*a*c)*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(x\left(a+bx^2+cx^4\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**5+b*x**3+a*x)**(3/2)/x**(1/2),x)
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^5+b*x^3+a*x)^(3/2)/x^(1/2),x, algorithm="giac")
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*sqrt(x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\left(c\,x^5+b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.120 $\int \frac{1}{x^{3/2}(ax+bx^3+cx^5)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}$$

```
[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(5
/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x^(3/2)/(c*x^5+b*x^3+a*x)^(1/2)-1/2*
(-8*a*c+3*b^2)*(c*x^5+b*x^3+a*x)^(1/2)/a^2/(-4*a*c+b^2)/x^(5/2)
```

Rubi [A]

time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1938, 1965, 12, 1927, 212}

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{4a^{5/2}}-\frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2x^{5/2}\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^{3/2}\left(b^2-4ac\right)\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^(3/2)*Sqrt[a*x + b*x^3 + c*x^5])
 - ((3*b^2 - 8*a*c)*Sqrt[a*x + b*x^3 + c*x^5])/(2*a^2*(b^2 - 4*a*c)*x^(5/2)
) + (3*b*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^
5])])/(4*a^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
```

```
qQ[m, q/2 - 1]
```

Rule 1938

```
Int[(x_)^(m_.)*((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(p_
), x_Symbol] :> Simp[(-x^(m - q + 1))*(b^2 - 2*a*c + b*c*x^(n - q))*((a*x^q
 + b*x^n + c*x^(2*n - q))^(p + 1)/(a*(n - q)*(p + 1)*(b^2 - 4*a*c))), x] +
Dist[1/(a*(n - q)*(p + 1)*(b^2 - 4*a*c)), Int[x^(m - q)*(b^2*(m + p*q + (n
- q)*(p + 1) + 1) - 2*a*c*(m + p*q + 2*(n - q)*(p + 1) + 1) + b*c*(m + p*q
+ (n - q)*(2*p + 3) + 1)*x^(n - q))*(a*x^q + b*x^n + c*x^(2*n - q))^(p + 1)
, x], x] /; FreeQ[{a, b, c}, x] && EqQ[r, 2*n - q] && PosQ[n - q] &&  !Inte
gerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[p, -1] && RationalQ[m,
q] && LtQ[m + p*q + 1, n - q]
```

Rule 1965

```
Int[(x_)^(m_.)*((c_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.))^(p_
.)*((A_) + (B_.)*(x_)^(r_.)), x_Symbol] :> Simp[A*x^(m - q + 1)*((a*x^q + b
*x^n + c*x^(2*n - q))^(p + 1)/(a*(m + p*q + 1))), x] + Dist[1/(a*(m + p*q +
 1)), Int[x^(m + n - q)*Simp[a*B*(m + p*q + 1) - A*b*(m + p*q + (n - q)*(p
+ 1) + 1) - A*c*(m + p*q + 2*(n - q)*(p + 1) + 1)*x^(n - q), x]*(a*x^q + b*
x^n + c*x^(2*n - q))^p, x], x] /; FreeQ[{a, b, c, A, B}, x] && EqQ[r, n - q
] && EqQ[j, 2*n - q] &&  !IntegerQ[p] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
&& RationalQ[m, p, q] && ((GeQ[p, -1] && LtQ[p, 0]) || EqQ[m + p*q + (n - q
)*(2*p + 1) + 1, 0]) && LeQ[m + p*q, -(n - q)] && NeQ[m + p*q + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3+cx^5\right)^{3/2}}\,dx = \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\int \frac{-3b^2+8ac-2bcx^2}{x^{5/2}\sqrt{ax+bx^3+cx^5}}\,dx}{a\left(b^2-4ac\right)}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{\int -}{}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} - \frac{(3b)}{}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{(3b)S}{}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)x^{3/2}\sqrt{ax+bx^3+cx^5}} - \frac{\left(3b^2-8ac\right)\sqrt{ax+bx^3+cx^5}}{2a^2\left(b^2-4ac\right)x^{5/2}} + \frac{3b\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.52, size = 159, normalized size = 1.03

$$\frac{\sqrt{a}\left(-4a^2c+3b^2x^2(b+cx^2)+a(b^2-10bcx^2-8c^2x^4)\right)+3b(b^2-4ac)\,x^2\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{5/2}\left(-b^2+4ac\right)x^{3/2}\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x^
4)) + 3*b*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 -
Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(5/2)*(-b^2 + 4*a*c)*x^(3/2)*Sqrt[x
*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.08, size = 220, normalized size = 1.43

method	result
default	$\frac{\sqrt{x\left(c\,x^4+b\,x^2+a\right)}\left(-16a^{\frac{3}{2}}c^2x^4+6b^2c\,x^4\sqrt{a}+12\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)abc\,x^2\sqrt{c\,x^4+b\,x^2} \ldots\right.}{4a^{\frac{5}{2}}x^{\frac{5}{2}}\left(c\,x^4+b\,x \ldots\right.}$
risch	$-\frac{c\,x^4+b\,x^2+a}{2a^2x^{\frac{3}{2}}\sqrt{x\left(c\,x^4+b\,x^2+a\right)}}+\frac{\left(\frac{b^2x^2c}{a^2\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}-\frac{2c^2x^2}{a\left(4ac-b^2\right)\sqrt{c\,x^4+b\,x^2+a}}+\frac{\ldots}{4a^2\left(4ac-b^2\right) \ldots}\right. \ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(x*(c*x^4+b*x^2+a))^(1/2)/a^(5/2)*(-16*a^(3/2)*c^2*x^4+6*b^2*c*x^4*a^(1
/2)+12*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*a*b*c*x^2*(c*x^4
+b*x^2+a)^(1/2)-3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)*b^3*x
^2*(c*x^4+b*x^2+a)^(1/2)-20*a^(3/2)*b*c*x^2+6*b^3*x^2*a^(1/2)-8*a^(5/2)*c+2
*a^(3/2)*b^2)/x^(5/2)/(c*x^4+b*x^2+a)/(4*a*c-b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^5 + b*x^3 + a*x)^(3/2)*x^(3/2)), x)
```

Fricas [A]
time = 0.39, size = 508, normalized size = 3.30

$$\left[\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{7}+\left(b^{4}-4\,ab^{2}c\right)x^{5}+\left(ab^{3}-4\,a^{2}bc\right)x^{3}\right)\sqrt{a}\log\left(-\frac{\left(b^{2}+4\,ac\right)x^{5}+8\,abx^{3}+8\,a^{2}x+4\,\sqrt{cx^{5}+bx^{3}+ax}\left(bx^{2}+2\,a\right)\sqrt{a}\sqrt{x}}{x^{5}}\right)-4\,\sqrt{cx^{5}+bx^{3}+ax}\left(\left(3\,ab^{2}c-8\,a^{2}c^{2}\right)x^{4}+a^{2}b^{2}-4\,a^{3}c+\left(3\,ab^{3}-10\,a^{2}bc\right)x^{2}\right)\sqrt{x}}{8\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)x^{7}+\left(a^{3}b^{3}-4\,a^{4}bc\right)x^{5}+\left(a^{4}b^{2}-4\,a^{5}c\right)x^{3}\right)},-\frac{3\left(\left(b^{3}c-4\,abc^{2}\right)x^{7}+\left(b^{4}-4\,ab^{2}c\right)x^{5}+\left(ab^{3}-4\,a^{2}bc\right)x^{3}\right)\sqrt{-a}\arctan\left(\frac{\sqrt{cx^{5}+bx^{3}+ax}\left(bx^{2}+2\,a\right)\sqrt{-a}\sqrt{x}}{2\left(acx^{5}+abx^{3}+a^{2}x\right)}\right)+2\,\sqrt{cx^{5}+bx^{3}+ax}\left(\left(3\,ab^{2}c-8\,a^{2}c^{2}\right)x^{4}+a^{2}b^{2}-4\,a^{3}c+\left(3\,ab^{3}-10\,a^{2}bc\right)x^{2}\right)\sqrt{x}}{4\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)x^{7}+\left(a^{3}b^{3}-4\,a^{4}bc\right)x^{5}+\left(a^{4}b^{2}-4\,a^{5}c\right)x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b
*c)*x^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x + 4*sqrt(c*x
^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5) - 4*sqrt(c*x^5 + b*x^
3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (3*a*b^3 - 10*a
^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*
x^5 + (a^4*b^2 - 4*a^5*c)*x^3), -1/4*(3*((b^3*c - 4*a*b*c^2)*x^7 + (b^4 - 4
*a*b^2*c)*x^5 + (a*b^3 - 4*a^2*b*c)*x^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b
*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x)) + 2
*sqrt(c*x^5 + b*x^3 + a*x)*((3*a*b^2*c - 8*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c
 + (3*a*b^3 - 10*a^2*b*c)*x^2)*sqrt(x))/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3
*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^{2}+cx^{4}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(c*x**5+b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2 + c*x**4))**(3/2)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(c*x^5+b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(c\,x^5+b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)),x)
[Out] int(1/(x^(3/2)*(a*x + b*x^3 + c*x^5)^(3/2)), x)
```

3.121 $\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

```
[Out] -2*x^(-1/2+1/2*n)*(2*c*x+b)/(-4*a*c+b^2)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1929}

$$-\frac{2x^{\frac{n-1}{2}}(b+2cx)}{(b^2-4ac)\sqrt{ax^{n-1}+bx^n+cx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a*x^(-1 + n) + b*x^n + 
c*x^(1 + n)])
```

Rule 1929

```
Int[(x_)^(m_.)/((b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(3/
2), x_Symbol] :> Simp[-2*x^((n - 1)/2)*((b + 2*c*x)/((b^2 - 4*a*c)*Sqrt[a*x
^(n - 1) + b*x^n + c*x^(n + 1)])), x] /; FreeQ[{a, b, c, n}, x] && EqQ[m, 3
*((n - 1)/2)] && EqQ[q, n - 1] && EqQ[r, n + 1] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{\frac{3}{2}(-1+n)}}{\left(ax^{-1+n}+bx^n+cx^{1+n}\right)^{3/2}}\,dx = -\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{ax^{-1+n}+bx^n+cx^{1+n}}}$$

Mathematica [A]
time = 0.05, size = 46, normalized size = 0.90

$$-\frac{2x^{\frac{1}{2}(-1+n)}(b+2cx)}{(b^2-4ac)\sqrt{x^{-1+n}(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((3*(-1 + n))/2)/(a*x^(-1 + n) + b*x^n + c*x^(1 + n))^(3/2),x]
```

```
[Out] (-2*x^((-1 + n)/2)*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[x^(-1 + n)*(a + x*(b +
c*x))])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{-\frac{3}{2}+\frac{3n}{2}}}{\left(a\,x^{-1+n}+b\,x^{n}+c\,x^{1+n}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

```
[Out] int(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

Fricas [A]
time = 0.41, size = 83, normalized size = 1.63

$$-\frac{2\,\left(2\,cx^{2}+bx\right)\sqrt{\frac{\left(cx^{2}+bx+a\right)x^{n+1}}{x^{2}}}}{\left(ab^{2}-4\,a^{2}c+\left(b^{2}c-4\,ac^{2}\right)x^{2}+\left(b^{3}-4\,abc\right)x\right)x^{\frac{1}{2}\,n+\frac{1}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="f
ricas")
```

```
[Out] -2*(2*c*x^2 + b*x)*sqrt((c*x^2 + b*x + a)*x^(n + 1)/x^2)/((a*b^2 - 4*a^2*c
+ (b^2*c - 4*a*c^2)*x^2 + (b^3 - 4*a*b*c)*x)*x^(1/2*n + 1/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3/2+3/2*n)/(a*x**(-1+n)+b*x**n+c*x**(1+n))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3/2+3/2*n)/(a*x^(-1+n)+b*x^n+c*x^(1+n))^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate(x^(3/2*n - 3/2)/(c*x^(n + 1) + a*x^(n - 1) + b*x^n)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{\frac{3\,n}{2}-\frac{3}{2}}}{{\left(b\,x^n+a\,x^{n-1}+c\,x^{n+1}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2),x)
```

```
[Out] int(x^((3*n)/2 - 3/2)/(b*x^n + a*x^(n - 1) + c*x^(n + 1))^(3/2), x)
```

3.122 $\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx$

Optimal. Leaf size=287

$$\frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{}}{}$$

```
[Out] 2/3*d*x^2*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)+2/7*e*x^4*AppellF1(
7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(
1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(c*x^5+b*x^3+a*x)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1968, 1349, 1155, 524}

$$\frac{2dx^2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3\sqrt{ax+bx^3+cx^5}}+\frac{2ex^4\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7\sqrt{ax+bx^3+cx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*d*x^2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*Sqrt[a*x + b*x^3 + c*x^
5]) + (2*e*x^4*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b - S
qrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*Sqrt[a*x + b*x^3
 + c*x^5])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1968

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] &&  !IntegerQ[p] && Po
sQ[k - j]
```

Rubi steps

$$\int \frac{x(d+ex^2)}{\sqrt{ax+bx^3+cx^5}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\left(\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \left(\frac{d\sqrt{x}}{\sqrt{a+bx^2+cx^4}}+\frac{ex^{5/2}}{\sqrt{a+bx^2+cx^4}}\right)dx}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\left(d\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{ax+bx^3+cx^5}} + \frac{\left(e\sqrt{x}\,\sqrt{a+bx^2+cx^4}\right)\int \cdots}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{\left(d\sqrt{x}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\cdots}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\cdots}}{\sqrt{ax+bx^3+cx^5}}$$

$$= \frac{2dx^2\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},\cdots\right)}{3\sqrt{ax+bx^3+cx^5}}$$

Mathematica [A]

time = 15.12, size = 239, normalized size = 0.83

$$\frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dx^2F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^4F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21\sqrt{x\left(a+bx^2+cx^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a*x + b*x^3 + c*x^5],x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x^2*AppellF1[3
/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + 3*e*x^4*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[x*(a + b*x^
2 + c*x^4)])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{x(e\,x^2+d)}{\sqrt{c\,x^5+b\,x^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

```
[Out] int(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^5 + b*x^3 + a*x)*(x^2*e + d)/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex^2)}{\sqrt{x\left(a+bx^2+cx^4\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(c*x**5+b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/sqrt(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(c*x^5+b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)*x/sqrt(c*x^5 + b*x^3 + a*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(e\,x^2+d\right)}{\sqrt{c\,x^5+b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2),x)
```

```
[Out] int((x*(d + e*x^2))/(a*x + b*x^3 + c*x^5)^(1/2), x)
```

3.123 $\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]

[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx &= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 66, normalized size = 1.47

$$\frac{x\sqrt{3-3x^2+x^4}\,\tanh^{-1}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[3*x^2 - 3*x^4 + x^6],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.22, size = 58, normalized size = 1.29

method	result	size
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-3\right)x^3-2\,\mathrm{RootOf}\left(\_Z^2-3\right)x+2\sqrt{x^6-3x^4+3x^2}}{x^3}\right)}{6}$	52
default	$\frac{x\sqrt{x^4-3x^2+3}\,\sqrt{3}\,\mathrm{arctanh}\left(\frac{\left(x^2-2\right)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3\,x^3+2\sqrt{3}\,(x^3-2\,x)+2\sqrt{x^6-3\,x^4+3\,x^2}\left(\sqrt{3}+2\right)-6\,x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^6 - 3x^4 + 3x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Giac [**A**]
time = 5.94, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2 + \sqrt{3} - \sqrt{x^4 - 3\,x^2 + 3}\right) - \sqrt{3}\log\left(-x^2 + \sqrt{3} + \sqrt{x^4 - 3\,x^2 + 3}\right)}{6\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^6 - 3\,x^4 + 3\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int(1/(3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.124 $\int \frac{1}{\sqrt{x^2 (3-3x^2+x^4)}} \, dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2021, 1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]

[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.47

$$\frac{x\sqrt{3-3x^2+x^4}\tanh^{-1}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.20, size = 58, normalized size = 1.29

method	result	size
trager	$-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-\text{RootOf}\left(\_Z^2-3\right)x^3+2\,\text{RootOf}\left(\_Z^2-3\right)x+2\sqrt{x^6-3x^4+3x^2}}{x^3}\right)}{6}$	53
default	$\frac{\sqrt{x^4-3x^2+3}\,x\sqrt{3}\,\text{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^2(x^4-3x^2+3)}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*(x^4-3*x^2+3)^(1/2)*x*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A]
time = 0.34, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}(x^3-2x)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2(x^4-3x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Giac [A]
time = 6.38, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.125 $\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2021, 1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - (1 - x^2)^3],x]

[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-(1-x^2)^3}}\,dx = \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.47

$$\frac{x\sqrt{3-3x^2+x^4}\tanh^{-1}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - (1 - x^2)^3],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.18, size = 58, normalized size = 1.29

method	result	size
trager	$-\frac{\text{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-\text{RootOf}\left(\_Z^2-3\right)x^3+2\,\text{RootOf}\left(\_Z^2-3\right)x+2\sqrt{x^6-3x^4+3x^2}}{x^3}\right)}{6}$	53
default	$\frac{x\sqrt{x^4-3x^2+3}\,\sqrt{3}\,\text{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^6-3x^4+3x^2}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6/(x^6-3*x^4+3*x^2)^(1/2)*x*(x^4-3*x^2+3)^(1/2)*3^(1/2)*arctanh(1/2*(x^2-
2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^2 - 1)^3 + 1), x)
```

Fricas [A]
time = 0.41, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}(x^3-2x)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{1-(1-x^2)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - (1 - x**2)**3), x)
```

Giac [A]
time = 5.52, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{(x^2-1)^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^2 - 1)^3 + 1)^(1/2),x)
[Out] int(1/((x^2 - 1)^3 + 1)^(1/2), x)
```

3.126 $\int \sqrt{3x^2-3x^4+x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1917, 1121, 626, 633, 221}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rubi steps

$$\int \sqrt{3x^2-3x^4+x^6}\,dx = \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}$$

$$= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,d\right.}{16x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx\right.}{16x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

Mathematica [**A**]
time = 0.07, size = 80, normalized size = 0.93

$$\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3*x^2 - 3*x^4 + x^6],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.13, size = 81, normalized size = 0.94

method	result	size
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}-\frac{3\ln\left(\frac{-2x^3+2\sqrt{x^6-3x^4+3x^2}+3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6-3*x^4+3*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(1/
2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^6 - 3*x^4 + 3*x^2), x)
```

Fricas [A]
time = 0.35, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\,(2\,x^2-3)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^6-3x^4+3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**6-3*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**6 - 3*x**4 + 3*x**2), x)
```

Giac [A]
time = 3.72, size = 69, normalized size = 0.80

$$\frac{1}{16}\left(2\sqrt{x^4-3x^2+3}\,(2x^2-3)-3\log\left(-2x^2+2\sqrt{x^4-3x^2+3}+3\right)\right)\operatorname{sgn}(x)+\frac{3}{16}\left(2\sqrt{3}+\log\left(2\sqrt{3}+3\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^6-3*x^4+3*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^6-3x^4+3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 - 3*x^4 + x^6)^(1/2),x)
```

```
[Out] int((3*x^2 - 3*x^4 + x^6)^(1/2), x)
```

3.127 $\int \sqrt{x^2\left(3-3x^2+x^4\right)}\,dx$

Optimal. Leaf size=86

$$-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2021, 1917, 1121, 626, 633, 221}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \sqrt{x^2\left(3-3x^2+x^4\right)}\,dx = \int \sqrt{3x^2-3x^4+x^6}\,dx$$

$$= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}}$$

$$= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\right.}{16x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\right.}{16x\sqrt{3-3x^2+x^4}}$$

$$= -\frac{\left(3-2x^2\right)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

Mathematica [A]
time = 0.00, size = 80, normalized size = 0.93

$$\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\,\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2\left(3-3x^2+x^4\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.11, size = 81, normalized size = 0.94

method	result	size
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\ln\left(\frac{2x^3+2\sqrt{x^6-3x^4+3x^2}-3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^2(x^4-3x^2+3)}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(x^2*(x^4-3*x^2+3))^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(
1/2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A]
time = 0.34, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\,(2\,x^2-3)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2 (x^4 - 3x^2 + 3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Giac [A]
time = 3.19, size = 69, normalized size = 0.80

$$\frac{1}{16} \left(2 \sqrt{x^4 - 3 x^2 + 3} (2 x^2 - 3) - 3 \log \left(-2 x^2 + 2 \sqrt{x^4 - 3 x^2 + 3} + 3\right)\right) \mathrm{sgn}(x) + \frac{3}{16} \left(2 \sqrt{3} + \log \left(2 \sqrt{3} + 3\right)\right) \mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2\ (x^4 - 3\,x^2 + 3)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int((x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.128 $\int \sqrt{1-\left(1-x^2\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x}-\frac{3\sqrt{3x^2-3x^4+x^6}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}$$

```
[Out] -1/8*(-2*x^2+3)*(x^6-3*x^4+3*x^2)^(1/2)/x-3/16*arcsinh(1/3*(-2*x^2+3)*3^(1/
2))*(x^6-3*x^4+3*x^2)^(1/2)/x/(x^4-3*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2021, 1917, 1121, 626, 633, 221}

$$-\frac{\sqrt{x^6-3x^4+3x^2}\,(3-2x^2)}{8x}-\frac{3\sqrt{x^6-3x^4+3x^2}\,\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] -1/8*((3 - 2*x^2)*Sqrt[3*x^2 - 3*x^4 + x^6])/x - (3*Sqrt[3*x^2 - 3*x^4 + x^
6]*ArcSinh[(3 - 2*x^2)/Sqrt[3]])/(16*x*Sqrt[3 - 3*x^2 + x^4])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1121

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1917

```
Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol]
 :> Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q)
+ c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x
], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-(1-x^2)^3}\,dx &= \int \sqrt{3x^2-3x^4+x^6}\,dx \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\int x\sqrt{3-3x^2+x^4}\,dx}{x\sqrt{3-3x^2+x^4}} \\
&= \frac{\sqrt{3x^2-3x^4+x^6}\,\text{Subst}\left(\int \sqrt{3-3x+x^2}\,dx, x, x^2\right)}{2x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(3\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{3-3x+x^2}}\,dx \right.}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} + \frac{\left(\sqrt{3}\sqrt{3x^2-3x^4+x^6}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, \right.}{16x\sqrt{3-3x^2+x^4}} \\
&= -\frac{(3-2x^2)\sqrt{3x^2-3x^4+x^6}}{8x} - \frac{3\sqrt{3x^2-3x^4+x^6}\sinh^{-1}\left(\frac{3-2x^2}{\sqrt{3}}\right)}{16x\sqrt{3-3x^2+x^4}}
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 80, normalized size = 0.93

$$\frac{x\left(-18+30x^2-18x^4+4x^6-3\sqrt{3-3x^2+x^4}\log\left(3-2x^2+2\sqrt{3-3x^2+x^4}\right)\right)}{16\sqrt{x^2(3-3x^2+x^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - (1 - x^2)^3],x]
```

```
[Out] (x*(-18 + 30*x^2 - 18*x^4 + 4*x^6 - 3*Sqrt[3 - 3*x^2 + x^4]*Log[3 - 2*x^2 +
 2*Sqrt[3 - 3*x^2 + x^4]]))/(16*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.11, size = 81, normalized size = 0.94

method	result	size
trager	$\frac{(2x^2-3)\sqrt{x^6-3x^4+3x^2}}{8x}+\frac{3\ln\left(\frac{2x^3+2\sqrt{x^6-3x^4+3x^2}-3x}{x}\right)}{16}$	64
risch	$\frac{(2x^2-3)\sqrt{x^2(x^4-3x^2+3)}}{8x}+\frac{3\,\mathrm{arcsinh}\left(\frac{2\sqrt{3}\,(x^2-\frac{3}{2})}{3}\right)\sqrt{x^2(x^4-3x^2+3)}}{16\sqrt{x^4-3x^2+3}\,x}$	74
default	$\frac{\sqrt{x^6-3x^4+3x^2}\left(4\sqrt{x^4-3x^2+3}\,x^2+3\,\mathrm{arcsinh}\left(\frac{\sqrt{3}\,(2x^2-3)}{3}\right)-6\sqrt{x^4-3x^2+3}\right)}{16x\sqrt{x^4-3x^2+3}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(-x^2+1)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(x^6-3*x^4+3*x^2)^(1/2)*(4*(x^4-3*x^2+3)^(1/2)*x^2+3*arcsinh(1/3*3^(1/
2)*(2*x^2-3))-6*(x^4-3*x^2+3)^(1/2))/x/(x^4-3*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x^2 - 1)^3 + 1), x)
```

Fricas [A]
time = 0.33, size = 70, normalized size = 0.81

$$-\frac{12\,x\log\left(-\frac{2\,x^3-3\,x-2\,\sqrt{x^6-3\,x^4+3\,x^2}}{x}\right)-8\,\sqrt{x^6-3\,x^4+3\,x^2}\,(2\,x^2-3)-9\,x}{64\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*x*log(-(2*x^3 - 3*x - 2*sqrt(x^6 - 3*x^4 + 3*x^2))/x) - 8*sqrt(x^
6 - 3*x^4 + 3*x^2)*(2*x^2 - 3) - 9*x)/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1-(1-x^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x**2+1)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - (1 - x**2)**3), x)
```

Giac [A]
time = 3.28, size = 69, normalized size = 0.80

$$\frac{1}{16}\left(2\sqrt{x^4-3x^2+3}\,(2x^2-3)-3\log\left(-2x^2+2\sqrt{x^4-3x^2+3}+3\right)\right)\mathrm{sgn}(x)+\frac{3}{16}\left(2\sqrt{3}+\log\left(2\sqrt{3}+3\right)\right)\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(-x^2+1)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(x^4 - 3*x^2 + 3)*(2*x^2 - 3) - 3*log(-2*x^2 + 2*sqrt(x^4 - 3*x
^2 + 3) + 3))*sgn(x) + 3/16*(2*sqrt(3) + log(2*sqrt(3) + 3))*sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{(x^2-1)^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 - 1)^3 + 1)^(1/2),x)
```

```
[Out] int(((x^2 - 1)^3 + 1)^(1/2), x)
```

3.129 $\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/Sqrt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 0.97

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.24, size = 35, normalized size = 0.92

method	result	size
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 111, normalized size = 2.92

$$\left[\frac{\log\left(-\frac{8\,abx+(b^2+4\,ac)x^2-4\,\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{a}+8\,a^2}{x^2}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^2+bx+a}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^2+abx+a^2)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a
)*sqrt(a) + 8*a^2)/x^2)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*
(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)), x)
```

Giac [**A**]
time = 3.71, size = 35, normalized size = 0.92

$$\frac{2\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a)
```

Mupad [**B**]
time = 0.08, size = 34, normalized size = 0.89

$$-\frac{\ln\left(\frac{b}{2}+\frac{a}{x}+\frac{\sqrt{a}\,\sqrt{c\,x^2+b\,x+a}}{x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] -log(b/2 + a/x + (a^(1/2)*(a + b*x + c*x^2)^(1/2))/x)/a^(1/2)
```

3.130 $\int \frac{1}{\sqrt{x^2(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

[Out] -arctanh(1/2*x*(b*x+2*a)/a^(1/2)/(c*x^4+b*x^3+a*x^2)^(1/2))/a^(1/2)

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2021, 1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(a + b*x + c*x^2)],x]

[Out] -(ArcTanh[(x*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^3 + c*x^4])]/Sqrt[a])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] && !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2\left(a+bx+cx^2\right)}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3+cx^4}}\,dx$$
$$= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx)}{\sqrt{ax^2+bx^3+cx^4}}\right)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x(2a+bx)}{2\sqrt{a}\sqrt{ax^2+bx^3+cx^4}}\right)}{\sqrt{a}}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 1.51

$$\frac{2x\sqrt{a+x(b+cx)}\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x + c*x^2)],x]
```

```
[Out] (2*x*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt
[a]])/(Sqrt[a]*Sqrt[x^2*(a + x*(b + c*x))])
```

Maple [A]
time = 0.03, size = 64, normalized size = 1.42

method	result	size
default	$-\frac{x\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x^2\left(c\,x^2+bx+a\right)}\sqrt{a}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^2*(c*x^2+b*x+a))^(1/2)*x*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^2 + b*x + a)*x^2), x)
```

Fricas [A]
time = 0.37, size = 130, normalized size = 2.89

$$\left[\frac{\log\left(-\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{a}}{x^3}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^3+ax^2}\,(bx+2\,a)\sqrt{-a}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^4 + b*x^3 +
 a*x^2)*(b*x + 2*a)*sqrt(a))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^3 + a*x^2)*(b*x + 2*a)*sqrt(-a)/(a*c*x^3 + a*b*x^2 + a^2*x))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\,(a+bx+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a + b*x + c*x**2)), x)
```

Giac [A]
time = 4.25, size = 59, normalized size = 1.31

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(-(sqrt(c)*x - sqrt(c
*x^2 + b*x + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x + c*x^2))^(1/2),x)
```

```
[Out] int(1/(x^2*(a + b*x + c*x^2))^(1/2), x)
```

3.131 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx+cx^2)}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx)}{2\sqrt{a}\,\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)*x^(1/2)/a^(1/2)/(c*x^3+b*x^2+a*x)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2022, 1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx)}{2\sqrt{a}\,\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[x]*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x + b*x^2 + c*x^3])]/Sqrt
[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,\sqrt{x\,(a+bx+cx^2)}}\,dx &= \int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^2+cx^3}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\,(2a+bx)}{\sqrt{ax+bx^2+cx^3}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx)}{2\sqrt{a}\,\sqrt{ax+bx^2+cx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 70, normalized size = 1.49

$$\frac{2\sqrt{x}\,\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x + c*x^2)]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + x*(b + c*x))])
```

Maple [**A**]
time = 0.02, size = 64, normalized size = 1.36

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x\,(c\,x^2+bx+a)}\,\sqrt{a}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -x^(1/2)/(x*(c*x^2+b*x+a))^(1/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*x+2*
a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((c*x^2 + b*x + a)*x)*sqrt(x)), x)
```

Fricas [A]
time = 0.41, size = 131, normalized size = 2.79

$$\left[\frac{\log\left(\frac{8\,abx^2+(b^2+4\,ac)x^3+8\,a^2x-4\,\sqrt{cx^3+bx^2+ax}\,(bx+2\,a)\sqrt{a}\,\sqrt{x}}{x^3}\right)}{2\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^3+bx^2+ax}\,(bx+2\,a)\sqrt{-a}\,\sqrt{x}}{2\,(acx^3+abx^2+a^2x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^2 + (b^2 + 4*a*c)*x^3 + 8*a^2*x - 4*sqrt(c*x^3 + b*x^2 +
a*x)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^3)/sqrt(a), sqrt(-a)*arctan(1/2*sqrt(c*
x^3 + b*x^2 + a*x)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^3 + a*b*x^2 + a^2*x)
)/a]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.34, size = 53, normalized size = 1.13

$$\frac{2\,\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - 2*arctan
(sqrt(a)/sqrt(-a))/sqrt(-a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(a + b*x + c*x^2))^(1/2)), x)
```

3.132 $\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx+cx^2)}} \, dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

```
[Out] -arctanh(1/2*x^(3/2)*(b*x+2*a)/a^(1/2)/(c*x^5+b*x^4+a*x^3)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2022, 1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] -(ArcTanh[(x^(3/2)*(2*a + b*x))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^4 + c*x^5])]/Sq
rt[a])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{x^3\left(a+bx+cx^2\right)}}\,dx=\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^4+cx^5}}\,dx$$
$$=-\left(2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{x^{3/2}(2a+bx)}{\sqrt{ax^3+bx^4+cx^5}}\right)\right)$$
$$=-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx)}{2\sqrt{a}\,\sqrt{ax^3+bx^4+cx^5}}\right)}{\sqrt{a}}$$

Mathematica [**A**]
time = 0.02, size = 72, normalized size = 1.47

$$\frac{2x^{3/2}\sqrt{a+x(b+cx)}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x^3(a+x(b+cx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x + c*x^2)],x]
```

```
[Out] (2*x^(3/2)*Sqrt[a + x*(b + c*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + x*(b + c*x))])
```

Maple [**A**]
time = 0.02, size = 66, normalized size = 1.35

method	result	size
default	$-\frac{x^{\frac{3}{2}}\sqrt{c\,x^2+bx+a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{x^3\left(c\,x^2+bx+a\right)}\,\sqrt{a}}$	66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(x^3*(c*x^2+b*x+a))^(1/2)*x^(3/2)*(c*x^2+b*x+a)^(1/2)/a^(1/2)*ln((2*a+b*
x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^2 + b*x + a)*x^3), x)
```

Fricas [A]
time = 0.40, size = 139, normalized size = 2.84

$$\left[\frac{\log\left(\frac{8\,abx^3+(b^2+4\,ac)x^4+8\,a^2x^2-4\,\sqrt{cx^5+bx^4+ax^3}\,(bx+2\,a)\sqrt{a}\,\sqrt{x}}{x^4}\right)}{2\,\sqrt{a}}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^5+bx^4+ax^3}\,(bx+2\,a)\sqrt{-a}\,\sqrt{x}}{2\,(acx^4+abx^3+a^2x^2)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a*b*x^3 + (b^2 + 4*a*c)*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^5 + b*x^4
+ a*x^3)*(b*x + 2*a)*sqrt(a)*sqrt(x))/x^4)/sqrt(a), sqrt(-a)*arctan(1/2*sqr
t(c*x^5 + b*x^4 + a*x^3)*(b*x + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^4 + a*b*x^3 +
a^2*x^2))/a]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**2+b*x+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.10, size = 58, normalized size = 1.18

$$\frac{2\left(\frac{\arctan\left(-\frac{\sqrt{c}\,x-\sqrt{cx^2+bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}\right)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^2+b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] 2*(arctan(-(sqrt(c)*x - sqrt(c*x^2 + b*x + a))/sqrt(-a))/sqrt(-a) - arctan(
sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^2+b\,x+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2),x)
[Out] int(x^(1/2)/(x^3*(a + b*x + c*x^2))^(1/2), x)
```

3.133 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1128, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Mathematica [**A**]
time = 0.00, size = 41, normalized size = 0.93

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a]
```

Maple [**A**]
time = 0.04, size = 39, normalized size = 0.89

method	result	size
default	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39
elliptic	$-\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{a}}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 0.36, size = 124, normalized size = 2.82

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)}{4\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(a) + 8*a^2)/x^4)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 +
 b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))/a]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**A**]
time = 4.60, size = 38, normalized size = 0.86

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a)
```

Mupad [B]
time = 2.23, size = 44, normalized size = 1.00

$$-\frac{\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}-\frac{\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] - log(1/x^2)/(2*a^(1/2)) - log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) +
b*x^2)/(2*a^(1/2))
```

3.134 $\int \frac{1}{\sqrt{x^2 (a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=49

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x*(b*x^2+2*a)/a^(1/2)/(c*x^6+b*x^4+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2021, 1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^2 + b*x^4 + c*x^6])]/Sqr
t[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^4+cx^6}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x(2a+bx^2)}{\sqrt{ax^2+bx^4+cx^6}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x(2a+bx^2)}{2\sqrt{a}\sqrt{ax^2+bx^4+cx^6}}\right)}{2\sqrt{a}}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 1.59

$$\frac{x\sqrt{a+bx^2+cx^4}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(a + b*x^2 + c*x^4)],x]
[Out] (x*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/
Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.06, size = 72, normalized size = 1.47

method	result	size
default	$-\frac{x\sqrt{c\,x^4+b\,x^2+a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x^2(c\,x^4+b\,x^2+a)}\,\sqrt{a}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2/(x^2*(c*x^4+b*x^2+a))^(1/2)*x*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [A]
time = 0.37, size = 135, normalized size = 2.76

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{a}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^6+bx^4+ax^2}(bx^2+2a)\sqrt{-a}}{2(acx^5+abx^3+a^2x)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^6 + b*x^4 +
 a*x^2)*(b*x^2 + 2*a)*sqrt(a))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c
*x^6 + b*x^4 + a*x^2)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^5 + a*b*x^3 + a^2*x))/a
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2(a+bx^2+cx^4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 5.01, size = 62, normalized size = 1.27

$$-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}}+\frac{\arctan\left(-\frac{\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + arctan(-(sqrt(c)*x^2 - sqrt(c*x
^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*sgn(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^2\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2),x)
[Out] int(1/(x^2*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.135 $\int \frac{1}{\sqrt{x}\sqrt{x(a+bx^2+cx^4)}}\,dx$

Optimal. Leaf size=51

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)*x^(1/2)/a^(1/2)/(c*x^5+b*x^3+a*x)^(1/2))/a^(1/
2)
```

Rubi [A]
time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2022, 1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[x]*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x + b*x^3 + c*x^5])]
/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,\sqrt{x\,(a+bx^2+cx^4)}}\,dx &= \int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^3+cx^5}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{\sqrt{x}\,(2a+bx^2)}{\sqrt{ax+bx^3+cx^5}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{x}\,(2a+bx^2)}{2\sqrt{a}\,\sqrt{ax+bx^3+cx^5}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 80, normalized size = 1.57

$$\frac{\sqrt{x}\,\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x\,(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(a + b*x^2 + c*x^4)]),x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.02, size = 72, normalized size = 1.41

method	result	size
default	$-\frac{\sqrt{x}\,\sqrt{c\,x^4+b\,x^2+a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x\,(c\,x^4+b\,x^2+a)}\,\sqrt{a}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x^4 + b*x^2 + a)*x)*sqrt(x)), x)

Fricas [A]
time = 0.34, size = 137, normalized size = 2.69

$$\left[\frac{\log\left(-\frac{(b^2+4ac)x^5+8abx^3+8a^2x-4\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{a}\sqrt{x}}{x^5}\right)}{4\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{cx^5+bx^3+ax}\,(bx^2+2a)\sqrt{-a}\sqrt{x}}{2(acx^5+abx^3+a^2x)}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")

[Out] [1/4*log(-((b^2 + 4*a*c)*x^5 + 8*a*b*x^3 + 8*a^2*x - 4*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^5)/sqrt(a), 1/2*sqrt(-a)*arctan(1/2*sqrt(c*x^5 + b*x^3 + a*x)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^5 + a*b*x^3 + a^2*x))/a]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(x*(c*x**4+b*x**2+a))**(1/2),x)

[Out] Timed out

Giac [A]
time = 4.03, size = 56, normalized size = 1.10

$$\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(x*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")

[Out] arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arctan(sqrt(a)/sqrt(-a))/sqrt(-a)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,(c\,x^4+b\,x^2+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)),x)
[Out] int(1/(x^(1/2)*(x*(a + b*x^2 + c*x^4))^(1/2)), x)
```

3.136 $\int \frac{\sqrt{x}}{\sqrt{x^3 (a+bx^2+cx^4)}} \, dx$

Optimal. Leaf size=53

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*arctanh(1/2*x^(3/2)*(b*x^2+2*a)/a^(1/2)/(c*x^7+b*x^5+a*x^3)^(1/2))/a^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2022, 1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x^(3/2)*(2*a + b*x^2))/(2*Sqrt[a]*Sqrt[a*x^3 + b*x^5 + c*x^7]
)]/Sqrt[a]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] &&  !General
```

izedTrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{x^3(a+bx^2+cx^4)}}\,dx &= \int \frac{\sqrt{x}}{\sqrt{ax^3+bx^5+cx^7}}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{3/2}(2a+bx^2)}{\sqrt{ax^3+bx^5+cx^7}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{x^{3/2}(2a+bx^2)}{2\sqrt{a}\sqrt{ax^3+bx^5+cx^7}}\right)}{2\sqrt{a}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 82, normalized size = 1.55

$$\frac{x^{3/2}\sqrt{a+bx^2+cx^4}\,\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x^3(a+bx^2+cx^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[x^3*(a + b*x^2 + c*x^4)],x]
```

```
[Out] (x^(3/2)*Sqrt[a + b*x^2 + c*x^4]*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*
x^4])/Sqrt[a]])/(Sqrt[a]*Sqrt[x^3*(a + b*x^2 + c*x^4)])
```

Maple [A]
time = 0.02, size = 74, normalized size = 1.40

method	result	size
default	$-\frac{x^{\frac{3}{2}}\sqrt{c\,x^4+b\,x^2+a}\,\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2\sqrt{x^3\left(c\,x^4+b\,x^2+a\right)}\,\sqrt{a}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(x^3*(c*x^4+b*x^2+a))^(1/2)*x^(3/2)*(c*x^4+b*x^2+a)^(1/2)/a^(1/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [A]
time = 0.35, size = 145, normalized size = 2.74

$$\left[\frac{\log\left(-\frac{(b^2+4\,ac)x^6+8\,abx^4+8\,a^2x^2-4\,\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{a}\,\sqrt{x}}{x^6}\right)}{4\,\sqrt{a}},\ \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{cx^7+bx^5+ax^3}\,(bx^2+2\,a)\sqrt{-a}\,\sqrt{x}}{2\,(acx^6+abx^4+a^2x^2)}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-((b^2 + 4*a*c)*x^6 + 8*a*b*x^4 + 8*a^2*x^2 - 4*sqrt(c*x^7 + b*x^5
 + a*x^3)*(b*x^2 + 2*a)*sqrt(a)*sqrt(x))/x^6)/sqrt(a), 1/2*sqrt(-a)*arctan(
1/2*sqrt(c*x^7 + b*x^5 + a*x^3)*(b*x^2 + 2*a)*sqrt(-a)*sqrt(x)/(a*c*x^6 + a
*b*x^4 + a^2*x^2))/a]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**3*(c*x**4+b*x**2+a))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.41, size = 61, normalized size = 1.15

$$\frac{\frac{\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^3*(c*x^4+b*x^2+a))^(1/2),x, algorithm="giac")
```

```
[Out] (arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/sqrt(-a) - arcta
n(sqrt(a)/sqrt(-a))/sqrt(-a))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{\sqrt{x^3\,\left(c\,x^4+b\,x^2+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2),x)
[Out] int(x^(1/2)/(x^3*(a + b*x^2 + c*x^4))^(1/2), x)
```

3.137 $\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/2*(-x^2+2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1128, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(2-x^2)}{2\sqrt{x^4-3x^2+3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[3]*(2 - x^2))/(2*Sqrt[3 - 3*x^2 + x^4])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1128

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{3-3x^2+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{3-3x+x^2}}\,dx, x, x^2\right)$$
$$= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{3(2-x^2)}{\sqrt{3-3x^2+x^4}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(2-x^2)}{2\sqrt{3-3x^2+x^4}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 33, normalized size = 0.82

$$\frac{\tanh^{-1}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[3 - 3*x^2 + x^4]),x]
[Out] ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]]/Sqrt[3]
```

Maple [A]
time = 0.19, size = 31, normalized size = 0.78

method	result	size
default	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^2+6\right)\sqrt{3}}{6\sqrt{x^4-3x^2+3}}\right)}{6}$	31
elliptic	$-\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\left(-3x^2+6\right)\sqrt{3}}{6\sqrt{x^4-3x^2+3}}\right)}{6}$	31
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^2-3\right)x^2+2\sqrt{x^4-3x^2+3}+2\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{6}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4-3*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/6*3^(1/2)*arctanh(1/6*(-3*x^2+6)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [A]
time = 0.46, size = 20, normalized size = 0.50

$$-\frac{1}{6}\sqrt{3}\,\operatorname{arsinh}\left(-\sqrt{3}+\frac{2\sqrt{3}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsinh(-sqrt(3) + 2*sqrt(3)/x^2)
```

Fricas [A]
time = 0.34, size = 47, normalized size = 1.18

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^2+2\sqrt{3}(x^2-2)+2\sqrt{x^4-3x^2+3}\left(\sqrt{3}+2\right)-6}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^2 + 2*sqrt(3)*(x^2 - 2) + 2*sqrt(x^4 - 3*x^2 + 3)*(sq
rt(3) + 2) - 6)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x\sqrt{x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**4 - 3*x**2 + 3)), x)
```

Giac [A]
time = 4.90, size = 55, normalized size = 1.38

$$\frac{1}{6}\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\frac{1}{6}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4-3*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - 1/6*sqrt(3)*log(-x
^2 + sqrt(3) + sqrt(x^4 - 3*x^2 + 3))
```

Mupad [B]
time = 0.43, size = 33, normalized size = 0.82

$$-\frac{\sqrt{3}\left(\ln\left(x^2-\frac{2\sqrt{3}\sqrt{x^4-3x^2+3}}{3}-2\right)+\ln\left(\frac{1}{x^2}\right)\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x^4 - 3*x^2 + 3)^(1/2)),x)
[Out] -(3^(1/2)*(log(x^2 - (2*3^(1/2)*(x^4 - 3*x^2 + 3)^(1/2))/3 - 2) + log(1/x^2
)))/6
```

3.138 $\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx$

Optimal. Leaf size=45

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

```
[Out] -1/6*arctanh(1/6*x*(-3*x^2+6)*3^(1/2)/(x^6-3*x^4+3*x^2)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2021, 1918, 212}

$$-\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{x^6-3x^4+3x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
```

```
[Out] -1/2*ArcTanh[(x*(6 - 3*x^2))/(2*Sqrt[3]*Sqrt[3*x^2 - 3*x^4 + x^6])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1918

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(r_.)], x_Symbol] :
> Dist[-2/(n - 2), Subst[Int[1/(4*a - x^2), x], x, x*((2*a + b*x^(n - 2))/S
qrt[a*x^2 + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, n, r}, x] && EqQ[r, 2*n
 - 2] && PosQ[n - 2] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2021

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedTrinomialQ[u, x] &&  !GeneralizedTrinomialMatchQ[u, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{x^2(3-3x^2+x^4)}}\,dx = \int \frac{1}{\sqrt{3x^2-3x^4+x^6}}\,dx$$
$$= -\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{x(6-3x^2)}{\sqrt{3x^2-3x^4+x^6}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x(6-3x^2)}{2\sqrt{3}\sqrt{3x^2-3x^4+x^6}}\right)}{2\sqrt{3}}$$

Mathematica [A]
time = 0.00, size = 66, normalized size = 1.47

$$\frac{x\sqrt{3-3x^2+x^4}\tanh^{-1}\left(\frac{x^2-\sqrt{3-3x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x^2(3-3x^2+x^4)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(3 - 3*x^2 + x^4)],x]
[Out] (x*Sqrt[3 - 3*x^2 + x^4]*ArcTanh[(x^2 - Sqrt[3 - 3*x^2 + x^4])/Sqrt[3]])/(S
qrt[3]*Sqrt[x^2*(3 - 3*x^2 + x^4)])
```

Maple [A]
time = 0.18, size = 58, normalized size = 1.29

method	result	size
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2-3\right)x^3+2\,\mathrm{RootOf}\left(\_Z^2-3\right)x+2\sqrt{x^6-3x^4+3x^2}}{x^3}\right)}{6}$	53
default	$\frac{\sqrt{x^4-3x^2+3}\,x\sqrt{3}\,\mathrm{arctanh}\left(\frac{(x^2-2)\sqrt{3}}{2\sqrt{x^4-3x^2+3}}\right)}{6\sqrt{x^2(x^4-3x^2+3)}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4-3*x^2+3))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6/(x^2*(x^4-3*x^2+3))^(1/2)*(x^4-3*x^2+3)^(1/2)*x*3^(1/2)*arctanh(1/2*(x^
2-2)*3^(1/2)/(x^4-3*x^2+3)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((x^4 - 3*x^2 + 3)*x^2), x)
```

Fricas [A]
time = 0.36, size = 55, normalized size = 1.22

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{3x^3+2\sqrt{3}(x^3-2x)+2\sqrt{x^6-3x^4+3x^2}\left(\sqrt{3}+2\right)-6x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(3*x^3 + 2*sqrt(3)*(x^3 - 2*x) + 2*sqrt(x^6 - 3*x^4 + 3*x^
2)*(sqrt(3) + 2) - 6*x)/x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{x^2(x^4-3x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(x**4-3*x**2+3))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(x**4 - 3*x**2 + 3)), x)
```

Giac [A]
time = 4.12, size = 60, normalized size = 1.33

$$\frac{\sqrt{3}\log\left(x^2+\sqrt{3}-\sqrt{x^4-3x^2+3}\right)-\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4-3x^2+3}\right)}{6\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(x^4-3*x^2+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3)*log(x^2 + sqrt(3) - sqrt(x^4 - 3*x^2 + 3)) - sqrt(3)*log(-x^2
+ sqrt(3) + sqrt(x^4 - 3*x^2 + 3)))/sgn(x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\sqrt{x^2\,\left(x^4-3\,x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2),x)
```

```
[Out] int(1/(x^2*(x^4 - 3*x^2 + 3))^(1/2), x)
```

3.139 $\int \frac{1}{\sqrt{x}\,\sqrt{x(3-3x+x^2)}}\,dx$

Optimal. Leaf size=43

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}$$

```
[Out] -1/3*arctanh(1/2*(2-x)*3^(1/2)*x^(1/2)/(x^3-3*x^2+3*x)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2022, 1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{3}\,(2-x)\sqrt{x}}{2\sqrt{x^3-3x^2+3x}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] -(ArcTanh[(Sqrt[3]*(2 - x)*Sqrt[x])/(2*Sqrt[3*x - 3*x^2 + x^3])]/Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rule 2022

```
Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{d, m, p}, x] && GeneralizedTrinomialQ[u, x] && !General
izedTrinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{x(3-3x+x^2)}}\,dx &= \int \frac{1}{\sqrt{x}\sqrt{3x-3x^2+x^3}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx, x, \frac{(6-3x)\sqrt{x}}{\sqrt{3x-3x^2+x^3}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3}(2-x)\sqrt{x}}{2\sqrt{3x-3x^2+x^3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 61, normalized size = 1.42

$$\frac{2\sqrt{x}\sqrt{3-3x+x^2}\tanh^{-1}\left(\frac{x-\sqrt{3-3x+x^2}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{x(3-3x+x^2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[x*(3 - 3*x + x^2)]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[3 - 3*x + x^2]*ArcTanh[(x - Sqrt[3 - 3*x + x^2])/Sqrt[3]])/
(Sqrt[3]*Sqrt[x*(3 - 3*x + x^2)])
```

Maple [A]
time = 0.02, size = 50, normalized size = 1.16

method	result	size
default	$\frac{\sqrt{x}\sqrt{x^2-3x+3}\sqrt{3}\,\text{arctanh}\left(\frac{(x-2)\sqrt{3}}{2\sqrt{x^2-3x+3}}\right)}{3\sqrt{x(x^2-3x+3)}}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^(1/2)/(x*(x^2-3*x+3))^(1/2)*(x^2-3*x+3)^(1/2)*3^(1/2)*arctanh(1/2*(x-
2)*3^(1/2)/(x^2-3*x+3)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt((x^2 - 3*x + 3)*x)*sqrt(x)), x)
```

Fricas [**A**]
time = 0.38, size = 49, normalized size = 1.14

$$\frac{1}{6}\sqrt{3}\log\left(\frac{7x^3+4\sqrt{3}\sqrt{x^3-3x^2+3x}\,(x-2)\sqrt{x}-24x^2+24x}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((7*x^3 + 4*sqrt(3)*sqrt(x^3 - 3*x^2 + 3*x)*(x - 2)*sqrt(x)
- 24*x^2 + 24*x)/x^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(x*(x**2-3*x+3))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.15, size = 47, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(x+\sqrt{3}-\sqrt{x^2-3x+3}\right)-\frac{1}{3}\sqrt{3}\log\left(-x+\sqrt{3}+\sqrt{x^2-3x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(x*(x^2-3*x+3))^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log(x + sqrt(3) - sqrt(x^2 - 3*x + 3)) - 1/3*sqrt(3)*log(-x + s
qrt(3) + sqrt(x^2 - 3*x + 3))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x}\,\sqrt{x\,\left(x^2-3x+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)),x)
```

```
[Out] int(1/(x^(1/2)*(x*(x^2 - 3*x + 3))^(1/2)), x)
```

3.140 $\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$

Optimal. Leaf size=70

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\,\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}\,(n-q)}$$

```
[Out] -arctanh(1/2*x^(1/2*q)*(2*a+b*x^(n-q))/a^(1/2)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1
/2))/(n-q)/a^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1927, 212}

$$-\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\,\sqrt{ax^q+bx^n+cx^{2n-q}}}\right)}{\sqrt{a}\,(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
```

```
[Out] -(ArcTanh[(x^(q/2)*(2*a + b*x^(n - q)))/(2*Sqrt[a]*Sqrt[b*x^n + c*x^(2*n -
q) + a*x^q])]/(Sqrt[a]*(n - q)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1927

```
Int[(x_)^(m_.)/Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)]
, x_Symbol] :> Dist[-2/(n - q), Subst[Int[1/(4*a - x^2), x], x, x^(m + 1)*(
(2*a + b*x^(n - q))/Sqrt[a*x^q + b*x^n + c*x^r])], x] /; FreeQ[{a, b, c, m,
 n, q, r}, x] && EqQ[r, 2*n - q] && PosQ[n - q] && NeQ[b^2 - 4*a*c, 0] && E
qQ[m, q/2 - 1]
```

Rubi steps

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx = -\frac{2\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{x^{q/2}(2a+bx^{n-q})}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{n-q}$$
$$= -\frac{\tanh^{-1}\left(\frac{x^{q/2}(2a+bx^{n-q})}{2\sqrt{a}\sqrt{bx^n+cx^{2n-q}+ax^q}}\right)}{\sqrt{a}\,(n-q)}$$

Mathematica [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{bx^n+cx^{2n-q}+ax^q}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q],x]
[Out] Integrate[x^(-1 + q/2)/Sqrt[b*x^n + c*x^(2*n - q) + a*x^q], x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{b\,x^n+c\,x^{2n-q}+a\,x^q}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
[Out] int(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="maxima
")
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+1/2*q)/(b*x**n+c*x**(2*n-q)+a*x**q)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8011 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+1/2*q)/(b*x^n+c*x^(2*n-q)+a*x^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(1/2*q - 1)/sqrt(c*x^(2*n - q) + b*x^n + a*x^q), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{\frac{q}{2}-1}}{\sqrt{b\,x^n+a\,x^q+c\,x^{2\,n-q}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2),x)
```

```
[Out] int(x^(q/2 - 1)/(b*x^n + a*x^q + c*x^(2*n - q))^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```